GED® Math Test Tutor (1st ed.)

Test-Alignment Supplement

This supplement, prepared under the supervision of the GED Testing Service®, picks up where REA's *GED® Math Test Tutor* leaves off to provide complete alignment to GED® test objectives. Any revision or addition is strictly designed to give the reader complete, accurate coverage of all test objectives. The document presents a series of inserts, including text, examples, charts, and figures, that the testing service deems necessary to achieve 100% alignment with test objectives. These inserts, each of which is labeled by page number (and, as necessary, by other location references) represent improvements to the first edition.

We recommend that readers print this supplement and keep it handy as they use the book.

Page 27: Insert the following after the third paragraph.

USE LOGIC! Calculations are secondary in importance to finding a logical solution pathway that focuses on problem solving, modeling, thinking, and reasoning. The calculations, if done correctly, will always give you the correct solution, but if you don't know what the problem is asking for, or if you cannot model it, or if you cannot reason through a solution pathway, how will you know what calculations to make? This book will help you with the calculations and give you ways to improve your logical skills—skills that you were "born knowing."

Sometimes it is possible to get mathematically correct results even if you use incorrect lines of reasoning. This can be because two "wrongs" made a "right" or due to several other scenarios. And sometimes you can use what seems to be a correct line of reasoning but come up with an incorrect answer. Be sure to check whether your answer makes sense.

<<NOTE>>An essential last step for each problem should be to always check whether the answer makes sense. <<END OF NOTE>>

The following popular riddle shows the importance of checking your answers. If you bought a cupcake and a cookie at a bake sale for $1.10, and the cupcake costs a dollar more than the cookie, how much did the cookie cost? A surprising number of people answer this right away as 10 cents. But if you checked that answer, you would see right away that it was wrong because then that total is $1.20, not $1.10. (The correct answer is 5 cents.)

When starting a problem, ask yourself first what is the problem asking? What information is it giving? Is any of this information unnecessary or missing? As an example of a problem with unnecessary information (which should just be ignored), suppose Tim has twelve coins. He tells you that five of them are dimes and seven are worth five cents or more. He will give you all of the coins if you can tell him how many quarters he has. The problem is asking how many quarters there are. You know that five are dimes and seven are nickels, dimes, and quarters (and perhaps half dollars). You can figure out that two of them must be coins that are nickels, quarters, or half dollars. But you don't have enough information to say how many of each, so you can't take Tim's challenge. You have to have another piece of information, such as that he as two nickels, in which case you know there aren't any quarters. If Tim told you the total value of the coins, you could figure out how many are dimes, even if it is by trial and error. But without the last piece of information, this is an impossible question to answer correctly, except by a guess. Can you see that the answer must be 0, 1, or 2?

Suppose Tim asked instead how many pennies he had. Could you answer that? Of course you can because you know that seven of the coins are nickels or greater, so there must be (12 – 7 =) five pennies.

Let's try another example: Three people, Adele, Benjamin, and Contessa, are playing a card game, which ends when the totals of all points equals 50. Can you tell who is the winner? No, of course not—you don't have enough information. What more do you have to know? If you know that Contessa has 15 points, will that help? No, not yet, because all that tells you is that Adele and Benjamin have a total of (50 – 15 =) 35 points, but you don't know how that is distributed. Will knowing just two scores, but not all three, help? Yes, because you can then add them and subtract the total from 50 to get the third score, and then you can tell who the winner is. And, of course, if you are given all three scores, you can tell right away who won.

In fact, if you are given all three scores and the total to end the game, what would be the extra, unnecessary information? To solve the problem, if you knew two scores and the fact that 50 ends the game, you can figure out the third score to compare to the other two. Conversely, if you knew all three scores, there is hardly any challenge in finding which is higher, and you don't have to know that a total of 50 ends the game.

A final example has to do with horse racing. The Kentucky Derby is a 2 kilometer track. How fast on average was the winning horse going? You cannot tell if you only know the length of the course. What other information do you need? You need the time it took for the horse to finish the race. Without that, you cannot tell how fast the horse was running.

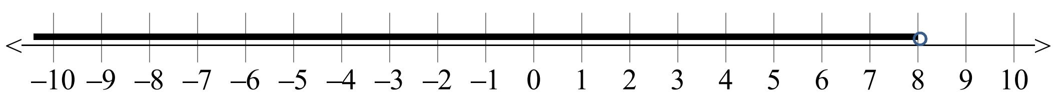
The point is that rather than jumping in to solve a problem or choose a multiple-choice answer, take a moment to see what is given and what is asked for and what you have to figure out yourself. Then, when you have answered the problem, take another moment to make sure the answer makes sense.

Page 29: Insert the following under the heading "The Number Line."

A brief history of how numbers evolved is interesting. About three millennia ago, the only numbers that were used were the positive whole numbers, 1, 2, 3, . . . These were **counting numbers**, also called **natural numbers**. There was no number to indicate "nothing," such as if a farmer had three cows and sold them all, how many did he now have? There was a concept of zero, but no symbol for it. The symbol "0" came about as a placeholder, so "3" no longer meant 3 or 30 or 300. These numbers are called **whole numbers**, 0, 1, 2, 3, . . . (natural numbers plus 0). Eventually, the idea of negatives came to be; **integers** are the numbers shown on the number line below, . . . –3, –2, –1, 0, 1, 2, 3, . . . All of the numbers (including fractions, decimals, etc.) between any two integers plus all of the integers together are called **real** numbers.

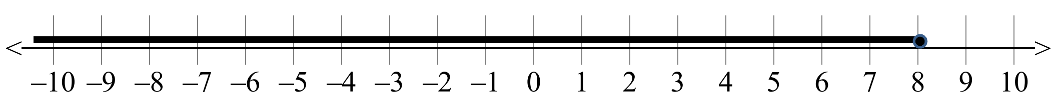
Page 29: Insert the following at the bottom of the page.

Combining an equal sign with an inequality sign yields the symbols ≥ and ≤, which include the number. So all the whole numbers less than 8 (< 8) include . . . 3, 4, 5, 6, and 7; whereas all the whole numbers less than or equal to 8 include . . . 3, 4, 5, 6, 7, 8. When graphed on the number line,



<8

Notice that the circle at 8 is an open circle for <8.



