#### **REA: SERIOUS PREP FOR THE GRE**

# **GRE® PHYSICS TEST**



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### **About the Book and TestWare**<sup>®</sup>

This book provides an accurate and complete representation of the GRE Physics Subject Test. The four practice exams and review section are based on the format of the most recently administered GRE Physics Test. Each exam is two hours and fifty minutes in length and includes every type of question that can be expected on the actual exam. Following each exam is an answer key, complete with detailed explanations designed to clarify the material for the student. Take the two practice tests on CD-ROM first These will give you a good sense of the areas that require the most review. By studying the review section, completing all four exams in the book, and studying the explanations that follow, students can discover their strengths and weaknesses and thereby become well prepared for the actual exam.

## **About the Test**

The GRE Physics Test is offered three times a year by the Educational Testing Service, under the direction of the Graduate Record Examinations Board. Applicants for graduate school submit GRE test results together with other undergraduate records as part of the highly competitive admission process to graduate school. The GRE tests are intended to provide the graduate school admissions committee with a means of evaluating your competence in certain subject areas. Scores on the test are intended to indicate mastery of the subject matter emphasized in an undergraduate program.

The test consists of about 100 multiple-choice questions, some of which are grouped in sets and based on such materials as diagrams, experimental data, graphs, and descriptions of physical situations. Emphasis is placed on the ability to grasp fundamental principles of physics as well as the ability to apply these principles. Most test questions can be answered on the basis of a mastery of the first three years of undergraduate physics. Emphasis is placed on the following major areas of physics and occur in the percentages indicated. These percentages reflect the relative emphasis placed on these topics in most undergraduate curricula.

- 1. Fundamentals of electromagnetism, including Maxwell's equations (18%)
- 2. Classical mechanics (20%)
- 3. Atomic physics (10%)
- 4. Quantum mechanics (12%)
- 5. Physical optics and wave phenomena (9%)
- 6. Special relativity (6%)
- 7. Thermodynamics and statistical mechanics (10%)
- 8. Laboratory methods (6%)
- 9. Advanced topics: Lagrangian and Hamiltonian mechanics, solid state physics, nuclear and particle physics, and miscellaneous (9%)

#### SSD ACCOMMODATIONS FOR STUDENTS WITH DISABILITIES

Many students qualify for extra time to take the GRE Physics Test. For information on how ETS meets disability needs, contact:

ETS Disability Services Educational Testing Service P.O. Box 6054 Princeton, NJ 08541-6054 Phone: 1-866-387-8602 (toll free) *Monday–Friday 8:30 a.m. to 5:00 p.m. Eastern Time (New York)* TTY: 1-609-771-7714 Fax: 1-973-735-1892 E-mail: stassd@ets.org

Our TestWare<sup>®</sup> can be adapted to accomodate your time extension. This allows you to practice under the same extended-time accomodations that you will receive on the actual test day. To customize your TestWare<sup>®</sup> to suit the most common extensions, visit our website at *www.rea.com/ssd*.

## **About the Authors**

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## **About the Subject Review**

The subject review in this book is designed to further your understanding of the test material. It includes techniques you can use to enhance your knowledge of physics and to earn higher scores on the exam. The review includes extensive discussions and examples to refresh your knowledge. Topics covered in the review are:

#### **Classical Mechanics**

- Vectors
- Linear Motion
- Two-Dimensional Motion
- Newton's Laws
- Momentum
- Energy and Work
- Harmonic Motion
- Collisions
- Lagrangian Mechanics

#### Electromagnetism

- Electric Fields
- Capacitors
- Current and Resistance
- Circuits
- Magnetism
- Maxwell's Equations and Electromagnetic Waves

#### **Atomic Physics**

- Rutherford Scattering
- Atomic Spectra
- The Bohr Atom
- The Laser

#### Thermodynamics

- Temperature
- Coefficient of Thermal Expansion
- Heat Capacity, Specific Heat, and  $C_p$
- Heat of Vaporization and Heat of Fusion
- Conduction, Convection, and Radiation
- · Heat, Work, and the Laws of Thermodynamics
- Entropy

#### **GRE PHYSICS**

#### **Quantum Mechanics**

- Wave Functions and Equations
- Schrödinger's Equation
- Potential Wells and Energy Levels
- Harmonic Oscillator
- · Reflection and Transmission by a Barrier

#### **Special Relativity**

- Time Dilation and Length Contraction
- Dynamics
- Lorentz Transformations

#### **Optics and Wave Phenomena**

- Mechanical Waves
- Ray Optics
- Thin Lenses
- Interference
- Diffraction

## **Scoring the Exam**

Two types of scores are obtained from your results on the GRE Physics examination: a raw score and a scaled score. The raw score is determined first and is then converted into the scaled score.

To determine the raw score, a number of things must be done. The following equation represents the process:

R - W/4 = Raw Score (round off if necessary)

First calculate the total number of wrong (W) answers. Next, calculate the total number of right (R) answers. Unanswered questions are not counted. At this point, divide the total number of wrong answers by four and subtract his result from the total number of right answers. This adjustment is made to compensate for guessing. Finally, take the last result and round it off to the nearest whole number, which will be the raw score.

To determine the scaled score, find the number that corresponds to the raw score in the table on the following page.

It is important to note that the raw score can vary slightly and still result in the same scaled score. This is because various forms of the test may be administered.

For more information, please visit the official GRE Web site at www.gre.org.

#### INTRODUCTION

#### **GRE** Physics—Total Score

Raw Score	Scaled Score	Raw Score	Scaled Score
84–100	990	41–42	670
83	980	40	660
81-82	970	39	650
80	960	37–38	640
79	950	36	630
77–78	940	35	620
76	930	33–34	610
75	920	32	600
73–74	910	30–31	590
72	900	29	580
71	890	28	570
69–70	880	26–27	560
68	870	25	550
67	860	24	540
65–66	850	22–23	530
64	840	21	520
63	830	20	510
61–62	820	18–19	500
60	810	17	490
59	800	16	480
57–58	790	14–15	470
56	780	13	460
55	770	12	450
53–54	760	10-11	440
52	750	9	430
51	740	8	420
49–50	730	6–7	410
48	720	5	400
47	710	4	390
45–46	700	2–3	380
44	690	1	370
43	680	0	360

## **GRE Physics Pro Study Plan**

#### Making the Most of REA's 12-Week Pro Study Plan

As a busy student or professional, you may be wondering how you will find time to work, have a social life, take care of family obligations, and prepare for the GRE Physics. But don't worry, our GRE Pro Study Schedule gives you a complete road map from now until test day. This flexible schedule allows you to work at your own pace and shows you how to prepare for the GRE as efficiently as possible. You'll get organized, stay on track, and make the most of your valuable study time.

You will need to make time *every day* to study for the GRE Physics. How and when you study is up to you, but consistency is the key to completing your GRE preparation. Make studying a priority and consider it a "job" until you take the GRE exam.

Keep in mind that a high score on the GRE doesn't just help you get into a better grad school; it helps move your career forward. Therefore, a solid commitment to daily study is worth every minute of your time. The results will pay off in the long run!

This study schedule will help you become thoroughly prepared for the GRE Physics exam. Although the schedule is designed as a 12-week study program, it can be compressed into a 6-week plan by combining two weeks of study into one. If you choose the 12-week schedule, you should plan to study for at *least* one hour a day. GRE examinees who are following a 6-week program need to set aside at least two hours for studying every day.

Find a study routine that works for you and stick to it! Some people like to get up early and study for an hour or two before going to work. Others might choose to study while commuting, on their lunch hour, at the library, or at home after work. Whatever schedule you choose, make a commitment to study every day – even on weekends.

Remember, daily focused concentration on the GRE Physics subject matter will help you retain more information, fully grasp the material tested, and improve your overall score.

#### INTRODUCTION

Week	Activity
1	Read and study our introduction to the GRE Physics on the following pages. Be sure you under- stand the format of the GRE Physics and know exactly what is tested on the exam. Now would be a good time to plan a study schedule. When and where will you study? How will you balance your responsibilities with studying? Decide when and where you study best, and get ready to tackle the GRE Physics!
2	Start your study routine by taking the GRE Physics Practice Test 1 on CD to determine your strengths and weaknesses. Make sure you give yourself at least 3 hours after work, on a week- end, or at another convenient time to take the exam. You will need to focus and concentrate, so take the practice test at a time and place where you will not be disturbed. When you take the test, try to do your best, even on sections where you may be confused. After you have finished the test, record your scores. This will help you track your progress as you study. If you score low in any area, thoroughly review the relevant chapters. Later in the week, study the detailed explanations for the questions you answered incorrectly. In the cases where you erred, find out why. Take notes and pay attention to sections where you missed a lot of questions. You will need to spend more time reviewing the related material.
3	Make a firm commitment to study for at least an hour a day, every day for the next few weeks. It may seem hard to find time in your busy schedule, but remember: the more you study, the better prepared you will be for the GRE Physics. This week, study chapters 1 and 2. Be sure you are comfortable with the material you've just covered before continuing to next week's study agenda. Give yourself more time to practice and review areas in which you feel you need improvement.
4	This week, focus your study on chapters 3 and 4. Make sure you fully understand the top- ics and have a good grasp of the formulas and theories discussed. Pay close attention to the examples outlined in chapter four; they will show you how to solve questions you may en- counter on the actual GRE exam. If you find yourself in need of extra review or clarification on a subject, you may want to consult your physics textbook or ask a classmate or professor for additional help.
5	Take Practice Test 3 in the book. Record your score and see how well you did. After you've evaluated your test results, go back through each chapter and brush up on the topics, theories, or formulas that you need to review. Later this week, study the detailed explanations of answers for any questions you answered incorrectly. Make sure you understand why you answered the question wrong, so you can improve your test-taking skills.

#### **GRE PHYSICS**

Week	Activity
6	Congratulations! You've reached the halfway point! Success is just around the corner, but keep going, there's more work to be done. Study chapters 5 and 6 this week. If you feel you have a good grasp of the material, review your notes and restudy any areas of weakness from previous chapters.
7	Take Practice Test 4 in the book. After scoring the exam, record your score and measure your progress. How much have you improved since you took Practice Test 1? Go back and study the detailed explanations of answers to any questions you answered incorrectly. Give yourself a break and do something fun – after you've finished studying for the day.
8	Your hard work is paying off! By the time you've finished studying for the GRE, you'll have an excellent understanding of all the topics tested on the exam – and that will help you get a high score! Take time this week to study chapters 7 and 8. Pay close attention to the examples and formulas discussed!
9	This week, review any questions you answered incorrectly on the previous practice tests. Read through the detailed answer explanations to clarify any subjects or areas in which you are weak. If you feel you need extra GRE Physics practice, why not review a chapter during your lunch hour or during your commute?
10	Now you're ready for Practice Test 2 on CD. Allow yourself at least 3 hours to take the practice test. Take your time, remember what you've learned, and answer every question to the best of your ability. After the test, thoroughly review all the explanations for the questions you answered incorrectly. Review any areas in which you still need improvement. Remember, your primary goal right now is to get a high score on the GRE Physics, so keep working!
11	You've just about reached your goal! Study chapter 9, the last review chapter in the book. Make sure you have a good grasp of these specialized topics, so you're ready for any question on the GRE Physics exam.
12	Use this time to restudy any sections in which you need improvement. You may want to re- read our review chapters or retake practice tests to help you boost your skills. Just for fun, why not take Practice Test 1 again? After you've completed the exam, compare your two scores and see how far you've come in just a matter of weeks!

## Congratulations! You've worked hard and you're ready for the GRE Physics!

## **About Research & Education Association**

Founded in 1959, Research & Education Association is dedicated to publishing the finest and most effective educational materials—including software, study guides, and test preps—for students in elementary school, middle school, high school, college, graduate school, and beyond.

REA's Test Preparation series includes books and software for all academic levels in almost all disciplines. Research & Education Association publishes test preps for students who have not yet entered high school, as well as high school students preparing to enter college. Students from countries around the world seeking to attend college in the United States will find the assistance they need in REA's publications. For college students seeking advanced degrees, REA publishes test preps for many major graduate school admission examinations in a wide variety of disciplines, including engineering, law, and medicine. Students at every level, in every field, with every ambition can find what they are looking for among REA's publications.