≤8

Notice that the circle at 8 is filled in for ≤8.

Page 30: Insert the following after Answer 2.1.

<<EXAMPLE>> Example

The number –29 belongs to which number sets? Choose all that apply.

A. Natural numbers

B. Integers

C. Whole numbers

D. Real numbers

<<ANSWER>>Answer 2.2.

B and D, integers and real numbers

Page 42: Insert the following before the heading "Multiplying Numbers Greater Than 10."

<<HEAD>>Counterexamples

<<TEXT>> This is a good time to introduce the concept of counterexamples. A **counterexample** is an example that disproves a statement. A popular one involves the statement that all primes are odd. We call such a statement a **conjecture**—we think a statement is correct, and we are willing to accept it if we can't come up with any counterexamples. This concept comes up a lot in geometry, but you probably can think of conjectures that you believed because you couldn't come up with a counterexample. For example, a friend tells you a lie (conjecture) and you can't think on the spot of something to show it isn't true (counterexample).

Back to math: Suppose Sam, a high school student, says he knows several prime numbers: 7, 11, 13, 17, 19, and 23, and he then says, "All prime numbers must be odd numbers." That's a conjecture, and it looks like it might be true. But then his little sister, who is in the fourth grade says, "What about 2?" Sam is busted by his little sister's counterexample, since 2 is considered to be prime (it can be evenly divided only by itself and 1).

Now consider this conjecture: "The opposite of a number is always negative." On first looking at this sentence, it seems plausible, but it isn't. Can you think of a counterexample to disprove it? (Hint: What's the opposite of a negative number?)

Page 93: Insert the following as the first paragraph under the heading "Simplifying Roots."

Note that $\sqrt{2}$ here cannot be broken down any further. The solution is written in terms of $\sqrt{2}$, which is called an **irrational** number since it cannot be written as a fraction, decimal, or percentage.

Page 96: Insert the following before the heading "Scientific Notation."

For example, if you are asked to evaluate $6 \times 4 + 14 \div 2$ and you don't follow PEMDAS and instead jump right in, going left to right, you will get $24 + 14 \div 2 = 38 \div 2 = 19$. In actuality, your solution pathway should lead you to using PEMDAS, and this simple expression is equal to $30 + 2 = 32$. (Remember that PEMDAS says to do multiplication and division left to right before addition and subtraction left to right.)

<<EXAMPLE>>Example

A farrier (a person who takes care of shoeing horses) is asked to make horseshoes for two groups of horses. The first group of 6 horses all need shoes, but only half of the other group of 8 horses need shoes. How many horseshoes does the farrier have to make?

<<ANSWER>>Answer

40. Do you agree that the equation should be $(6 \times 4) + [(8 \div 2) \times 4]$? Using logic before calculation, the first group needs $6 \times 4 = 24$ shoes total. The second group needs only $4 \times 4 = 16$ shoes because half (4 of the 8) don't need new shoes. So altogether the farrier needs to make

$$24 + 16 = 40 \text{ shoes.}$$

However, jumping right in (without using PEMDAS) might give an answer of

$$6 \times 4 + 8 \div 2 \times 4 = 24 + 8 \div 2 \times 4 = 32 \div 2 \times 4 = 16 \times 4 = 64 \text{ shoes.}$$

Now, let's use the essential last step: *does this answer make sense?* How many horses, even including those that don't need shoes, are there? $6 + 8 = 14$. This second (and incorrect) answer of 64 gives an answer that involves 16 horses ($64 \div 4$), and we haven't yet eliminated the half that don't need shoes. Clearly, something is wrong.

A more direct way to look at this problem is just to total how many horses need shoes and multiply that number by 4, the number of feet each horse has. The number of horses is $6 + 4$ (half of the 8), which equals 10. Then the farrier needs to make $10 \times 4 = 40$ shoes.

<<EXAMPLE>>Example

What is the error in the following calculation to find the value of $8 \div 2(1 + 3)$?

$$8 \div 2(1 + 3) =$$

$$8 \div 2(4) =$$

$$8 \div 8 = 1$$

<<ANSWER>>Answer

PEMDAS tells us to first do the calculations in parentheses (P). Since there are no exponents (E), we go on to multiplication and division (MD) *in order left to right*. Now we see that the correct answer is 16:

$$8 \div 2(1 + 3) =$$

$$8 \div 2(4) =$$

$$8 \div 2 \times 4 =$$

$4 \times 4 = 16$ <<END OF ANSWER>>

<<TEXT>>This example shows two important rules in PEMDAS:

<<NUMBER LIST>>1. Once the value in the parentheses is determined, if there are no + or – signs before the parentheses, they indicate multiplication.

2. MD means to multiply and divide as they are presented, left to right. (The same is true for AS: add and subtract left to right.) <<END NUMBER LIST>>

So the error in the calculation with the answer of 1 is that the multiplication was done before the division when division is indicated prior to multiplication when reading left to right.

Page 103: Insert the following before the heading "Setting up Equations."

A key concept in algebra (and even more so in geometry) involves a caution: Don't jump right in without thinking the problem through! Too often, when presented with a problem, we tend to try to solve it right away, when time and frustration would be saved if we analyzed it first.

An example would be pulling the chain on a new ceiling fan and finding that it doesn't turn (this is a true story). So you pull the other chain, like you did for the previous fan—it still doesn't turn (but the light goes on, so you know there is power). You push the blades—no luck. Finally, you call a repairman. Luckily, before the repairman shows up and you are out a chunk of money, you realize this new fan is operated by a remote—and the remote has the fan turned off. If a few minutes were taken at the outset to analyze the problem instead of trying all of the things that didn't work, a lot of time and frustration would have been saved.

What this has to do with algebra and geometry is that planning a solution pathway or outlining a line of reasoning (especially helpful in geometry, see Chapter 7) goes a long way toward a quicker and easier solution. First determine what it is you are asked to find. Then, it may help to list what information you are given and what each piece of information tells you. Also, recognize any missing information that is needed to arrive at a solution, and see whether you can find the missing information from what you are given. This will help you to select the pathway you must use to solve the problem. Although it seems like a lot of work, a solution pathway actually will cut down on your work. A lot of this can be done mentally before you even pick up a pencil or go to your calculator.

Page 107: Replace Example 5.5 and Answer 5.5 with the following.

<<EXAMPLE>>Example 5.5

Twelve students are in the art class. The art teacher's budget for a particular project is no more than \$60. How much can she spend per student on this art project?

A. < \$5

B. ≤ \$5

C. \$5

D. Cannot tell from information given.

<<ANSWER>>Answer 5.5 (B) ≤ \$5. This is an inequality because it contains the phrase "no more than." Translating from words to algebra,

How much per student = budget

$x \quad (12) \quad \leq \$60$

$12x \leq \$60$, or $x \leq \$5$

Page 108: Insert the following at top of the page (before Example 5.6).

<<EXAMPLE>>Example

Find the errors in the following incorrect solutions to the same problem:

A. $7x + 4 = 5x - 3(2)^2$

$7x + 4 = 5x - (6)^2$

$7x + 4 = 5x - 36$

$2x + 4 = -36$

$2x = -40$

$x = -20$

B. $7x + 4 = 5x - 3(2)^2$

$7x + 4 = 5x - 3(4)$

$7x + 4 = 5x - 12$

$2x + 4 = -12$

$2x = -8$

$x = -4$

C. $7x + 4 = 5x - 3(2)^2$

$7x + 4 = 5x - 3(4)$

$7x + 4 = 5x - 12$

$12x = -16$

$x = -\frac{16}{12} = -1\frac{4}{12} = -1\frac{1}{3}$

<<ANSWER>>Answer

A. In the second line, the quantity 3(2) is squared instead of just the (2).

B. In the fifth line, 4 should have been subtracted from both sides of the equation, but it is added to the right side.

C. In the fourth line, $5x$ should have been subtracted from both sides of the equation, but it is added to the left side.

Page 110: Replace Example 5.10 with the following.

a. Write in simplest form: $4x + 5xy - 6y + 6xy - 2x - xy$.

b. Evaluate the expression in part (a) if $x = 2$ and $y = -3$.

Page 111: Insert the following after the first paragraph on the page.

Simplifying algebraic expressions is similar to simplifying numerical fractions (see Chapter 3). The idea is to factor the expressions and cancel like factors from the numerator and denominator. Factoring the expressions usually means finding a common factor in each term of the expression. We can factor a simple expression, such as $6y + 48$ by recognizing a common factor of 2: $6y + 48 = 2(3y + 24)$, or a common factor of 3: $6y + 48 = 3(2y + 16)$, but each of those cases leaves a factor that can be factored further. The best factoring involves the greatest common factor, which in this case is 6:

$$6y + 48 = 6(y + 8)$$

Another example of simplifying an algebraic expression by using factoring is shown below.

$$\frac{9x+3}{12x+4} = \frac{3(3x+1)}{4(3x+1)} = \frac{3}{4},$$

by canceling out the common factor $3x + 1$.

<<NOTE>>

Note: All the rules for operations on algebraic expressions (addition, subtraction, multiplication, and division), as well as factoring and using PEMDAS, are the same as those for numbers.

The rules don’t change!

<<END OF NOTE>>

Page 117: Insert the following before the last paragraph on the page.

If you have two points, you can find the equation of a line that goes through those points, by using these two formulas. First, find the slope of the line that would go through the two points by using the slope formula, than proceed to the point-slope formula to get the equation of the line. For example, for the line that goes through the points (5, 1) and (1, –3), first find the slope. Designate (5, 2) as (x_1, y_1) and (1, –3) as (x_2, y_2). Then the slope between the points (or the slope of the line through them) is

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{(-3) - 1}{1 - 5} = \frac{-4}{-4} = 1$$

Now use the point-slope formula, but this time let (x_2, y_2) be just (x, y), representing any point on the line. Either point can be used for (x_1, y_1); let's use (5, 1). Then, substituting into $(y_2 - y_1) = m(x_2 - x_1)$, we get

$$(y - 1) = 1(x - 5)$$

$$(y - 1) = (x - 5)$$

$$y = x - 4$$

As a check, let's see whether the other point (1, –3) is a point on this line:

$$-3 = 1 - 4$$

It checks, so the line through the two points (5, 1) and (1, –3) is $y = x - 4$.

Page 136: Insert the following at the bottom of the page, after Question 15.

<<EXERCISE>>

16. Which of the following is the equation of a line with intercepts at (0, 3) and (6, 0)?

A. $x + 2y = 3$

B. $x + 2y = 6$

C. $x - 2y = 6$

D. none of the above

Page 139: Insert the following after Answer 15.

<<EXERCISE ANSWER>>

Answer 16. (B) $x + 2y = 6$. The y-intercept is $b = 3$ because (0, 3) is a point on the graph. The slope is given by $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0-3}{6-0} = \frac{-3}{6} = -\frac{1}{2}$. Therefore, the equation in slope-intercept form is $y = mx + b = y = -\frac{1}{2}x + 3$. Putting this in point-slope form, this is

$$y = -\frac{1}{2}x + 3$$

$$2y = -x + 6$$

$$x + 2y = 6$$

Page 145: Insert the following to replace the last sentence on the page.

Then we end up with at most two roots (solutions) for the quadratic equation. These roots can both be rational (can be written as a fraction) or irrational (cannot be written as a fraction, for example $\sqrt{2}$ or $\sqrt{40}$).

Page 146: Replace the last sentence in the third paragraph of Item No. 1 with the following.

The GED® test includes only **real numbers**, that is, numbers that are either rational (fractions, decimals, or percentages) or irrational.

Page 147: Replace the second full paragraph with the following.

The quadratic formula involves a square root. The value under the radical is called the **discriminant**. The $\pm$ sign in the equation gives two different roots unless $b^2 = 4ac$, in which case there two identical roots, considered as one root (sometimes called a double root). If $b^2 - 4ac$ is negative, the roots are imaginary. So quadratic equations always have either 2 real roots, 1 real root, or no real roots.

On the GED® test, the discriminant will usually be a common square, such as 16 or 25, but it can also be 7. Remember that you don't have to know how to find a square root for the GED® test (other than on the calculator), but you should be able to recognize common squares and square roots (see Chapter 4).