REA's series presents tests that accurately depict the official exams in both degree of difficulty and types of questions. REA's practice tests are always based upon the most recently administered exams, and include every type of question that can be expected on the actual exams.

REA's publications and educational materials are highly regarded and continually receive an unprecedented amount of praise from professionals, instructors, librarians, parents, and students. Our authors are as diverse as the fields represented in the books we publish. They are well-known in their respective disciplines and serve on the faculties of prestigious high schools, colleges, and universities throughout the United States and Canada.

We invite you to visit us at www.rea.com to find out how "REA is making the world smarter."

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## **GRE Physics Review**

## CHAPTER

## Classical Mechanics

# 1

#### A. VECTORS

A vector is a measure of both direction and magnitude. Vector variables are usually indicated in **boldface** or with an arrow, such as  $\vec{v}$ .

#### The Components of a Vector



 $a_x$  and  $a_y$  are the components of a vector **a**. The angle  $\theta$  is measured counterclockwise from the positive *x*-axis. The components are formed when we draw perpendicular lines to the chosen axes.

The Formation of Vector Components on the Positive x – y axis

The components of a vector are given by

$$a_x = a \cos \theta$$
$$a_y = a \sin \theta$$

A component is equal to the product of the magnitude of vector A and the cosine of the angle between the positive x axis and the vector.

The magnitude can be expressed in terms of the components

$$|\boldsymbol{a}| = \sqrt{a_x^2 + a_y^2}$$



#### **CLASSICAL MECHANICS**

Finally, the angle  $\theta$  is given by

$$\theta = \tan^{-1} \frac{a_y}{a_x}$$

Like scalars, which are measures of magnitude, vectors can be added, subtracted, and multiplied.

To add or subtract vectors, simply add or subtract the respective x and y coordinates. For example,

$$\mathbf{a} - \mathbf{b}$$
 implies  $a_x - b_x = c_x$ ,  
 $a_y - b_y = c_y$ 

Therefore, **c** is the difference of **a** and **b** and is given by  $c = \sqrt{c_x^2 + c_y^2}$ .

There are two forms of multiplication: the dot product and the vector, or cross product. The dot product yields a scalar value:

$$\mathbf{a} \cdot \mathbf{b} = ab \cos \theta$$

The cross product of two vectors yields a vector:

$$\mathbf{a} \times \mathbf{b} = \mathbf{c}$$
  
and  
 $|\mathbf{c}| = ab \sin \theta$ 



#### **Vector Multiplication**

The direction of the vector product  $\mathbf{a} \times \mathbf{b} = \mathbf{c}$  is given by the "right-hand rule":

- 1) With **a** and **b** tail-to-tail, draw the angle  $\theta$  from **a** to **b**.
- 2) With your right hand, curl your fingers in the direction of the angle drawn. The extended thumb points in the direction of **c**.



Properties of the Cross Product:

$$\mathbf{a} \times \mathbf{b} = -\mathbf{b} \times \mathbf{a}$$
  
 $\mathbf{a} \times (\mathbf{b} + \mathbf{c}) = (\mathbf{a} \times \mathbf{b}) + (\mathbf{a} \times \mathbf{c})$   
 $c(\mathbf{a} \times \mathbf{b}) = (c\mathbf{a}) \times \mathbf{b} = \mathbf{a} \times (c\mathbf{b})$ , where c is a scalar  
 $|\mathbf{a} \times \mathbf{b}|^2 = a^2b^2 - (\mathbf{a} \cdot \mathbf{b})^2$ 

#### **B. LINEAR MOTION**

Any object in motion has an average and an instantaneous velocity:

a) Average Velocity

$$V = \frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{t_2 - t_1}$$
 Units: Meters

b) Instantaneous Velocity

$$V = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt} = v(t)$$
 Units: Meters Sec

Just as the average and instantaneous velocities are the rate of change of position with respect to time, acceleration is the rate of change of velocity with respect to time.

$$\frac{dx}{dt} = v; \ \frac{dv}{dt} = a$$

From this, the following basic kinematic equations of motion can be derived:

1. 
$$v = v_0 + at$$

2. 
$$v^2 = v_0^2 + 2a(x - x_0)$$

3. 
$$x = x_0 + v_0 t + \frac{1}{2} at$$

3.  $x = x_0 + v_0 t + \frac{1}{2} a t^2$ 4.  $x = x_0 + \frac{1}{2} (v_0 + v) t$ 

where  $v_0$  and  $x_0$  are initial values.

#### **C. TWO-DIMENSIONAL MOTION**

For two-dimensional, or planar, motion, simply break the velocity and acceleration vectors down into their x and y components. Once this is done, the preceding one-dimensional equations can apply.



A special case of two-dimensional motion is **uniform circular motion**. For a particle to be held on a circular path, a radial force must be applied. This force is called **centripetal force**.



Centripetal Acceleration: This is also radial

$$a = \frac{v^2}{r}$$

where a = acceleration

Chapter

v = tangential component of velocity

r = radius of the path

For uniform circular motion, *a* can also be written as

$$a = \frac{4\pi^2 r}{T^2}$$

where *T*, the period or time for one revolution, is given by

$$T = \frac{2\pi r}{v}$$

The tangential component of the acceleration is the rate at which the particle speed changes; for uniform circular motion, this acceleration is zero.

When dealing with circular motion or other situations involving motion relative to a central force field, it is often appropriate to use polar coordinates, where the position is a function of radius and angle  $(r, \theta)$ .

In the case of the three dimensions, the coordinates become  $(r, \theta, z)$ , where the *z*-coordinate is identical to the respective Cartesian *z*-coordinate. This is known as the cylindrical coordinate system.



In describing two-dimensional motion,  $\alpha$  represents angular acceleration

$$\left(\frac{d\omega}{dt}\right)$$

and  $\omega$  represents angular velocity

$$\left(\frac{d\theta}{dt}\right)$$

If angular acceleration,  $\alpha$ , is constant, then equations correlating to those previously stated for linear motion can be shown to apply as given in the table below.

Rotational Motion	Linear Motion Equivalent
$\alpha = \text{constant}$	a = constant
$\omega = \omega_0 + \alpha t$	$v = v_0 + at$
$ heta=rac{\omega_0+\omega}{2}$	$x = \frac{v_0 + v}{2}t$
$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$	$x = v_0 t + \frac{1}{2}at^2$
$\omega^2 = \omega_0^2 + 2\alpha\theta$	$v^2 = v_0^2 + 2ax$
$\theta_0, \theta = \text{initial and final angular displacements}$	
$\omega_0, \omega = \text{initial and final angular velocities}$	

#### **Comparison of Rotational and Linear Motion Equations**

Another type of coordinate system used is the spherical coordinate system, with components  $(\rho, \varphi, \theta)$ .



#### **Circular Orbits**

Chapter

Gravitational pull is the centripetal force on the satellite:

$$G\frac{mM}{r^2} = \frac{mv^2}{r}$$

The speed of a satellite in circular orbit about a body of mass M:

$$v = \sqrt{\frac{GM}{r}}$$

The time period (*T*) of the satellite is proportional to  $r^{3/2}$  (Kepler's Third Law)

$$T^2 = \frac{4\pi^2}{GM}r^3$$

#### **Elliptic Orbits**

#### **Kepler's Laws for Elliptic Orbits**

#### **First Law**

The orbit of every planet is an ellipse with the sun at a focus.



#### **TWO-DIMENSIONAL MOTION**

#### Second Law

A line joining a planet and the sun sweeps out equal areas during equal intervals of time.



#### **Third Law**

"The square of the orbital period of a planet is directly proportional to the cube of the semi-major axis of its orbit."

 $T^2 \propto a^3$ 

and

$$T^2 = \frac{4\pi^2 a^3}{GM}$$

 $T \rightarrow$  period of the planet

 $G \rightarrow$  universal gravitational constant

 $M \rightarrow$  mass of the sun

 $a \rightarrow semi-major axis$ 



#### **Conservation of Angular Momentum in Elliptic Orbit**

 $mv_1r_1 = mv_2r_2$ 

#### **D. NEWTON'S LAWS**

#### **First Law**

Every body remains in its state of rest or uniform linear motion, unless a force is applied to change that state.

#### Second Law

If the vector sum of the forces  $\mathbf{F}$  acting on a particle of mass *m* is different from zero, then the particle will have an acceleration **a** directly proportional to, and in the same direction as,  $\mathbf{F}$ , but inversely proportional to mass *m*. Symbolically,

 $\mathbf{F} = m\mathbf{a}$ Units: Newtons =  $kgm/s^2$ 

(if mass is constant)

#### **Third Law**

For every action, there exists a corresponding equal and opposing reaction, or the mutual actions of two bodies are always equal and opposing.



Newton's Laws all refer to the effects of forces on particles or bodies. These forces are often represented with vectors and can be added/subtracted vectorially. In addition, certain mathematical operations can be used on vectors to obtain components, unit vectors, directional cosines, and resultants.



 $\mathbf{OP} = d_x \mathbf{x} + d_y \mathbf{y} + d_z \mathbf{z}$ 

**Unit Vector:** 

$$\mathbf{u} = \frac{\mathbf{OP}}{OP} = \frac{1}{d} \left( d_x \mathbf{x} + d_y \mathbf{y} + d_z \mathbf{z} \right)$$

Force F:

$$\mathbf{F} = F\mathbf{u} = \frac{F}{d} \left( d_x \mathbf{x} + d_y \mathbf{y} + d_z \mathbf{z} \right)$$

**Components:** 

$$F_x = \frac{Fd_x}{d}, \ F_y = \frac{Fd_y}{d}, \ F_z = \frac{Fd_z}{d}$$

**Distance**:

$$d = \sqrt{d_x^2 + d_y^2 + d_z^2}$$

**Directional Cosines of F:** 

$$\alpha = \cos^{-1} \frac{d_x}{d}$$
$$\beta = \cos^{-1} \frac{d_y}{d}$$
$$\gamma = \cos^{-1} \frac{d_z}{d}$$

#### **Unit Vector Expressed in Terms of Angles:**

 $\mathbf{u} = \cos \alpha \mathbf{x} + \cos \beta \mathbf{y} + \cos \gamma \mathbf{z}$ 

#### **Relationship Between Angles:**

 $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$ 

#### **Non-Inertial Frame of Reference**

A frame of reference for which acceleration a = 0 is called an inertial frame. A frame of reference that is accelerating is a non-inertial frame.

The velocity of an inertial frame is dependent on the frame of reference of the observer. The velocity of an inertial frame cannot be determined from any experiment performed within the frame of reference. The acceleration of a non-inertial frame can be determined by making localized measurement within it. Newton's First Law is apparently violated in a non-inertial frame because an object in it may accelerate without application of a force. For example, a rider in an amusement park ride called Gravitron feels pinned to the wall of the well as the floor beneath her drops away. This force that a person feels in rotating frame of reference directed away from the center is referred to as "centrifugal force" and is said to be pseudo force.

A person can determine the acceleration of a train compartment from within the train. One way this can be done is to hang a simple pendulum from the ceiling as in the figure below.



As the train accelerates, the pendulum makes an angle of  $\theta$  with the vertical.

$$T\cos\theta = mg$$
$$T\sin\theta = ma$$

Hence,  $T\sin\theta/T\cos\theta = mg/(ma) \rightarrow \tan\theta = g/a \rightarrow a = g \tan\theta$ 

The acceleration of the train is given by the equation

 $\mathbf{a} = \mathbf{g} \tan \boldsymbol{\theta}$ 

#### **E. MOMENTUM**

#### LINEAR MOMENTUM

p = mv units :  $\frac{\text{kg} \times \text{m}}{\text{sec}}$ 

where p = linear momentum of particle

m = mass of particle

v = velocity of particle

#### **NEWTON'S SECOND LAW**

$$\mathbf{F} = \frac{d\mathbf{p}}{dt} = \frac{d(m\mathbf{v})}{dt}$$

where  $\mathbf{F} =$  the net force on the particle

#### LINEAR MOMENTUM OF A SYSTEM OF PARTICLES

#### **Total Linear Momentum**

$$P = \sum_{i=1}^{n} p_1 = p_1 + p_2 + \dots + p_n$$
  
=  $m_1 v_1 + m_2 v_2 + \dots + m_n v_n$ 

where

 $\mathbf{P}$  = total linear momentum of system  $\mathbf{p}_i, m_i, \mathbf{v}_i$  = linear momentum, mass, and velocity of the *i*th particle, respectively.

#### Newton's Second Law for a System of Particles (Momentum Form)

$$\mathbf{F}_{\text{ext}} = \frac{d\mathbf{P}}{dt}$$

where  $\mathbf{F}_{ext} = \text{sum of all external forces}$ 

Momentum is conserved. The total linear momentum of the system remains unchanged if the *sum* of all forces acting on the system is zero.

According to Newton's Second Law:

$$\frac{d\mathbf{P}}{dt} = \sum F_{\text{ext}} = 0$$

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#### ANGULAR MOMENTUM

#### **Vector Equation**

 $\mathbf{L} = \mathbf{r} \times \mathbf{P} = \mathbf{I}\boldsymbol{\omega}$  Units:  $kgm^2/s$ 

#### **Scalar Equation**

 $l = rP \sin \theta$ 

#### where $\mathbf{L} =$ angular momentum

 $\mathbf{r}$  = radius vector from axis of rotation to  $\mathbf{p}$ 

- $\mathbf{p} = \text{linear momentum vector}$
- $\mathbf{I}$  = moment of inertia about axis of rotation
- $\omega$  = angular velocity



Angular Momentum

Note that these two equations are equivalent with producing a vector result that is perpendicular to the plane formed by  $\mathbf{r}$  and  $\mathbf{p}$ , as shown in the figure. Using cross-product form, the magnitude is found by

$$L = rp \sin \theta$$

with the direction and sense given by the right-hand rule.

For the second form, the same result is achieved with

$$L = I\omega$$

where I is the moment of inertia about any selected axis of rotation and is defined by

$$I = \int r^2 dm$$

and is a measure of rotational inertia.

Torque can be defined as

$$\tau = \mathbf{r} \times \mathbf{F}$$
 Units: Nm

and is a vector with magnitude given by cross-product operation and direction/sense as determined by the right-hand rule. An equivalent form is

$$\tau = d\mathbf{L}/dt$$

Determination of I:

#### **Integration Method**

Area—General Formula:

$$I = \int_A s^2 \, dA$$

where s = perpendicular distance from the axis to the area element.

#### **Example:**

For a rectangular area,



This is the moment of inertia with respect to an axis passing through the base of the rectangle.

Moments of Inertia of Masses:



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In Polar Coordinates, the polar moment of inertia is noted as J.

Polar Moment of Inertia:

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$$J_0 = \int r^2 \, dA$$

In terms of rectangular moments of inertia:

$$J_0 = I_x + I_y$$

The Radius of Gyration can be determined once the moment of inertia and the area are known.

$$I_x = k_x^2 A; \quad I_y = k_y^2 A$$

Rectangular component form:

$$k_x = \sqrt{\frac{I_x}{A}}$$
$$k_y = \sqrt{\frac{I_y}{A}}$$

Polar form:

$$k_0 = \sqrt{\frac{J_0}{A}}$$

Relation between rectangular component form and polar form:

$$k_0^2 = k_x^2 + k_y^2$$

#### Masses

$$I = k^2 m$$
$$k = \sqrt{\frac{I}{m}}$$

#### **Impulse and Momentum**

**Impulse-Momentum Method**—An alternate method to solving problems in which forces are expressed as a function of time. It is applicable to situations wherein forces act over a small interval of time.

Linear Impulse-Momentum Equation:

$$\int_{1}^{2} \mathbf{F} dt = \text{impulse} = m\mathbf{v}_{2} - m\mathbf{v}_{1}$$

Ideal impulse produces an instantaneous change in momentum and velocity of the particle without producing any displacement.

$$M\mathbf{v}_1 + \Sigma \mathbf{F} \Delta t = M\mathbf{v}_2$$

Any force that is non-impulsive may be neglected, e.g., weight or small forces.

#### F. ENERGY AND WORK

The work done by a force **F** through a displacement  $d\mathbf{r}$  is defined as:

 $dw \equiv \mathbf{F} \cdot d\mathbf{r}$  in Joules (SI units)

Over a finite distance from point 1 to point 2:

$$W_{1-2} = \int_1^2 \mathbf{F} \times d\mathbf{r}$$

#### **Work-Energy Principle**

Kinetic energy for a particle of mass M and velocity v is defined as

$$\text{K.E.} = \frac{1}{2}mv^2$$

Kinetic energy is the energy possessed by a particle by virtue of its motion.



#### **CLASSICAL MECHANICS**

**Principle of Work and Energy**—Given that a particle undergoes a displacement under the influence of a force **F**, the work done by **F** equals the change in kinetic energy of the particle.

$$W_{1-2} = (KE)_2 - (KE)_1$$

Results of the Principle of Work and Energy:

- A) Acceleration is not necessary and may not be obtained directly by this principle.
- B) The principle may be applied to a system of particles if each particle is considered separately.
- C) Those forces that do not contribute work are eliminated.

Kinetic Energy and Newton's Law:

$$F = mv\frac{dv}{dx} = \frac{d}{dx}(\text{KE})$$

where KE = a function of *x*.

(This applied only in an inertial reference frame.)

#### **Power and Efficiency**

Power is defined as the time-rate of change of work and is denoted by dw/dt,

Power = 
$$\frac{dw}{dt} = \mathbf{F} \cdot \mathbf{v}$$

Mechanical Efficiency:

$$\eta = \frac{\text{Power out}}{\text{Power in}}$$

**NOTE**:  $\eta$  is *always* < 1.

#### **Potential Energy**

**Potential Energy**  $\equiv$  The stored energy of a body or particle in a force field associated with its position from a reference frame.

If PE represents potential energy,

$$PE = mgh$$
$$U_{2-1} = (PE)_1 - (PE)_2$$

A negative value would indicate an increase in potential energy.

Types of potential energy include:

Gravitational Potential Energy:

$$PE_g = -G\frac{M_1M_2}{r}$$

Spring Potential Energy:

$$PE = \frac{1}{2}ky^2$$

#### **Conservation of Energy**

#### **Conservative Case**

For a particle under the action of conservative forces:

$$(KE)_1 + (PE)_1 = (KE)_2 + (PE)_2 = E$$
(1)

The sum of kinetic and potential energy at a given point is constant.

Equation (1) can be written as:

$$E = \frac{1}{2} mv^2 + (PE)$$

The potential energy must be less than or equal to the total energy.

In a conservative system, if PE = E, then velocity = 0.

In a non-conservative system, relating potential and kinetic energy with the nonconservative force  $\mathbf{F}'$ 

$$d(\mathbf{PE} + \mathbf{KE}) = \int \mathbf{F}' \cdot d\mathbf{r}$$

- 1) The direction of  $\mathbf{F}'$  is opposite to that of  $d\mathbf{r}$ .
- 2) Total energy *E* decreases with motion.
- 3) Friction forces are nonconservative.

#### **G. HARMONIC MOTION**

**Simple Harmonic Motion**—Linear motion of a body where the acceleration is proportional to the displacement from a fixed origin and is always directed towards the origin. The direction of acceleration is always opposite to that of the displacement.

Equation of Motion:

or

$$mx'' + kx = 0$$
  
$$x'' + p^2 x = 0$$
 (2)

where  $p^2 = k/m$ .

General Solution of Equation (2):

 $x = c_1 \sin pt + c_2 \cos pt$ 

where  $c_1$  and  $c_2$  may be obtained from initial conditions.

An alternate form of Equation (2):

$$x = x_m \sin\left(pt + \phi\right)$$

where  $x_m =$  the amplitude  $\phi =$  the phase angle



For small angles of vibration, the motion of a simple pendulum can be approximated by simple harmonic motion.



For small angles of vibration,

$$\phi = s/l$$

Equation of Motion:

$$\phi'' + \frac{g}{l}\phi = 0$$

The solution is:

$$\phi = \phi_0 \cos\left(\omega_0 t + \alpha_0\right)$$

where

 $\omega_0 = \sqrt{g/l}$   $\phi_0 = \max \text{ amplitude of oscillation}$  $\alpha_0 = \text{ phase factor}$ 

The period of oscillation is:

$$T_0 = \frac{2\pi}{\omega_0} = 2\pi \sqrt{\frac{l}{g}}$$

The Spherical Pendulum refers to the simple pendulum-like arrangement, but with motion in three dimensions.

The equations of motion become:

$$x'' + (g/l)x = 0$$
$$y'' + (g/l)y = 0$$

with solutions:

$$x = A \cos (\omega t + r)$$
$$y = B \cos (\omega t + \delta)$$

where  $\omega = \sqrt{g/l}$ 

On the x-y plane, the motion is an ellipse.

Spherical Coordinates—More accurate than the previous solution.

Equations of Motion:

$$ma_r = F_r = Mg \cos \theta - T$$
$$ma_{\phi} = F_{\phi} = -Mg \sin \phi$$
$$ma_{\phi\theta} = F_{\phi\theta} = 0$$

In spherical coordinates:

$$\phi'' - \Psi'^2 \sin \phi \cos \phi + g \sin \phi = 0$$
$$\frac{1}{\sin \phi} \frac{d}{dt} (\alpha' \sin^2 \phi) = 0$$

#### Forced (Driven) Harmonic Oscillator

For periodic driving force:

Equation of Motion:

$$\frac{d^2x}{dt^2} = -\omega^2 x + \frac{F^0}{m} \cos \omega_f t$$

where  $\omega_f = \text{driving frequency}$  $F_0 = \text{max. magnitude of the driving force}$ 

The solution will be equal to the sum of the complementary solution and the particular solution.

$$X = A \cos \omega t + B \sin \omega t + \frac{F^{o} / m}{\omega^{2} - \omega_{f}^{2}} \cos \omega_{f} t$$

Resonance occurs when  $\omega_f \approx \omega$ .

#### **Damped Oscillator**

A common damping force is

$$\mathbf{F} = c \frac{dx}{dt}$$

If an object's motion is damped in this manner, then the equation of motion is:
$$m\frac{d^2x}{dt^2} + c\frac{dx}{dt} + kx = 0$$
$$a_1 = \frac{-c}{2m} - \sqrt{\left(\frac{c}{2m}\right)^2 - \frac{k}{m}}$$
$$a_2 = \frac{-c}{2m} + \sqrt{\left(\frac{c}{2m}\right)^2 - \frac{k}{m}}$$

The critical damping coefficient is defined as

$$C_{\text{critical}} = 2m\sqrt{\frac{k}{m}}$$

Now, three cases must be considered with respect to  $C_{\text{critical}}$ :

A) If  $c > C_{\text{critical}}$ ,  $a_1$  and  $a_2$  are both real, the motion is nonoscillating, and the system is overdamped. The general solution is given by

$$x = Ae^{a_1t} + Be^{a_2t}$$

B) If  $c = C_{\text{critical}}$ ,  $a_1 = a_2$ , and the system is critically damped with general solution:

$$x = (A + Bt)e^{-\left(\frac{C_{\text{critical}}}{2m}\right)t}$$

C) If  $c < C_{\text{critical}} a_1$  and  $a_2$ , are complex and imaginary, and the system is underdamped with the solution given by:

$$X = E\left[e^{-(c/2m)t}\sin\left(\sqrt{\frac{k}{m} - \left(\frac{C}{2m}\right)^2}T + \Psi\right)\right]$$

where constants A, B, E, and  $\psi$  are determined from initial conditions. The graph representing the above equation is shown in the following figure

$$D = \begin{bmatrix} \exp\left(-\left(\frac{c}{2M}\right)t\right) \end{bmatrix} \sin\left(\sqrt{\frac{k}{M} - \left(\frac{c}{2M}\right)^{2}}t + \Psi\right) \end{bmatrix}$$

$$0 = \begin{bmatrix} \exp\left(-\left(\frac{c}{2M}\right)t\right) \end{bmatrix} \sin\left(\sqrt{\frac{k}{M} - \left(\frac{c}{2M}\right)^{2}}t + \Psi\right) \end{bmatrix}$$

$$D = \begin{bmatrix} \exp\left(-\left(\frac{c}{2M}\right)t\right) \end{bmatrix}$$

$$E = \exp\left(-\left(\frac{c}{2M}\right)t\right)$$



#### **Damped Force (Driven) Vibration**

The equation of motion becomes:

$$m = \frac{d^2x}{dt^2} + c\frac{dx}{dt} + kx = F\sin\omega t$$

Assuming  $X_p = A \sin(\omega t - \psi)$ 

$$A = \frac{F/k}{\sqrt{\left[1 - \frac{4m^2\omega^2}{C_{\text{critical}}^2}\right]^2 + \left[\frac{4m\omega C}{C_{\text{critical}}^2}\right]^2}}$$
$$\Psi = \tan^{-1}\left[\frac{c\omega/k}{\left(1 - \frac{m\omega^2}{k}\right)}\right]$$

**NOTE**: Angle  $\psi$  is the phase difference between the resulting steady-state vibration and the applied force.

The magnification factor is defined as:

Magnification Factor = 
$$\frac{A}{F/k} = \frac{1}{\sqrt{\left[1 - \frac{4m^2\omega^2}{C_{\text{critical}}}\right]^2 \left[\frac{4m\omega c}{C_{\text{critical}}}\right]^2}}$$

and the graph is shown in the following figure. Resonance occurs only when the damping is zero and the frequency ratio is one.