Page 149: Insert the following as the new first paragraph in Answer 6.6.

5 is a double root. If you recognize that the two numbers whose product is 25 and sum is –10 are –5 and –5, the factors are $(x - 5)(x - 5) = 0$, and two roots are 5. Otherwise, the trusty quadratic formula works as follows.

Page 149: Insert the following after Answer 6.6.

<<EXAMPLE>>Example

The sum of the squares of two numbers equals 13. The difference of the squares of the same two numbers equals 5. What is the value of the difference between the fourth powers of the two numbers?

<<ANSWER>>Answer

Let the two numbers be x and y. The first sentence says $x^2 + y^2 = 12$, and the second sentence says $x^2 - y^2 = 8$. The problem is to find the value of $x^4 - y^4$, which is also the difference of two squares, as follows:

$$x^4 - y^4 = (x^2 + y^2)(x^2 - y^2) = 12 \times 8 = 96.$$

Page 152: Insert the following directly above Example 6.9.

Think of the graph of a straight line—every x value has only one $f(x)$ (or y) value, and it is a function. What about the readout of an electrocardiogram? Again, as the x value (input) changes, there is only one point on the output, so it also is a function. Now, what about a circle? Each x value seems to have two two y values, so a circle is not a function. In fact, there is a graphic "test" to see whether a relationship is a function. It is called the vertical-line test, which states that if a vertical line anywhere on the graph hits the graph in more than one place (such as for a circle), the graph fails the vertical-line test and is therefore not a function.

Page 153: Insert the following directly above the heading "Describing Graphs."

Now suppose that we don't have a graph drawn but we know several points of the relationship. Let's look at the points (1, 5), (2, 6), and (2, 7). Right away we know that this does not represent a function because there are two *y* values for $x = 2$. It's okay if there is more than one *x* for the same *y* value, but not the other way around.

Because every *x* value in a function always gives only one *y* value, it is reliable, and that is a very important property. To appreciate this attribute, suppose each time you called your friend, you weren't sure whether you would get her phone or someone else's. Telephone numbers and the phones connected to each one form a function, which is a good thing.

Page 153: Replace the first paragraph under the heading "Describing Graphs" with the following.

We already talked about parabolas, the $\bigcup$-shaped curves of quadratic equations. A parabola is a function because for every *x* there is one and only one *y*. Another function is an **exponential** function, which is typically a slow growth followed by an accelerated growth, or a slow decay (the opposite of growth) followed by an accelerated decay. A typical equation for an exponential function is $f(x) = Ab^x$, where the *x* is the exponent of *b* and *A* can be any constant. If $b > 1$, the equation and graph show exponential growth, as shown in the first graph below, where $f(x) = 2^x$. If $b < 1$ (including all negative numbers), the function and graph show exponential decay, as shown in the second graph below, where $f(x) = 64\left(\frac{1}{2}\right)^x$.

Page 181: Insert the following directly above the heading "Quadrilaterals."

<<TEXT>>If you know the perimeter or area of a triangle, can you find the sides? Yes and no. Yes if you have a little more information, such as the length of the other two sides or the height of the triangle. No, if you don't have that information.

You can see how this works if you consider two triangles, each with a perimeter of 16. One can have sides of length 5, 2, and 9, and another can have sides of 3, 4, and 9. Even if you knew one side (like 9 here), you still don't know the other two sides. Can you think of another triangle with perimeter 16 and one side 9? To find a side of a triangle if you know the perimeter and *two* side lengths, call the missing side *x*, and use the formula $p = a + b + c$, and fill in the values for *p*, *a*, and *b* and solve the equation for *x*.

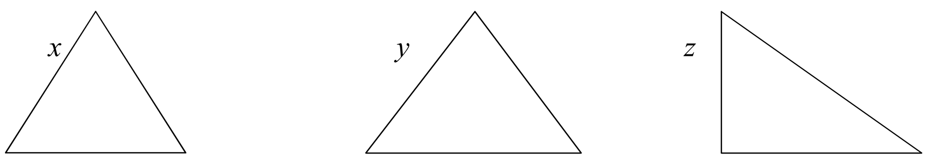
An exception to this rule for finding a side of a triangle if you know the perimeter is the equilateral triangle. For example, if you know the triangle is equilateral and the perimeter is 21, you don't have to be given the other two sides. You can immediately show that each side is 7 because all of the sides are equal. The perimeter formula is $p = a + a + a = 3a$, or $21 = 3a$, and $a = 7$.

What about if you know the area of a triangle? Can you find any of the sides? You still have to have another measurement, either two sides or the height of the triangle. The Pythagorean Theorem comes into play here, and finding the side when you know the area isn't as straightforward as finding a side when you know the perimeter. Let's look at a scalene triangle with area 10. If you know the height of the triangle, you can find the base by using algebra. The formula is $A = \frac{1}{2}bh$, so plug in the values for A and h to find the base, b. But to find the other two sides takes some knowledge of trigonometry, so we won't discuss it here, but it is possible.

Again, certain types of triangles make it easier. If you have a *right* triangle, the legs are the base and the height, and now you *can* find the third side with the Pythagorean Theorem. If you know the area of an *isosceles* triangle and either the base or the height, you can find the other missing value in the formula $A = \frac{1}{2}bh$. Knowing it is an isosceles triangle, you can even find the equal sides of the triangle because the height has divided the triangle into two right triangles with areas equal to half the original triangle. It is even easier for an equilateral triangle because you know the three sides are equal and if you know the height, you can find one of the sides by the same method as for an isosceles triangle.

<<EXAMPLE>>Example

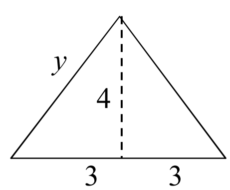
Using your knowledge of triangle perimeters and/or areas, find the values of x, y, and z for each of the triangles below given the following information. Each has height $h = 4$ and area 12. Triangle (a) is equilateral, triangle (b) is isosceles, and triangle (c) is a right triangle.



a. b. c.

<<ANSWER>>Answer **a.** $x = 6$. The base is found by the formula for the area of a triangle: $A = \frac{1}{2}bh$. With the given measurements of $A = 12$ and $h = 4$, than b must be 6. Since all sides are equal in an equilateral triangle, $x = 6$.

b. $y = 5$. Using the same procedure as in part (a), the base is 6. Draw the height to the base to get the following figure:



By using the Pythagorean Theorem, or remembering the 3-4-5 right triangle, the value of $y = 5$.

c. $z = 4$. In this figure, the height is given as 4, but because this is a right triangle, side z is the height.

<<EXAMPLE>>Example	The length of one leg of a right triangle is 2 inches less than the hypotenuse, and the other leg is 8 inches long. What is the area of the triangle?

A. 15 square inches

B. 60 inches

C. 60 square inches

D. 120 square inches

<<ANSWER>>Answer (C) 60 square inches. The area of a right triangle is $\frac{1}{2}$ (leg1 × leg2). One leg of this triangle is 8. Let the other leg be x. Then the hypotenuse is $x + 2$. The Pythagorean Theorem then can be written as

$$(x + 2)^2 = 8^2 + x^2$$

$$x^2 + 4x + 4 = 64 + x^2$$

$$4x = 60$$

$$x = 15$$

Then the two legs are 8 and 15, and the area of the triangle is $\frac{1}{2}(8)(15) = 60$ square inches. Note that answer choice (B) is incorrect because it has the wrong dimension.

Page 185: Insert the following before the heading "Square."

<<EXAMPLE>>Example

The sides of a rectangle are in the ratio of 1:3. The area of the rectangle is 75 square inches. What is the perimeter of the rectangle?

A. 5 inches

B. 15 inches

C. 20 inches

D. 40 inches

<<ANSWER>>Answer

(D) 40 inches. If the sides of the rectangle are in the ratio of 1:3, we can represent them as x and $3x$. The area of the rectangle is then $x(3x)$, and the equation to find the dimensions of the rectangle is

$$x(3x) = 75$$

$$3x^2 = 75$$

$$x^2 = 25$$

$$x = 5$$

So one side of the rectangle is 5, and the other side is 3 times that, or 15. But the problem asks for the perimeter, so we must add up all the sides: $p = 5 + 15 + 5 + 15 = 40$. Answer choices (A) and (B) are the lengths of the sides, and answer choice (C) didn't double the sides to find the perimeter.

Page 188: Insert the following after Answer 7.9.

<<EXAMPLE>>Example

A carpet installer was asked to install carpet in a room in a customer's house. The width of the room was 11 feet, and the installer was told to use 180 square feet of a 12-foot-wide roll of carpet. What is the length of the room?

A. 12 feet

B. 15 feet

C. 16 feet

D. 16.4 feet

<<ANSWER>>Answer

(B) Use the formula $A = lw$. The width of the roll is 12 feet, and the area of the carpet is 180 square feet, so $180 = 12l$, or $l = 15$ feet. This is a straightforward calculation. Answer choice (A) is too short, and answer choice (C) is too long. Answer choice (D) would be correct for an 11-foot-wide roll of carpet but this carpet is 12 feet wide. The information that the width of the room is 11 feet is extraneous to this problem.

<<HEAD>>Polygons

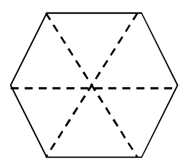
<<TEXT>>Triangles and quadrilaterals are three-sided and four-sided closed figures. But closed, straight-line figures can have more than four sides. Think of the Pentagon, the building that houses the headquarters of the U.S. Department of Defense. An aerial view of the building shows that it has five equal sides, and in fact its name derives from the geometric five-sided **pentagon**. A cell in a beehive or

even the cross-sections of most wooden pencils are six-sided **hexagons**. A stop sign has eight sides so it is in the shape of an **octagon**. Polygons are named for the number of their sides. Equilateral triangles and squares are examples of regular polygons of three and four sides, respectively.

Most polygons we see, such as those mentioned above, are **regular** polygons, meaning all the sides are equal. These are the kind of polygons we will discuss here. But closed figures don't have to be regular. You can draw a figure of connected straight lines that aren't of the same length, and it is also a polygon.

The perimeter of a polygon is the sum of the lengths of the sides—just add them up. The perimeter of a regular polygon is the length of a side multiplied by how many sides there are.

The area of a polygon is more complicated; it is derived from the fact that any regular polygon is made up of identical triangles, as shown below for a hexagon.



The area of this figure is 6 times the area of each triangle. But the formula for the area of a triangle involves not only the base (which is the side of the hexagon), but also the height to the base, or a perpendicular line from the center of the hexagon to a side. This line has the special name of **apothem**. So the area of any regular polygon is given by $A_n = n(\frac{1}{2}bh) = \frac{1}{2}ap$, where a is the apothem and p is the perimeter, which is equal to n times any side, or base. For the GED® test, this formula will be given to you, but it is helpful to know what it means and how it was derived.

Let's look at a few typical examples.

<<EXAMPLE>>Example

Each side of the Pentagon building is 921 feet. What is the perimeter of the Pentagon?

A. 3,684 feet

B. 4,605 feet

C. 5,526 feet

D. 5,526 square feet

<<ANSWER>>Answer (B) 4,605 feet. Since the Pentagon has five sides, its perimeter is 5 times the length of a side, or 5 × 921 = 4605. Answer choice (D) is incorrect because its dimension is square feet, which is area, not length.

<<EXAMPLE>>Example

A regular polygon is composed of equilateral triangles. What kind of regular polygon can it be?

A. Square

B. Trapezoid

C. Hexagon

D. 13-sided polygon

<<ANSWER>>Answer (C) Hexagon. This answer is best derived by elimination. If you draw the diagonal of a square, the triangles formed are right triangles, so answer choice (A) is incorrect. Answer choice (B) isn't a regular polygon. Answer choice (C) does, in fact, contain six equilateral triangles, as seen in the figure in the text above. Answer choice (D) is incorrect because a sketch will show that the greater the number of sides, the more "pointy" the top angle is for each triangle.

Page 188: Insert the following after Answer 7.10.

Suppose Mr. Chavez wants to fence a square patch of grass whose sides are positive whole numbers. The cost per foot to do this is $7. The first salesperson measured the patch and told him that the perimeter was 84 feet and would cost $588. The second salesperson told him that the perimeter was 79 feet, at a cost of $553. Mr. Chavez chose the first salesperson to do the job, even though it was more expensive, because the second salesperson gave him a measurement that wasn't possible. Do you know why?