#### **H. COLLISIONS**

When kinetic energy is conserved, the collision is **elastic**. Otherwise, the collision is said to be **inelastic**.

A) For an elastic collision,

$$\frac{1}{2}m_1v_{1_i}^2 + \frac{1}{2}m_2v_{2_i}^2 = \frac{1}{2}m_1v_{1_f}^2 + \frac{1}{2}m_2v_{2_f}^2$$

B) For an inelastic collision, some kinetic energy is transformed into internal energy. However, linear momentum is still conserved. If the two bodies stick and travel together with a common final velocity after collision, it is said to be completely inelastic. From conservation of momentum, we have

$$m_1 v_{1i} + m_2 v_{2i} = (m_1 + m_2) v_f$$

#### **Collisions in Two and Three Dimensions**

Since momentum is linearly conserved, the resultant components must be found and then the conservation laws applied in each direction.

A) The *x*-component

$$m_1 v_{1i} = m_1 v_{1i} \cos \theta_1 + m_2 v_{2i} \cos \theta_2 \tag{i}$$

B) The *y*–component

$$m_2 v_{2_i} = m_1 v_{1_f} \sin \theta_1 + m_2 v_{2_f} \sin \theta_2$$
(i)

where  $\theta_1$  = the angle of deflection, after the collision, of mass  $m_1$  $\theta_2$  = the angle of deflection, after the collision, of mass  $m_2$ 

C) For three dimensions, there would be an added *z*-component and an added angle,  $\theta_3$ .

For the above cases, *i* denotes initial value; *f* denotes final value.

#### I. LAGRANGIAN MECHANICS

#### **Generalized Coordinates and Forces**

The position of a particle is described by employing the concept of a coordinate system. Given, for example, a coordinate system such as the spherical or the oblate spherical coordinates, etc., a particle in space may be characterized as an ordered triple of numbers called coordinates.

A constrained particle in motion on a surface requires two coordinates, and a constrained particle on a curve, requires one coordinate to characterize its location.

Given a system of *m* particles, 3*M* coordinates are required to describe the location of each particle. This is the configuration of the system. (If constraints are imposed on the system, fewer coordinates are required.)

A rigid body requires six coordinates—three for orientation and three for the reference point, to completely locate its position.

Chapter



#### **CLASSICAL MECHANICS**

Generalized coordinates—A set of coordinates,  $q_1, q_2, \ldots, q_m$ , equal to the number of degrees of freedom of the system.

If each  $q_i$  is independent of the others, then it is known as holonomic.

The rectangular coordinates for a particle expressed in generalized coordinates:

- x = x(q) Motion on a curve (one degree of freedom)
- $x = x(q_1, q_2)$  Motion on a surface (two degrees of freedom)  $y = y(q_1, q_2)$
- $x = x(q_1, q_2, q_3)$  Spatial motion (three degrees of freedom)  $y = y(q_1, q_2, q_3)$  $z = z(q_1, q_2, q_3)$

Small changes in coordinates:

$$\delta x = \frac{\delta x}{\delta q_1} \delta q_1 + \frac{\delta x}{\delta q_2} \delta q_2 + \frac{\delta x}{\delta q_3} \delta q_3$$
$$\delta y = \frac{\delta y}{\delta q_1} \delta q_1 + \frac{\delta y}{\delta q_2} \delta q_2 + \frac{\delta y}{\delta q_3} \delta q_3$$
$$\delta x = \frac{\partial z}{\partial q_1} \delta q_1 + \frac{\partial z}{\partial q_2} \delta q_2 + \frac{\partial z}{\partial q_3} \delta q_3$$

For a system of *m* particles in generalized coordinates:

$$\delta x_i = \frac{\partial x_i}{\partial q_k} \delta q_k \qquad k = 1, 2, \dots, m$$

$$1 < i < m$$

$$\delta y_i = \frac{\partial y_i}{\partial q_k} \delta q_k \qquad k = 1, 2, \dots, m$$

$$1 < i < m$$

$$\delta z_i = \frac{\partial z_i}{\partial q_k} \delta q_k \qquad k = 1, 2, \dots, m$$

$$1 < i < m$$

expressed in tensor notation.

Generalized forces:

Work done
$$\delta w = \mathbf{F} \cdot \delta \mathbf{r} = F_i \cdot \delta x_i$$
For one particle, $1 < i < 3$ and for *m* particles, $1 < i < 3m$ 

In terms of generalized coordinates:

$$\delta w = F_i \frac{\partial x_i}{\partial q_k} \delta q_k$$
$$\delta w = Q_k \delta q_k, \text{ where } Q_k = F_i \frac{\partial x_i}{\partial q_k}$$

or

and is known as the generalized force.

#### **Conservative Systems**

Forces expressed in terms of the potential energy function:

$$F_i = \frac{-\partial v}{\partial x_i}$$

where v is the potential energy.

In terms of the generalized force,

$$Q_k = -\frac{\partial v}{\partial x_i} \frac{\partial x_i}{\partial q_k} = -\frac{\partial v}{\partial q_k}$$

#### Lagrange's Equation

For a system, kinetic energy KE is

$$T = KE = \frac{1}{2}m_i x_i^{\prime 2} \quad i = 1, 2, \dots, 3M$$
$$x_1' = \frac{\partial x_i}{\partial q_k} q_k' + \frac{\partial x_i}{\partial t} \quad k = 1, 2, \dots, M$$

where

The Lagrange equation of motion using the equations above is:

$$\frac{d}{dt}\left(\frac{\partial T}{\partial q'_k}\right) = Q_k + \frac{\partial T}{\partial q_k} \quad k = 1, \ 2, \dots, M$$

or if the motion is conservative and if the potential energy is a function of generalized coordinates, then the equation becomes

$$\frac{d}{dt}\left(\frac{\partial T}{\partial q'_k}\right) = \frac{\partial T}{\partial q_k} - \frac{\partial v}{\partial q_k} \quad k = 1, \ 2, \dots, M$$

#### Lagrange's Function (L)

Chapter

L = T - V, where T and V are in terms of generalized coordinates.

Lagrange's Equation in Terms of L:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial q'_k}\right) = \frac{\partial L}{\partial q_k} \quad k = 1, \ 2, \dots, M$$

Lagrange's equation for nonconservative generalized forces:

If 
$$Q_k = Q' - rac{\partial v}{\partial q_k}$$

where Q' is nonconservative, then Lagrange's equation becomes

$$\frac{d}{dt}\frac{\partial L}{\partial q'_k} = Q'_k + \frac{\partial L}{\partial q_k}$$

and is useful, for example, when frictional forces are present.

General Procedure for Obtaining the Equation of Motion:

- A) Choose a coordinate system.
- B) Write the kinetic energy equation as a function of these coordinates.
- C) Find the potential energy, if the system is conservative.
- D) Combining these terms in Lagrange's equation results in the equation of motion.

#### Lagrange's Equations with Constraints

Holonomic Constraint—Constraints of the form

$$\frac{\partial q}{\partial q_k}\delta q = 0$$

Non-holonomic Constraint—Constraints of the form

$$h_k \delta q_k = 0$$

Differential equations of motion by the method of undetermined multipliers (the non-holonomic case):

Multiply the equation by a constant  $\lambda$  and add the result to the integrand of

$$\int_{t_a}^{t_b} \left[ \frac{\partial L}{\partial q_k} - \frac{d}{dt} \frac{\partial L}{\partial q'_k} \right] \delta q_k \, dt = 0$$

Select  $\lambda$  such that the terms in the brackets equal zero,

$$\frac{\partial L}{\partial q_k} - \frac{d}{dt} \frac{\partial L}{\partial q'_k} + \lambda h_k = 0 \ (k = 1, \ 2, \dots, M)$$
$$h_k q'_k = 0$$

There now exists m + 1 equations to obtain m + 1 unknowns, i.e.,  $(q_1, q_2, \ldots, q_n, \lambda)$ .

This technique may be employed with moving constraints or with several constraints by having corresponding undetermined coefficients with corresponding *h*'s in the Lagrangian equations.

#### **Hamiltonian Mechanics**

The Hamiltonian mechanics is a reformulation of Lagrangian mechanics.

The Hamiltonian equations are generally written as follows:

$$\dot{p} = -\frac{\partial H}{\partial q}$$

$$\dot{q} = \frac{\partial H}{\partial p}$$



where p = p(t) is the generalized momenta and q = q(t) is the generalized coordinate. "Dot" represents derivative with respect to time. H = H(p,q,t) is called Hamiltonian.

The above equation can be written as

$$\frac{d}{dt}p(t) = -\frac{\partial}{\partial q}H(p(t), q(t), t)$$
$$\frac{d}{dt}q(t) = \frac{\partial}{\partial p}H(p(t), q(t), t)$$

When the terms from both sides of the equation are associated, the equations above yield **Hamiltonian Equations**:

$$\frac{\partial H}{\partial q_i} = F_j - \dot{p}_j, \ \frac{\partial H}{\partial p_i} = \dot{q}_j, \frac{\partial H}{\partial t} = -\frac{\partial L}{\partial t}$$

The canonical forms of the above equations are

$$\frac{\partial H}{\partial q_i} = -\dot{p}_j, \ \frac{\partial H}{\partial p_i} = \dot{q}_j, \ \frac{\partial H}{\partial t} = -\frac{\partial L}{\partial t}$$

Basic physical interpretation: The Hamiltonian H represents the energy of the system, which is the sum of kinetic (T) and potential energy (V):

$$H = T + V, \ T = \frac{p^2}{2m}, \ V = V(q) = V(x)$$

here T is the function of p alone, and V is a function of x or q alone.