Think of the formula for the perimeter of a square: $P = 4s$. As long as s is a positive integer, $4s$ will always be an even number, so a square perimeter of 79 feet is impossible.

We can extend this reasoning to a rectangular patch of grass whose sides again are positive integers. We are inclined to make the conjecture that perimeters of squares or rectangles are always even numbers. But what about measurements that aren't integers—can the perimeter ever be odd?

Here's just one counterexample to the conjecture that the perimeters of a rectangle are always even: A rectangle with sides of 7.25, and 9.25 has a perimeter of 33. Any combination of mixed numbers for which the fractional parts add up to ½ or 1 will serve as a counterexample for a rectangle. Another counterexample would be a rectangle with sides of $2\frac{1}{6}$ and $4\frac{1}{3}$.

Can you think of a counterexample to the statement, "The perimeters of squares are always even"? (Hint: The sides of a square don't have to be whole numbers for the perimeter to be a whole number.)

Page 191: Insert the following at the bottom of the page.

<<EXAMPLE>>Example

An expensive resort has a large circular swimming pool. Someone walking around the pool at a pace of 250 feet per minute takes 1 minute to circle the pool. What is the approximate diameter of the pool?

A. 40 feet

B. 60 feet

C. 80 feet

D. 100 feet

<<ANSWER>>Answer

(C) 80 feet. Someone walking around the pool at a pace of 250 feet per minute takes 1 minute to circle the pool, so the pool must be 250 feet in circumference. $C = \pi d$, so the equation for the diameter is $250 = 3.14d$, and d is closest to 80 feet, which is just short of the length of an Olympic-size pool.

<<EXAMPLE>>Example

Pizza sizes refer to their diameters. A personal pizza is an 8-inch pizza. What size pizza is close to double the size of the personal pizza?

A. 10-inch pizza

B. 11-inch pizza

C. 12-inch pizza

D. 16-inch pizza

<<ANSWER>>Answer (B), 11-inch pizza. The area of the 8-inch pizza (with a 4-inch radius) is $A = \pi r^2 = 16\pi$. So a pizza that is double this size would have an area of 32π. Use the same formula to solve for the new pizza radius and diameter:

$$A = 32\pi = \pi r^2$$

then $$r = \sqrt{32} \approx 5.7$$

and $$d \approx 11$$

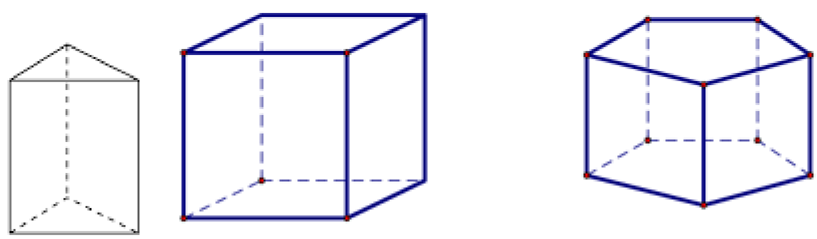
Pages 195-197: Replace the section on Right Prisms with the following.

<<HEAD>>Right Prisms

One type of three-dimensional figure is known as a **right prism**. The bases are identical polygons and the **faces** are perpendicular to the bases. Each set of three edges meets at a point called the **vertex** (plural, *vertices*).

Below are (a) a right triangular prism, (b) a right rectangular prism, and (c) a right pentagonal prism.

(a) (b) (c)



<<COMP: Please make the lines the same weight in these three figures.>>

Also obscured, but something we can envision, are the diagonals of a right prism, which are not the diagonals of the faces, but rather they are diagonals that run internally from one corner to another opposite corner. The *surface area* of a right prism is the sum of the areas of the faces. Every right prism has one pair of identical faces, which are the polygon bases, and one set of lateral faces that are perpendicular to the bases. The surface area of a right prism is thus the sum of the areas of all of the faces. There are 2 polygons and as many rectangles (the lateral faces) as there are sides in the polygon. The area of each base is usually designated as B (sometimes b), and the area of each lateral face is the height of the prism (h) times the length of the side of the polygon. Since there are as many of these lateral sides as there are sides of the polygon, when we add them all up we essentially have the perimeter of the base (p) times the height (h). Thus, the surface area becomes

$$SA_{\text{right prism}} = ph + 2B,$$

which is the formula given on the GED® formula sheet. Just remember what the B, p, and h stand for.

The *volume* of a right prism follows the rule that volume is the area of the base times the height. The equation for the area of the base depends on what kind of polygon it is. The volume is thus

$$V_{\text{right prism}} = Bh,$$

which also is the formula given on the GED® formula sheet. Just remember what the B and h are.

For example, for a 3 × 4 × 6-inch right rectangular prism, the surface area is

$$SA = 2[(3 \times 4) + (3 \times 6) + (4 \times 6)]$$

$$= 2(12 + 18 + 24) = 2(54) = 108 \text{ square inches.}$$

For the same right rectangular prism, the volume is

$$V = (3)(4)(6) = 72 \text{ cubic inches.}$$

<<EXAMPLE>>Example

Priority Mail shipping boxes at the post office are sold in specific sizes, such as the following (all are inside dimensions in inches; all boxes are right rectangular prisms):

12 × 12 × 5.5

5.38 × 8.63 ×1.63

11 × 8.5 × 5.5

a. What is the surface area of the biggest box?

b. What is the volume of the middle-size box?

<<ANSWER>>Answer

a. 354 square inches. First, which is the biggest box? Since all three boxes have one side that is about the same (5.5 or 5.38 inches), compare only the other two dimensions. It is clear that the 12 × 12 box will be larger than either 11 × 8.5 or 5.38 × 8.63, so we don't have to calculate the sizes of the boxes, just do a comparison of the dimensions. The surface area of the largest box, with dimensions 12 × 12 × 5.5, is $SA_{\text{right prism}} = ph + 2B$, according to the GED® formula sheet. B is the base area, so let's take the base as the largest side, 12 × 12 = 144 square inches. The perimeter of the base, since all edges are 12 inches, is 4 × 12 = 48 square inches. The height to the base is the remaining measure, 5.5 inches. Therefore, the surface area is

$$SA_{\text{right prism}} = ph + 2B$$

$$SA = (12)(5.5) + 2(144) = 66 + 288 = 354 \text{ square inches.}$$

b. We can determine which of the remaining two boxes is the medium-size one by observation. Two of their dimensions are close to the same, but the third dimension is 11 inches for one box and 1.63 inches for the other. There is no doubt which of these is larger, so the medium-size box has interior dimensions in inches of 11 × 8.5 × 5.5, and its volume is given by (see the GED® formula sheet)

$$V_{\text{right prism}} = lwh$$

$$V = 11 \times 8.5 \times 5.5 = 514.25 \text{ cubic inches.}$$

Page 200: Insert the following before the heading "Pyramid."

<<EXAMPLE>>Example

The radius of a right circular cylinder of height h and volume V is given by $r = \sqrt{\frac{V}{\pi h}}$. Use this formula to solve for h in terms of r and V. $h =$

A. $\frac{\pi r^2}{V}$

B. $\frac{V}{\pi r^2}$

C. $\frac{(\pi r)^2}{V}$

D. $\frac{V}{\pi r}$

<<ANSWER>>Answer

(B) $\frac{V}{\pi r^2}$. This is more of an algebra problem than a geometry problem, moving coefficients and variables around to find an expression for h. But it conveys an important fact in geometry, that any equation can be solved for any of the variables. It all depends on what values you are given. The calculation goes like this:

$$r = \sqrt{\frac{V}{\pi h}}.$$

$$r^2 = \frac{V}{\pi h} \qquad \text{(square both sides)}$$

$$r^2 \ \pi h = V \qquad \text{(multiply both sides by } \pi h \text{)}$$

$$h = \frac{V}{\pi r^2} \qquad \text{(divide both sides by } \pi r^2 \text{)}$$

If any of the above algebra seems tricky, review the section on "Setting up Equations" in Chapter 5. Remember that the same rules as were used with simple linear equations also work for more complicated equations: Use inverse operations to get the unknown variable on one side of the equation, or to get a quadratic equation that can be solved by using the quadratic formula.

Page 217: Insert the following after Answer 8.3.

<<EXAMPLE>>Example

A local store is considering offering credit cards. They plan to assign a four-digit PIN number for each card. How many cards with unique PIN numbers can be formed if zero is not an acceptable digit in the leftmost place (because then it would be a three-digit PIN) and repetition of digits is not allowed?

A. 3,360

B. 4,032

C. 4,536

D. 5,040

<<ANSWER>>Answer

(C) 4,536. This answer comes straight from the counting principle. The first digit can be picked in 9 ways (1-9, no 0 allowed); the second digit can be picked in 9 ways (now we include 0 but exclude the first digit); the third in 8 ways; and the fourth in 7 ways. So the number of unique PINs is

$$9 \times 9 \times 8 \times 7 = 4536$$

Page 219: Insert the following before Example 8.4.

<<EXAMPLE>>Example

How many different ways can the five letters of the word EIGHT be arranged by using each letter only once in each arrangement?

A. ${}_8P_5$

B. ${}_5C_5$

C. 5!

D. 5^5

<<ANSWER>>Answer

(C) 5!. First, you should have looked at the answer choices before starting to calculate a number. The question is simply asking which method you would use to calculate the number, not what the number actually is. Using the counting principle, the answer is $5 \times 4 \times 3 \times 2 \times 1 = 5!$

Because order makes a difference, it is also a permutation of 5 things being picked from a group of 5 (which is not an answer choice). The same result comes from using ${}_5P_5 = \frac{5!}{(5-5)!} = \frac{5!}{0!} = \frac{5!}{1} = 5!$.

Page 224: Insert the following directly above the heading "Data."

<<EXAMPLE>>Example

Two cards are drawn from a deck of 52 cards with the first card replaced before the second card is drawn. What is the probability that neither card is a spade?

A. $\frac{9}{16}$

B. $\frac{3}{4}$

C. $\frac{1}{16}$

D. $\frac{19}{34}$

<<ANSWER>>Answer

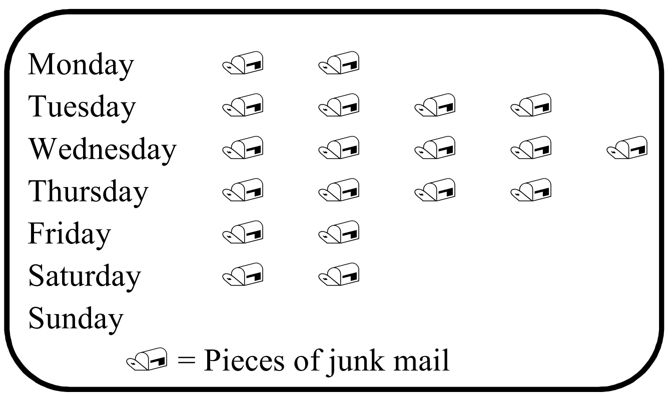
(A) $\frac{9}{16}$. The probability for the first pick is all non-spades, or 39 of the 52 cards: $\frac{39}{52} = \frac{3}{4}$. Since the card is replaced, the probability for the second pick is the same, so the probability that neither is a spade is $\frac{3}{4} \times \frac{3}{4} = \frac{9}{16}$.

Page 235: Insert the following at the bottom of the page, below the list.

<<HEAD>>Dot Plots

Dot plots are a way to show individual data points. The information from a box plot is similar to that shown for histograms, except that the data are usually individual values and not grouped. One dot (or symbol, as shown below) represents one instance of that variable. A box plot can be horizontal or vertical. A box plot for the following counts of pieces of junk mail received each day is shown below. It is easy to see how the dot plot allows you to compare days more easily than the digits. Dot plots are like the tick marks used when you are tallying a count of, for example, how many people are wearing certain colors to a concert.

Monday	2
Tuesday	4
Wednesday	5
Thursday	4
Friday	2
Saturday	2



Page 236: Insert the following to replace the first paragraph under the heading "Pie Charts."