Four steps to applying the Hamiltonian Equation:

- 1. Start out with Lagrangian L = T V with T and V expressed for a Lagrangian equation.
- 2. From the derivative of the Lagrangian with respect to velocity, obtain the momentums.
- 3. Invert the expressions in Step 2 above to express velocities in terms of momentums in Step 2
- 4. Calculate the Hamiltonian using the usual definition. Substitute for the velocities using the results in Step 3. Apply Hamilton equations.

$$H = \sum_{i} p_i q_i - l$$

**Determination of density using Archimedes Principle**: Density of a solid or liquid can be found using the Archimedes' principle with any of the following equations:

W<sub>air</sub>

t Fbuoyant

W<sub>liquid</sub>

 $F_{buoyant} = W_{air} - W_{liquid}$ 

$$\rho_s = \frac{W_{air}}{W_{air} - W_{liauid}} \rho_L$$

or

$$\rho_s = \frac{W_{air}}{F_{bouyant}} \rho_L$$

or

$$\rho_s = \frac{W_{air}}{\text{Weight of liquid displaced}}\rho_L$$

 $\mathbf{W}_{air}$  = weight of the body in air  $\mathbf{W}_{liquid}$  = weight of the body when it is immersed *completely* in a liquid  $\rho_{L}$  = density of the liquid

#### **Buoyancy on a Floating Body:**

For a body floating in a liquid



The weight of the floating body = the weight of the liquid displaced = the buoyant force on the floating body



Chapter

#### **CLASSICAL MECHANICS**

#### **Density of a Floating Body:**

Density of a floating body =  $\frac{V_i}{V}\rho_L$ 

 $V_i$  = Volume of the floating body inside the liquid V = Total volume of the floating body  $\rho_L$  = Density of the liquid

#### **Fluid Dynamics**

It is assumed here that the fluid is incompressible, nonviscous, and that any flow of the liquid is streamlined (irrotational).

**Continuity equation**: Let  $P_1$  and  $P_2$  be any two points in a liquid flowing through a pipe of a varying cross-sectional area. If the density of the liquid is constant, then



 $A_1V_1 = A_2V_2$ 

where  $A_1 = \text{area of cross section at point P}_1$   $v_1 = \text{velocity of the liquid at point P}_1$   $A_2 = \text{area of cross section at point P}_2$  $v_2 = \text{velocity of the liquid at point P}_2$ 

Rate of flow of liquid:

Volume flow rate through a pipe = Av  $(m^3/s)$ 

Mass flow rate through a pipe =  $\rho Av$  (kg/s)

**Bernoulli's Equation**: Let  $p_1$  and  $p_2$  be any two points in a streamlined flow of a liquid of density  $\rho$ . Then at these two points

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2$$



where  $P_1$  and  $P_2$  = the pressures at the points  $p_1$  and  $p_2$   $\rho$  = the density of the liquid  $y_1$  and  $y_2$  = the heights of points  $p_1$  and  $p_2$  above an arbitrary reference level  $v_1$  and  $v_2$  = the velocities of the fluid at points  $p_1$  and  $p_2$ 

Bernoulli's equation for horizontal flow:

If the points  $P_1$  and  $P_2$  are at the same height,  $y_1 = y_2$ . Hence, the Bernoulli's equation above reduces to

$$\mathbf{p}_1 + \frac{1}{2}\rho \mathbf{v}_1^2 = \mathbf{p}_2 + \frac{1}{2}\rho \mathbf{v}_2^2$$

Flow of liquid through a hole in a container:



A container with a large area of cross section is filled with a liquid. There is a small hole at distance **h** below the liquid level. The velocity of water through the hole is given by

 $v = \sqrt{2gh}$ 

### CHAPTER

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# Electromagnetism

### A. ELECTRIC FIELDS

#### **Definition of an Electric Field**

$$\mathbf{E} = \frac{\mathbf{F}}{q_0} \qquad \text{Units}: \frac{N}{\text{coul}}$$

where  $\mathbf{E} = \text{electric field}$  $\mathbf{F} = \text{electric force}$  $q_0 = \text{positive test charge}$ 

#### 1. Coulomb's Law

By definition, the force between two point charges of arbitrary positive or negative strengths is given by Coulomb's law as follows:

$$F = \frac{Q_1 Q_2}{4\pi\varepsilon_0 d^2}$$

where  $Q_1$  and  $Q_2$  = positive or negative charges on either object in coulombs d = distance separating the two point charges  $\varepsilon_0$  = permittivity in free space = 8.854 × 10<sup>-12</sup> F/m

**NOTE**:  $\varepsilon = \varepsilon_0 \varepsilon_r$  for media other than free space, where  $\varepsilon_r$  is the relative permittivity of the media.



#### **ELECTROMAGNETISM**

The force F can be expressed in vector form to indicate its direction as follows:

$$\mathbf{F} = \frac{Q_1 Q_2}{4\pi\varepsilon_0 d^2} \mathbf{a}_d$$

The unit vector  $\mathbf{a}_d$  is in the direction of *d*:

$$\mathbf{a}_d = \frac{\mathbf{d}}{|\mathbf{d}|} = \frac{\mathbf{d}}{d}$$

Naturally,  $Q_1$  and  $Q_2$  can each be either positive or negative. As a consequence, the resultant force can be either positive (repulsive) or negative (attractive).

#### 2. Gauss's Law

Gauss's law states that the net electric flux passing out of a closed surface is equal to the total charge within such surface.

Hence, since

$$d\psi = \mathbf{D} \cdot d\mathbf{s}$$
$$\psi = \int \mathbf{D} \cdot d\mathbf{s}$$

and by Gauss's law,

$$\psi_{\rm net} = \oint_s \mathbf{D} \cdot d\mathbf{s} = Q_s$$

where  $Q_s$  is the total number of charges enclosed by the surface.

#### **Application of Gauss's Law**

The following spherical surface is **chosen** to enclose a given charge to be determined:  $Q_s$ 





Applying Gauss's law:

The charge  $Q_s$  enclosed by the spherical surface is

$$Q_s = \oint_s \mathbf{D} \cdot \mathbf{ds}$$

where ds in this case is equal to  $4\pi r^2$  (**NOTE**: r is the radius of the sphere). Hence,

$$Q_{\rm s} = D4\pi r^2$$

and

$$\mathbf{D} = \frac{Q_s}{4\pi r^2} \cdot \mathbf{a}_r$$

Since electric field intensity E is equal to

$$\frac{Q}{4\pi\varepsilon_0 d^2}a_d$$

and d is equal to r in this case, then  $\mathbf{D} = \varepsilon_0 \mathbf{E}$ .

Some hints for choosing a special Gaussian surface:

- A) The surface must be closed.
- B) D remains constant through the surface and normal to the surface.
- C) D is either tangential or normal to the surface at any point on the surface.

It is easier in solving a problem if we can choose a special Gaussian surface. In other words, this surface should be chosen to conform to the flux at any given point on the closed surface about the charge.

#### 3. Electric Potential, Energy, and Work

Electric Potential Difference:

$$V_B - V_A = \frac{W_{AB}}{q_0} \rightarrow \text{units: Volts}$$

where

 $V_B$  = electric potential at point *B*  $V_A$  = electric potential at point *A*  $W_{AB}$  = work done by external force  $q_0$  = electrical test charge



More generally:

The potential difference between two points p and p', symbolized as  $V_{p'p}$  (or  $\phi p'p$ ) is defined as the work done in moving a unit positive charge by an external force from the initial point p to the final point p'.

$$V_{p'p} = -\int_{p}^{p'} \mathbf{E} \cdot d\mathbf{L} = V_{p'} - V_{p}$$

The unit for potential difference is the volt (V) which is Joules/coulomb.

#### **B. CAPACITORS**

The capacitance of two oppositely charged conductors in a uniform dielectric medium is

$$C = \frac{Q}{V_0}$$
 Units: Farads  $= \frac{\text{coul}}{v}$ 

where Q = the total charge in *either* conductor  $V_0$  = the potential difference between the two conductors

#### **Example:**

Capacitance of the parallel-plate capacitor:



$$\mathbf{E} = \frac{\boldsymbol{\rho}_s}{\boldsymbol{\varepsilon}} \mathbf{a}_z$$

 $\varepsilon$  is the permittivity of the homogeneous dielectric:

$$\mathbf{D} = \boldsymbol{\rho}_{\mathrm{s}} \cdot \mathbf{a}_{z}$$

On the lower plate:

$$D_n = D_z = \rho_S$$

 $D_n$  is the normal value of **D**.

On the upper plate:

$$D_n = -D_z$$

$$V_0$$
 = the potential difference

$$= -\int_{\text{upper}}^{\text{lower}} \mathbf{E} \cdot d\mathbf{L}$$
$$= -\int_{d}^{0} \frac{\rho_{s}}{\varepsilon} d_{z} = \frac{\rho_{s} d}{\varepsilon}$$
$$C = \frac{Q}{V_{0}} = \frac{\varepsilon s}{d}$$
$$Q = \rho_{s} S \text{ and } V_{0} = \frac{\rho_{s} d}{\varepsilon}$$

considering conductor planes of area S are of linear dimensions much greater than d.

Total energy stored in the capacitor:

$$W_E = \frac{1}{2} \int_{\text{vol}} \varepsilon E^2 dv = \frac{1}{2} \int_0^S \int_0^d \frac{\varepsilon \rho_s}{\varepsilon^2} dz \, ds$$
$$W_E = \frac{1}{2} C V_0^2 = \frac{1}{2} Q V_0 = \frac{1}{2} \frac{Q^2}{C}$$

#### **Multiple Dielectric Capacitors**



A parallel-plate capacitor containing two dielectrics with the dielectric interface parallel to the conducting plates;  $C = 1/\{(d_1/\epsilon_1 S) + (d_2/\epsilon_2 S)\}$ .

$$C = \frac{1}{\left(\frac{1}{C_1}\right) + \left(\frac{1}{C_2}\right)}$$

where

$$C_1 = \frac{\varepsilon_1 S}{d_1}$$

 $C_1 = \frac{\varepsilon_1 S}{d_1}$ 

 $V_0$  = a potential difference between the plates =  $E_1d_1 + E_2d_2$ 

$$E_1 = \frac{V_0}{d_1} + \left(\frac{\varepsilon_1}{\varepsilon_2}\right) d_2$$

 $\rho_{s_1}$  = the surface charge density =  $D_1 = \varepsilon_1 E_1$ 

$$= \frac{V_0}{\left(\frac{d_1}{\varepsilon_1}\right) + \left(\frac{d_2}{\varepsilon_2}\right)} = D_2$$

$$C = \frac{Q}{V_0} = \frac{\rho_s \cdot S}{V_0} = \frac{1}{\left(\frac{d_1}{\varepsilon_1 s}\right) + \left(\frac{d_2}{\varepsilon_2 s}\right)} = \frac{1}{\left(\frac{1}{C_1}\right) + \left(\frac{1}{C_2}\right)}$$

#### **C. CURRENT AND RESISTANCE**

#### Definitions

#### **Current:**

$$i = \frac{dq}{dt}$$
 amperes

where i = electric current q = net charge

$$t = time$$

#### **Current Density and Current:**

$$\mathbf{j} = \frac{i}{a}$$
 Amperes/m<sup>2</sup>

where  $\mathbf{j} = \text{current density}$ 

$$i = current$$

A =cross-sectional area

#### **Mean Drift Speed:**

$$\mathbf{v}_D = \frac{\mathbf{j}}{ne}$$



 $\mathbf{v}_{\mathrm{D}} = \text{mean drift speed} \\ \mathbf{j} = \text{current density}$ where

n = number of atoms per unit volume

#### **Resistance:**

$$R = \frac{V}{i} \text{ Ohms } (\Omega)$$

where R = resistanceV = potential difference i = current

#### **Resistivity:**

$$\rho = \frac{E}{\mathbf{j}} \text{ Ohms-meters}(\Omega \mathbf{m})$$

where  $\rho = \text{resistivity}$ 

E = electric field

 $\mathbf{j}$  = current density

#### **Power:**

$$P = VI = I^2 R = \frac{V^2}{R} \text{Watts (w)}$$

where P = powerI = currentV = potential difference R = resistance

#### **D. CIRCUITS**

#### Electromotive Force, $EMF(\varepsilon)$

$$\varepsilon = \frac{dw}{dq}$$

where  $\varepsilon$  = electromotive force

w = work done on charge

q = electric charge



#### **Current in a Simple Circuit**

$$i = \frac{\varepsilon}{R}$$

where i = current $\varepsilon$  = electromotive force R = resistance

#### Resistances



 $R_{\text{Total}} = (R_1 + R_2 + R_3) \Omega$  (in series)



$$\frac{i}{R_{\text{Total}}} = \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right) \text{(in parallel)}$$

#### **The Loop Theorem**

$$\Delta V_1 + \Delta V_2 + \Delta V_3 \dots = 0$$

For a complete circuit loop

#### **Example:**



Simple Circuit with Resistor

$$V_{ab} = \varepsilon - iR = + ir$$
$$\varepsilon - iR - ir = 0$$

Then

#### CIRCUITS

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**NOTE**: If a resistor is traversed in the direction of the current, the voltage change is represented as a voltage drop, -iR. A change in voltage while traversing the *EMF* (or battery) in the direction of the *EMF* is a voltage rise  $+\varepsilon$ .