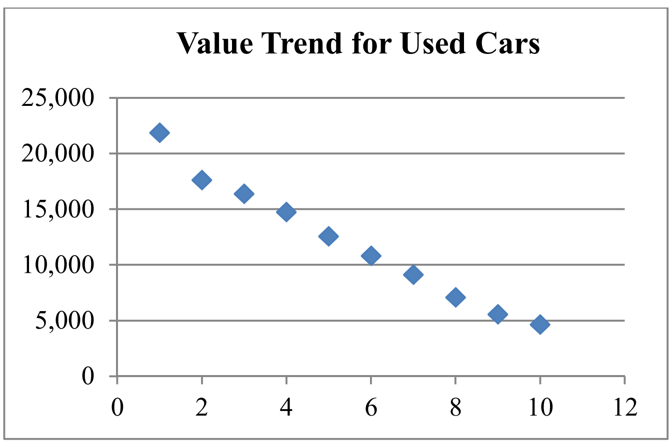
<<TEXT>> A **pie chart** (sometimes called a **circle graph**) is a circle divided into wedges that are proportional to the numbers in the data set. To present a pie chart of the data in the table on political party preference (shown below), we first must change each frequency to a percentage, which will determine how much of the "pie" is to be represented by each category. To do this, divide each frequency by the total. For these data points, the percentages are: Democrat $\left(\frac{368}{1000} = 36.8\%\right)$; Republican $\left(\frac{390}{1000} = 39.0\%\right)$; and Other/None $\left(\frac{242}{1000} = 24.2\%\right)$. The pie chart representing these percentages is shown below.

Page 238: Insert the following to replace the first two paragraphs on the page.

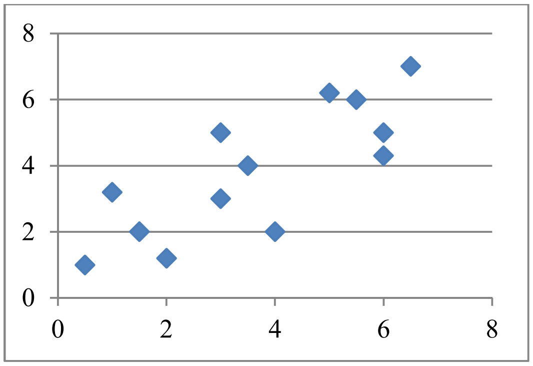
<<TEXT>> Let's see what the scatter plots for a set of data would look like and what we can predict about the correlation between the two variables. Initially, the data are in table form, but it is difficult to see a trend from these data. A scatter plot, which charts the data pairs in (x, y) form on a coordinate plane, reveals whether there is a correlation. Here are some data points representing the age of a used car versus its price.

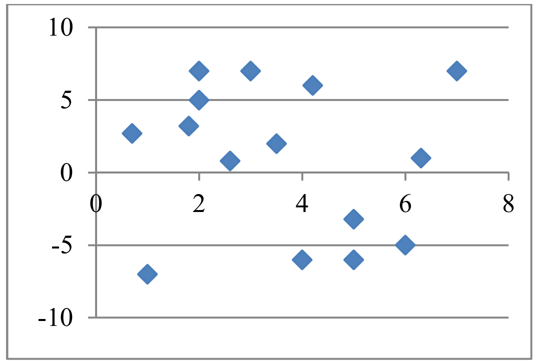
Age of a Used Car (years)	Price
1	$21,840
2	$17,600
3	$16,380
4	$14,735
5	$12,540
6	$10,805
7	$9,100
8	$7,075
9	$5,545
10	$4,640

The scatter plot shows a very strong negative correlation, since the points are close to a straight line, and the slope of the line is negative. This means the older a car is, the less it is worth.



Examples of other scatter plots are shown below. The left plot shows an average positive correlation, and the right plot shows no correlation at all. An example of two sets of data with no correlation would be the number of books read per year and the number of pets at home.


8
6
4
2
0
0
2
4
6
8


10
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-5
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Errata and Clarifications

Page 29: In the paragraph under the number line, replace the 2nd sentence with the following.

Integers are marked on the line, but there are infinitely many numbers between each two integers.

Page 48: Insert the following as a second sentence in the shaded rectangle.

If you enter a fraction with a denominator of 0 on the calculator, you will get an "Error" message.

Page 76: Insert "simple" in the beginning of the first sentence under the heading "Interest."

The formula $I = prt$, where I is simple interest, . . .

Page 129: In the second sentence in the last paragraph, change "with *x* being a whole number" to

with x being an integer,

Page 130: Replace the sentence under the Hint box with the following.

Inequalities are used if there is a limit. For example, if you have only \$10 to spend at a store, your total purchase, including tax, must be $\leq$ \$10.

Page 138: Change Answer 11 to the following.

(B) 60 pounds. When the seesaw is perfectly balanced, Marianne's weight times her distance from the pivot equals Jimmy's weight times his distance from the pivot. So the equation, where x is Jimmy's weight, is

Page 147: In the "Hint" box, last bullet item, change both instances of "quadratic equation" to

quadratic formula

Page 251: At the beginning of each of the choices for Question 9, change "There" to "Neglecting the origin, there"

Page 257: Question 31 should read as follows.

If Janet buys 6 dresses at 15% off the normal price *x* of each dress, which expression represents how much she has saved?

Page 279: Question 40. Change the two boxes with *x* in them to: $x +$

Page 289, boxplot: Change definition to:

A box drawn above a number line that provides ready information on the center (median) and variation (quartiles, range) in a data set.

Page 291, factorial. Change definition to:

Multiplication of integers from a given number down to 1.

Page 292, mutually exclusive. Change definition to:

Events that cannot happen *simultaneously*, such as passing a course and failing the same course.

Page 292, probability. Change definition to:

A value between 0 and 1 inclusive that indicates the chance that a given event will occur.

Page 293, regular polygon. Change definition to:

A polygon in which all the sides and angles are equal.

Page 294, square root. Change definition to:

A factor of a number that when squared gives the number; for example, the square root of 4 is 2.

Page 295, vertex. Change definition to:

The highest or lowest point of a parabola. Also, the point of an angle.

Page 295, whole numbers. Change definition to:

Any of the natural numbers (1, 2, 3, 4, 5, …) and 0.