#### **Circuit with Several Loops**

 $\sum_n i_n = 0$ 

**Example:** 



#### **RC Circuits (Resistors and Capacitors)**

RC charging and discharging

#### **Differential Equations**





#### **Charge in the Capacitor**

 $q = (C\varepsilon) \left(1 - e^{\frac{-t}{RC}}\right) \text{ (charging)}$  $q = (C\varepsilon)e^{\frac{-t}{RC}} \text{ (discharging)}$ 



#### **Current in the Resistor**

$$i = \left(\frac{\varepsilon}{R}\right) e^{\frac{-t}{RC}} \text{(charging)}$$
$$i = -\left(\frac{\varepsilon}{R}\right) e^{\frac{-t}{RC}} \text{(discharging)}$$

where e = 2.71828 (exponential constant)

#### **Kirchhoff's Current Law**

The algebraic sum of all currents entering a node equals the algebraic sum of all currents leaving it.

$$\sum_{n=1}^{N} i_n = 0$$

#### Kirchhoff's Voltage Law (Same as Loop Theorem)

The algebraic sum of all voltages around a closed loop is zero.

#### **Thevenin's Theorem**

In any linear network, it is possible to replace everything except the load resistor by an equivalent circuit containing only a single voltage source in series with a resistor ( $R_{th}$  Thevenin resistance), where the response measured at the load resistor will not be affected.



#### **Procedures to Find Thevenin Equivalent**

1) Solve for the open circuit voltage  $V_{oc}$  across the output terminals.

$$V_{oc} = V_{th}$$

2) Place this voltage  $V_{oc}$  in series with the Thevenin resistance, which is the resistance across the terminals found by setting all independent voltage and current sources to zero (i.e., short circuits and open circuits, respectively).

#### **RLC Circuits and Oscillations**

These oscillations are analogous to, and mathematically identical to, the case of mechanical harmonic motion in its various forms. (AC current is sinusoidal.)

#### CIRCUITS



#### Simple RL and RC Circuits

#### **Source Free RL Circuit**



**Properties:** Assume initially  $i(0) = I_0$ .

- A)  $v_R + v_L = Ri + L\frac{di}{dt} = 0$
- B)  $i(t) = I_0 e^{-Rt/L} = I_0 e^{-t/\tau}$ ,  $\tau = \text{time constant} = \frac{L}{R}$
- C) Power dissipated in the resistor =

$$P_R = i^2 R = I_0^2 R e^{-2Rt/L}$$

D) Total energy in terms of heat in the resistor =  $W_R = \frac{1}{2}LI_0^2$ 

#### **Source Free RC Circuit**



**Properties:** Assume initially  $v(0) = V_0$ 

A) 
$$C = \frac{dv}{dt} + \frac{v}{R} = 0$$
  
B) 
$$v(t) = v(0)e^{-t/RC} = V_0 e^{-t/RC}$$
  
C) 
$$\frac{1}{C} \int_{-\infty}^{t} i(d\tau) + i(t)R = 0$$
  

$$i(t) = i(0)e^{\frac{-t}{RC}}$$



#### **The RLC Circuits**

#### Parallel RLC Circuit (Source Free)

Circuit Diagram:



KCL equation for parallel RLC circuit:

$$\frac{v}{R} + \frac{1}{L} \int_{t_0}^t v \, dt - i(t_0) + C \, \frac{dv}{dt} = 0;$$

and the corresponding linear, second-order homogeneous differential equation is

$$C \frac{d^2v}{dt^2} + \frac{i}{T}\frac{dv}{dt} + \frac{v}{L} = 0$$

General Solution:

$$V = A_1 e^{S_1 t} + A_2 e^{S_2 t}$$

where

$$S_{1,2} = \frac{-1}{2RC} \pm \sqrt{\left(\frac{1}{2RC}\right)^2 - \frac{1}{LC}}$$
$$S_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2};$$

or

where  $\alpha$  = exponential damping coefficient neper frequency

$$=\frac{1}{2RC}$$

and  $\omega_0 =$  resonant frequency

$$=\frac{1}{\sqrt{LC}}$$



#### **Complete Response of RLC Circuit**

The general equation of a complete response of a second order system in terms of voltage for an RLC circuit is given by

$$v(t) = V_f + Ae^{S_I t} + Be^{S_2 t}$$
forced
response
natural response

(i.e., constant for DC excitation)

**NOTE:** *A* and *B* can be obtained by

- 1) Substituting *v* at  $t = 0^+$
- 2) Taking the derivative of the response, i.e.,

$$\frac{dv}{dt} = 0 + S_1 A e^{S_1 t} + S_2 B e^{S_2 t}$$
$$\frac{dv}{dt} \text{ at } t = 0^+ \text{ is known}$$

where

#### **E. MAGNETISM**

Force in a magnetic field

$$\mathbf{F}_b = q\mathbf{v} \times \mathbf{B}$$

where  $\mathbf{F}_{b} =$  force on particle due to field

 $\ddot{q}$  = charge on particle

 $\mathbf{v} =$  velocity of particle

 $\mathbf{B}$  = magnetic field

#### The Biot-Savart Law

$$dB = \frac{\mu_{0i}}{4\pi} \frac{\sin\theta dl}{r^2}$$

where  $\mathbf{B} = \text{magnetic field}$ 

 $\mu_0$  = permeability constant

i =current through a wire

l =length of wire

r = distance from assumed point charge to a point in the magnetic field

 $\theta$  = angle between *r* and the direction of the element

NOTE:

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$$\mu_0 = 4\pi \times 10^{-7} \frac{T \cdot m}{A}$$

Integral form:

$$\mathbf{B} = \frac{\mu_0 i}{4\pi} \oint_c \frac{d\mathbf{l}' \times \mathbf{a}_R}{r^2}$$

where primed terms refer to points along the source of the field.

#### Ampere's Law

The line integral of the tangential component of  $\mathbf{B}$  is exactly equal to the current enclosed by that path.

$$\oint \mathbf{B} \cdot d\mathbf{L} = I$$

#### **Curl of a Vector Field**

The curl of any vector is defined as a vector where the direction is given by the right-hand rule and the magnitude is given by the limit of the quotient of the closed line integral and the area of the enclosed path as the area approaches 0.

$$(\operatorname{curl} \mathbf{H})_n = \lim_{\Delta s_n \to 0} \frac{\oint \mathbf{H} \cdot d\mathbf{L}}{\Delta s_n}$$

 $\Delta s_n$  is the area enclosed by the closed line integral, and *n* is any component; this is normal to the surface enclosed by the closed path.

#### Stokes' Theorem

$$\oint_{l} \mathbf{F} \cdot d\mathbf{L} = \int_{S} (\nabla \times \mathbf{F}) \cdot d\mathbf{s}$$

**F** is any vector field; s is a surface bounded by l. It gives the relation between a closed line integral and surface integral.

By using the Divergence Theorem and Stokes' theorem, we can derive a very important identity:

$$\nabla \cdot \nabla \times \mathbf{A} \equiv \mathbf{0}$$

where  $\mathbf{A} =$ any vector field.

#### **Magnetic Flux and Magnetic Flux Density**

 $B = \mu_0 H$ , B is the magnetic flux density in free space.

Unit of B is webers per square meter  $(wb/m^2)$  or Tesla (T) a new unit.

 $\mu_0 = 4\pi \times 10^{-7}$  H/m (permeabiilty of free space)

*H* is in amperes per meter (A/m).

$$\oint_{S} \mathbf{B} \cdot d\mathbf{s} = 0$$

This is Gauss's law for the magnetic field

$$\nabla \cdot \mathbf{B} = 0$$

after application of the divergence theorem. This is the fourth and last equation of Maxwell.

#### Faraday's Law

Faraday's law can be stated as follows:

$$\operatorname{emf} = \oint \mathbf{E} \cdot d\mathbf{L} = -\frac{d\phi}{dt}(v)$$

The minus sign is by Lenz's law which indicates that the induced e.m.f. is always acting against the changing magnetic fields which produce that e.m.f.

Faraday's law describes the relationship between electric and magnetic fields.



#### **ELECTROMAGNETISM**

#### Inductance (L)

 $L = \frac{N\phi}{I} = \frac{\text{Total flux linkage}}{\text{Current link}}$ 

Unit of inductance is H, which is equivalent to wb/A.

#### Applications

Inductance per meter length of a coaxial cable of inner radius *a* and outer radius *b*.

$$L = \frac{\mu_0}{2\pi} \ln \frac{b}{a} \text{ H/m}$$

A toroidal coil of N turns and IA,

$$L = \frac{\mu_0 N^2 s}{2\pi R}$$

where R = mean radius of the toroid

Different expressions for inductance:

$$L = \frac{2W_H}{I^2}$$

$$L = \frac{\int_{\text{vol}} \mathbf{B} \cdot \mathbf{H} \, dv}{I^2}$$

$$= \frac{1}{I^2} \int_{\text{vol}} \mathbf{H} \cdot (\nabla \cdot \mathbf{A}) dv$$

$$L = \frac{1}{I^2} \left[ \int_{\text{vol}} \nabla \cdot (\mathbf{A} \cdot \mathbf{H}) dv + \int_{\text{vol}} \mathbf{A} \cdot (\nabla \cdot \mathbf{H}) dv \right]$$

$$L = \frac{1}{I^2} \int_{\text{vol}} \mathbf{A} \cdot \mathbf{J} \, dv$$

$$L = \frac{1}{I^2} \int_{\text{vol}} \left( \int_{\text{vol}} \frac{\mu \mathbf{J}}{4\pi R} dv \right) \cdot \mathbf{J} \, dv$$

Mutual inductance between circuits 1 and 2:

$$L_{12} = \frac{N_2 \cdot \phi_{12}}{I_1}$$

where N = the number of turns

$$L_{12} = \frac{1}{I_1 I_2} \int_{\text{vol}} (\mu \mathbf{H}_1 \times \mathbf{H}_2) dv$$
$$L_{12} = L_{21}$$

#### F. MAXWELL'S EQUATIONS AND ELECTROMAGNETIC WAVES

#### **Maxwell's Equations**

Maxwell's equation in differential form:

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$
$$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}$$
$$\nabla \cdot \mathbf{D} = \rho$$
$$\nabla \cdot \mathbf{B} = 0$$

Auxiliary equations relating **D** and **E**:

$$D = \varepsilon E \qquad D = \varepsilon_0 E + P$$
$$B = \mu H \qquad B = \mu_0 (H + M)$$
$$J = \sigma E \qquad J = \rho U$$

Lorentz force equation:

$$\mathbf{F} = \rho(\mathbf{E} + \mathbf{U} \times \mathbf{B})$$

Maxwell's equations in integral form:

$$\oint \mathbf{E} \cdot d\mathbf{L} = -\int_{s} \frac{\partial \mathbf{B}}{\partial t} \cdot d\mathbf{s}$$
$$\oint \mathbf{H} \cdot d\mathbf{L} = I + \int_{s} \frac{\partial \mathbf{D}}{\partial t} \cdot d\mathbf{s}$$
$$\oint_{s} \mathbf{D} \cdot d\mathbf{s} = \int_{\text{vol}} \rho \, dv$$
$$\oint \mathbf{B} \cdot ds = 0$$

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#### **ELECTROMAGNETISM**

These four integral equations enable us to find the boundary conditions on **B**, **D**, **H** and **E** which are necessary to evaluate the constants obtained in solving Maxwell's equations in partial differential form.

#### **Electromagnetic Waves**

Maxwell's equations in phasor form:

$$\nabla \times \mathbf{H}_{s} = j\omega\varepsilon_{0}\mathbf{E}_{s}$$
$$\nabla \times \mathbf{E}_{s} = -j\omega\mu_{0}\mathbf{H}_{s}$$
$$\nabla \cdot \mathbf{E}_{s} = 0$$
$$\nabla \cdot \mathbf{H}_{s} = 0$$

Wave equations:

$$\nabla \times \nabla \times \mathbf{E}_{s} = \nabla (\nabla \cdot \mathbf{E}_{s}) - \nabla^{2} \mathbf{E}_{s} = -j\omega\mu_{0}\nabla \times \mathbf{H}_{s}$$
$$= \omega^{2}\mu_{0}\varepsilon_{0}\mathbf{E}_{s} = -\nabla^{2}\mathbf{E}_{s}$$
$$\nabla^{2}\mathbf{E}_{s} = -\omega^{2}\mu_{0}\varepsilon_{0}\mathbf{E}_{s}$$
$$\nabla^{2} E_{xs} = \frac{\partial^{2}E_{xs}}{\partial x^{2}} + \frac{\partial^{2}E_{xs}}{\partial y^{2}} + \frac{\partial^{2}E_{xs}}{\partial z^{2}}$$
$$= -\omega^{2}\mu_{0}\varepsilon_{0} E_{xs}$$

For

$$\frac{\partial^2 \mathbf{E}_{xs}}{\partial y^2} = \frac{\partial^2 \mathbf{E}_{xs}}{\partial z^2} = 0$$

i.e.,  $E_{xs}$  is independent of x and y.

This can be simplified to

$$\frac{\partial^2 \mathbf{E}_{xs}}{\partial z^2} = -\omega^2 \mu_0 \varepsilon_0 E_{xs}$$
$$E_x = E_{x_0} \cos \left[\omega \left(t - z \sqrt{\mu_0 \varepsilon_0}\right)\right]$$
$$E_x = E_{x_0'} \cos \left[\omega \left(t + z \sqrt{\mu_0 \varepsilon_0}\right)\right]$$

and

$$E_{x0}$$
 = value of  $E_x$  at  $z = 0, t = 0$ 

Velocity of the traveling wave:

To find the velocity U, let us keep the value of  $E_x$  to be constant; therefore,

$$t - z \sqrt{\mu_0 \varepsilon_0} = \text{ constant}$$

Take differentials; we have

$$dt - \frac{1}{U}dz = 0$$
$$\frac{dz}{dt} = U$$

in free space.

Velocity of light  $= U = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = 3 \times 10^8$  m/s Wave length  $= \lambda = \frac{U}{f} = \frac{2\pi U}{\omega}$ 

The field is moving in the Z direction with velocity U. It is called a traveling wave.

Form of the *H* field:

If  $\mathbf{E}_s$  is given,  $\mathbf{H}_s$  can be obtained from

$$\nabla \times \mathbf{E}_{s} = -j\omega\mu_{0}\varepsilon_{0}\mathbf{H}_{s}$$
$$\frac{\partial E^{xs}}{\partial z} = -j\omega\mu_{0}\mathbf{H}_{ys}$$
$$= E_{x_{0}}\left(-j\omega\sqrt{\mu_{0}\varepsilon_{0}}\right)e^{-j\omega\sqrt{\mu_{0}\varepsilon_{0}z}}$$
$$H_{y} = E_{x_{0}}\sqrt{\frac{\varepsilon_{0}}{\mu_{0}}\cos\left[\omega\left(t-z\sqrt{\mu_{0}\varepsilon_{0}}\right)\right]}$$
$$\frac{E_{x}}{H_{y}} = \frac{\mu_{0}}{\varepsilon_{0}}$$

is a constant where

 $\eta = \sqrt{rac{\mu}{arepsilon}}$ 



where  $\eta$  = The intrinsic impedance: It is the square root of the ratio of permeability to permittivity and is measured in  $\Omega$ .

$$\eta_0 = \sqrt{\frac{\mu_0}{\epsilon_0}} = 377\Omega$$
  
 $\eta_0 = \eta$  of free space

The term *uniform plane wave* is used because the H and E fields are uniform throughout any plane, Z = constant, and it is also called a transverse electromagnetic (TEM) wave since both the E and H fields are perpendicular to the direction of propagation.





#### **Atoms in Electric and Magnetic Fields**

If one or more electrons are added to or removed from an atom, it can turn into a negative or positive ion, respectively. If the ions are traveling through a region that contains electric field E and magnetic field B, the force acting on the ion is given by the Lorentz force equation:

$$\mathbf{F} = q[\mathbf{E} + (\mathbf{v} \times \mathbf{B})],$$

## CHAPTER

# **Atomic Physics**

#### **A. RUTHERFORD SCATTERING**



 $\theta$  is the scattering angle and *b* is the impact parameter.

In this collision, it is assumed that the nucleus has sufficient mass so as to not be moved by the alpha particle. In this case, the energy of the alpha particle stays the same. It follows that the **magnitude** of the momentum remains constant:

$$P_1 = P_2 = mv$$

Therefore, by the Law of Sines:

$$\frac{\Delta p}{\sin \theta} = \frac{mv}{\sin \frac{\pi - \theta}{2}}$$

Solving for the change in momentum:

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$$\Delta p = 2mv \, \sin\left(\frac{\theta}{2}\right)$$

It can also be shown that the scattering angle  $\theta$  can be determined by the equation:

$$\cot\frac{\theta}{2} = \frac{4\pi\varepsilon_0 k}{Z_n e^2} b$$

where k = the alpha particle energy

#### **B. ATOMIC SPECTRA**

When an atomic gas is excited, it emits radiation at certain specific wavelengths. This produces the gas **emission line spectrum**.

When white light (all wavelengths) is passed through a gas, the gas will absorb certain specific wavelengths of the light. This produces the **absorption line spectrum**.

Wavelengths in atomic spectra fall into **spectral series**. There are five such series and five similar equations:

Lyman Series

$$\frac{1}{\lambda} = R\left(\frac{1}{1^2} - \frac{1}{n^2}\right)$$
  $n = 2, 3, 4, \dots$ 

**Balmer Series** 

$$\frac{1}{\lambda} = R\left(\frac{1}{2^2} - \frac{1}{n^2}\right)$$
  $n = 3, 4, 5, \dots$ 

**Paschen Series** 

$$\frac{1}{\lambda} = R\left(\frac{1}{3^2} - \frac{1}{n^2}\right)$$
  $n = 4, 5, 6, \dots$ 

#### **Brackett Series**

$$\frac{1}{\lambda} = R\left(\frac{1}{4^2} - \frac{1}{n^2}\right)$$
  $n = 5, 6, 7, \dots$ 

#### **Pfund Series**

$$\frac{1}{\lambda} = R\left(\frac{1}{5^2} - \frac{1}{n^2}\right)$$
  $n = 6, 7, 8, \dots$ 

In general, series k is

$$\frac{1}{\lambda} = R\left(\frac{1}{k^2} - \frac{1}{n^2}\right) \qquad n = k+1, \ k+2, \ k+3, \dots$$

*R* is known as the Rydberg constant:

$$R = 1.097 \times 10^7 \,\mathrm{m}^{-1}$$

#### **C. THE BOHR ATOM**

Classical physics is not adequate in describing the atom, due to the fact that in order to resist falling into the nucleus, an electron would have to whirl rapidly around the nucleus using the pull as centripetal acceleration. This motion would cause the electron to radiate electromagnetic energy, thereby rendering the system unstable. A quantum solution must therefore be sought.

#### Hydrogen Atom (1 Electron)

If viewed classically, the required electron velocity for stability, not considering electromagnetic generation, would be

$$v = \frac{c}{\sqrt{4\pi\varepsilon_0 mr}}$$

The DeBroglie wavelength for any object in motion is

$$\lambda = \frac{h}{mv}$$

. . Orbital electron wavelength

$$\lambda = \frac{h}{e} \sqrt{\frac{4\pi\varepsilon_0 r}{m}}$$

The conditions for stable orbit can be described as the situation when the circumference of the orbit contains an integral number of DeBroglie wavelengths.

#### **ATOMIC PHYSICS**

A condition for orbit stability

$$n\lambda = 2\pi r_n$$
  $n = 1, 2, 3, \ldots$ 

with possible radii

$$r_n = \frac{n^2 b^2 \varepsilon_0}{\pi m e^2} \quad n = 1, 2, 3, \dots$$

The aforementioned spectra correspond to differing discrete energy levels, one level for each electron radius  $r_n$ . Using the theory of the Bohr atom, an explanation of spectral series can be found.

$$E_n = -\frac{e^2}{8\pi\varepsilon_0 r_n} \left( E = \frac{1}{2}mv^2 + \frac{kqq}{r} \right)$$

Substituting  $r_n$  it can be shown that

$$E_1 = -\frac{me^4}{8\varepsilon_0^2 h^2}$$

and that

$$E_n = \frac{E_1}{n^2}$$

The Bohr model can be used to derive accurately the value of the Rydberg constant.

Note, however, that the Bohr theory is not in fact a complete and accurate description of what is occurring, and is limited in its application. For a more complete and accurate picture, quantum mechanics must be used.

#### **Energy Levels and Emission and Absorption of Photons**

According to the Bohr's model of H-atom, there are certain discrete orbits around the nucleus in which an electron does not radiate energy even though it is accelerating. These orbits are given quantum numbers n = 1, 2, 3... in increasing order of the size of the orbits. An atom has certain energy associated with the electron in such an orbit. Hence, different stationary orbits correspond to different energy levels for an atom.
The energy of the H-atom in any orbit is given by

$$E = \frac{-13.6}{n^2} eV$$

The orbit closest to the nucleus (smallest orbit) is said to be the ground state of the atom and has the lowest possible energy state. Theoretically, there are an infinite number of orbits around a nucleus; as one moves to higher orbits, the energy level increases.



Energy Level Diagram for H-Atom. (only six levels are shown)

#### **Emission of Radiation by an Atom**

When an electron is bumped to a higher orbit, the atom is said to be in an excited state. The electron later returns to the ground state directly or through one or more stationary orbits to the ground state. As the electron descends to lower energy states, the difference of energy between the two levels is radiated out as a photon. Thus,

$$hf = (E_i - E_f)$$

The diagram on the next page shows an excited atom of hydrogen in which an electron has been raised (by heating or passing an electric discharge through hydrogen gas) to the n = 3 state.

One can calculate the three possible wavelengths emitted by the hydrogen atoms in n = 3 excited states as on the next page.

# ATOMIC PHYSICS $E_{inf} = 0 \text{ eV}$ Continuumn = infinity $E_5 = -0.54 \text{ eV}$ n = 5 $E_4 = -0.85 \text{ eV}$ n = 4



For  $n = 3 \rightarrow n = 2$  transition:

Chapter

 $hf_1 = E_3 - E_2 \rightarrow 6.63 \times 10^{-34} f_1 = (-1.51) - (-3.4) = 1.89 \text{ eV} = 3.024 \times 10^{-19} \text{ J} \rightarrow f_1 = (3.024 \times 10^{-19}) / (6.63 \times 10^{-34}) = 4.56 \times 10^{14} \text{ Hz} \rightarrow \lambda_1 = c/f_1 = (3 \times 10^8) / (4.56 \times 10^{14}) \rightarrow \lambda_1 = 6.577 \times 10^{-7} \text{ m} = 657.7 \text{ nm}$ 

For  $n = 2 \rightarrow n = 1$  transition:

$$\begin{split} \mathrm{hf}_2 &= \mathrm{E}_2 - \mathrm{E}_1 \rightarrow 6.63 \times 10^{-34} \, \mathrm{f}_2 = (-3.4) - (-13.6) = 10.2 \, \mathrm{eV} = 16.32 \times 10^{-19} \, \mathrm{J} \rightarrow \mathrm{f}_2 = (16.32 \times 10^{-19}) / \\ (6.63 \times 10^{-34}) &= 2.46 \times 10^{15} \, \mathrm{Hz} \rightarrow \lambda_2 = \mathrm{c/f}_2 = (3 \times 10^8) / (4.56 \times 10^{15}) \rightarrow \lambda_2 = 1.219 \times 10^{-7} \, \mathrm{m} = 121.9 \, \mathrm{nm} \end{split}$$

For  $n = 3 \rightarrow n = 1$  transition:

 $\begin{array}{l} \mathrm{hf}_3 \,=\, \mathrm{E}_3 \,-\, \mathrm{E}_1 \,\rightarrow\, 6.63 \,\times\, 10^{-34} \,\, \mathrm{f}_3 \,=\, (-1.51) \,-\, (-13.6) \,=\, 12.09 \,\, \mathrm{eV} \,=\, 19.344 \,\times\, 10^{-19} \mathrm{J} \\ \rightarrow\, \mathrm{f}_3 \,=\, (19.344 \,\times\, 10^{-19}) \,/\, (6.63 \,\times\, 10^{-34}) \,=\, 2.918 \,\times\, 10^{15} \,\, \mathrm{Hz} \rightarrow \lambda_3 \,=\, \mathrm{c/f}_3 \,=\, (3 \,\times\, 10^8) / (2.918 \,\times\, 10^{15}) \\ \rightarrow\, \lambda_3 \,=\, \mathbf{1.028} \,\times\, \mathbf{10^{-7}} \,\, \mathbf{m} \,=\, \mathbf{102.8} \,\, \mathbf{nm} \end{array}$ 

#### Absorption of Radiation by an Atom

A reverse process of emission of photons is the absorption of an incident photon by an atom. For absorption to take place, the energy of the photon must be exactly equal to the energy difference between the two levels through which an electron goes up. If the energy of the photon is *more or less* than the energy difference between the two levels, the photon will pass through the gas without getting absorbed.

#### Atomic Structure—Electronic Configuration

Atomic #	Name	Symbol	<b>Electronic Config</b>
1	Hydrogen	Н	1s <sup>1</sup>
2	Helium	Не	1s <sup>2</sup>
3	Lithium	Li	[He] 2s <sup>1</sup>
4	Beryllium	Be	[He] 2s <sup>2</sup>
5	Boron	В	[He] 2s <sup>2</sup> 2p <sup>1</sup>
6	Carbon	С	[He] $2s^2 2p^2$
7	Nitrogen	Ν	[He] 2s <sup>2</sup> 2p <sup>3</sup>
8	Oxygen	0	[He] 2s <sup>2</sup> 2p <sup>4</sup>
9	Fluorine	F	[He] 2s <sup>2</sup> 2p <sup>5</sup>
10	Neon	Ne	[He] 2s <sup>2</sup> 2p <sup>6</sup>
11	Sodium	Na	[Ne] 3s <sup>1</sup>
12	Magnesium	Mg	[Ne] $3s^2$
13	Aluminum	Al	[Ne] $3s^2 3p^1$
14	Silicon	Si	[Ne] $3s^2 3p^2$
15	Phosphorus	Р	[Ne] $3s^2 3p^3$
16	Sulfur	S	[Ne] $3s^2 3p^4$
17	Chlorine	Cl	[Ne] $3s^2 3p^5$
18	Argon	Ar	[Ne] $3s^2 3p^6$
19	Potassium	Κ	[Ar] 4s <sup>1</sup>
20	Calcium	Ca	$[Ar] 4s^2$
21	Scandium	Sc	$[Ar] 3d^1 4s^2$
22	Titanium	Ti	$[Ar] 3d^2 4s^2$
23	Vanadium	V	$[Ar] 3d^3 4s^2$
24	Chromium	Cr	$[Ar] 3d^5 4s^1$
25	Manganese	Mn	$[Ar] 3d^5 4s^2$
26	Iron	Fe	$[Ar] 3d^6 4s^2$
27	Cobalt	Со	$[Ar] 3d^7 4s^2$
28	Nickel	Ni	$[Ar] 3d^8 4s^2$
29	Copper	Cu	$[Ar] 3d^{10} 4s^1$
30	Zinc	Zn	$[Ar] 3d^{10} 4s^2$
31	Gallium	Ga	$[Ar] \ 3d^{10} \ 4s^2 \ 4p^1$
32	Germanium	Ge	$[Ar] 3d^{10} 4s^2 4p^2$

#### **ATOMIC PHYSICS**

Atomic #	Name	Symbol	Electronic config
33	Arsenic	As	$[Ar] \ 3d^{10} \ 4s^2 \ 4p^3$
34	Selenium	Se	$[Ar] \ 3d^{10} \ 4s^2 \ 4p^4$
35	Bromine	Br	$[Ar] \ 3d^{10} \ 4s^2 \ 4p^5$
36	Krypton	Kr	$[{\rm Ar}]~3d^{10}~4s^2~4p^6$
37	Rubidium	Rb	[Kr] 5s <sup>1</sup>
38	Strontium	Sr	[Kr] 5s <sup>2</sup>
39	Yttrium	Y	$[Kr] 4d^1 5s^2$
40	Zirconium	Zr	$[Kr] 4d^2 5s^2$
41	Niobium	Nb	$[Kr] 4d^4 5s^1$
42	Molybdenum	Мо	[Kr] 4d <sup>5</sup> 5s <sup>1</sup>
43	Technetium	Tc	$[Kr] 4d^5 5s^2$
44	Ruthenium	Ru	[Kr] 4d <sup>7</sup> 5s <sup>1</sup>
45	Rhodium	Rh	[Kr] 4d <sup>8</sup> 5s <sup>1</sup>
46	Palladium	Pd	[Kr] 4d <sup>10</sup>
47	Silver	Ag	[Kr] 4d <sup>10</sup> 5s <sup>1</sup>
48	Cadmium	Cd	$[Kr] 4d^{10} 5s^2$
49	Indium	In	[Kr] 4d <sup>10</sup> 5s <sup>2</sup> 5p <sup>1</sup>
50	Tin	Sn	[Kr] 4d <sup>10</sup> 5s <sup>2</sup> 5p <sup>2</sup>
51	Antimony	Sb	[Kr] $4d^{10} 5s^2 5p^3$
52	Tellurium	Te	[Kr] $4d^{10} 5s^2 5p^4$
53	Iodine	Ι	[Kr] 4d <sup>10</sup> 5s <sup>2</sup> 5p <sup>5</sup>
54	Xenon	Xe	[Kr] 4d <sup>10</sup> 5s <sup>2</sup> 5p <sup>6</sup>
55	Cesium	Cs	[Xe] 6s <sup>1</sup>
56	Barium	Ba	[Xe] 6s <sup>2</sup>
57	Lanthanum	La	$[Xe] 5d^1 6s^2$
58	Cerium	Ce	$[Xe] 4f^1 5d^1 6s^2$
59	Praseodymium	Pr	$[Xe] 4f^3 6s^2$
60	Neodymium	Nd	$[Xe] 4f^4 6s^2$
61	Promethium	Pm	$[Xe] 4f^5 6s^2$
62	Samarium	Sm	$[Xe] 4f^6 6s^2$
63	Europium	Eu	$[Xe] 4f^7 6s^2$
64	Gadolinium	Gd	$[Xe] 4f^7 5d^1 6s^2$
65	Terbium	Tb	$[Xe] 4f^9 6s^2$
66	Dysprosium	Dy	$[Xe] 4f^{10} 6s^2$

Atomic #	Name	Symbol	Electronic config
67	Holmium	Но	[Xe] 4f <sup>11</sup> 6s <sup>2</sup>
68	Erbium	Er	[Xe] 4f <sup>12</sup> 6s <sup>2</sup>
69	Thulium	Tm	[Xe] 4f <sup>13</sup> 6s <sup>2</sup>
70	Ytterbium	Yb	[Xe] 4f <sup>14</sup> 6s <sup>2</sup>
71	Lutetium	Lu	[Xe] 4f <sup>14</sup> 5d <sup>1</sup> 6s <sup>2</sup>
72	Hafnium	Hf	$[Xe] 4f^{14} 5d^2 6s^2$
73	Tantalum	Та	[Xe] 4f <sup>14</sup> 5d <sup>3</sup> 6s <sup>2</sup>
74	Tungsten	W	[Xe] 4f <sup>14</sup> 5d <sup>4</sup> 6s <sup>2</sup>
75	Rhenium	Re	[Xe] 4f <sup>14</sup> 5d <sup>5</sup> 6s <sup>2</sup>
76	Osmium	Os	[Xe] 4f <sup>14</sup> 5d <sup>6</sup> 6s <sup>2</sup>
77	Iridium	Ir	[Xe] 4f <sup>14</sup> 5d <sup>7</sup> 6s <sup>2</sup>
78	Platinum	Pt	[Xe] 4f <sup>14</sup> 5d <sup>9</sup> 6s <sup>1</sup>
79	Gold	Au	[Xe] 4f <sup>14</sup> 5d <sup>10</sup> 6s <sup>1</sup>
80	Mercury	Hg	[Xe] $4f^{14} 5d^{10} 6s^2$
81	Thallium	T1	[Xe] 4f <sup>14</sup> 5d <sup>10</sup> 6s <sup>2</sup> 6p <sup>1</sup>
82	Lead	Pb	$[Xe] \ 4f^{14} \ 5d^{10} \ 6s^2 \ 6p^2$
83	Bismuth	Bi	[Xe] 4f <sup>14</sup> 5d <sup>10</sup> 6s <sup>2</sup> 6p <sup>3</sup>
84	Polonium	Ро	$[Xe] \ 4f^{14} \ 5d^{10} \ 6s^2 \ 6p^4$
85	Astatine	At	[Xe] 4f <sup>14</sup> 5d <sup>10</sup> 6s <sup>2</sup> 6p <sup>5</sup>
86	Radon	Rn	$[Xe]  4f^{14}  5d^{10}  6s^2  6p^6$
87	Francium	Fr	[Rn] 7s <sup>1</sup>
88	Radium	Ra	[Rn] 7s <sup>2</sup>
89	Actinium	Ac	$[Rn] 6d^1 7s^2$
90	Thorium	Th	$[Rn] 6d^2 7s^2$
91	Protactinium	Ра	[Rn] $5f^2 6d^1 7s^2$
92	Uranium	U	[Rn] $5f^3 6d^1 7s^2$
93	Neptunium	Np	[Rn] $5f^4 6d^1 7s^2$
94	Plutonium	Pu	$[Rn] 5f^6 7s^2$
95	Americium	Am	[Rn] $5f^7 7s^2$

#### **Selection Rules**

A selection rule is a quantum mechanical rule that describes types of quantum mechanical transitions that are permitted. Transitions not permitted by selection rules are said to be "forbidden," although in practice, such transitions are generally forbidden "to first order" only, which means they may occur in practice but with low probabilities.

#### **Selection Rules for Electronic Spectra of Transition Metal Complexes**

The selection rules governing transitions between electronic energy levels of transition metal complexes are:

- 1.  $\Delta S = 0$  The Spin Rule
- 2.  $\Delta l = +/- 1$  The Orbital Rule (Laporte)

#### **D. THE LASER**

Light Amplification by Stimulated Emission of Radiation

#### Properties

- 1. The light is coherent. (Waves are in phase.)
- 2. Light is nearly monochromatic (one wavelength).
- 3. Minimal divergence.
- 4. Highest intensity of any light source.

Many atoms have excited energy levels which have relatively long lifetimes  $(10^{-3} \text{ s instead of } 10^{-8} \text{ s})$ . These levels are known as **metastable**.

Through a process known as **population inversion**, the majority of an assembly of atoms is brought to an excited state.

Population inversion can be accomplished through a process known as **optical pumping**, where atoms of a specific substance, such as ruby, are exposed to a given wavelength of light. This wavelength is enough to excite the ruby atoms just above metastable light. The atoms rapidly lose energy and fall to the metastable level.

#### Ruby



#### THE LASER

Once population inversion has been obtained, **induced emission** can occur from photons dropping from an excited metastable state to ground state. The photons have a wavelength equal to the wavelength of photons produced by each individual atom. The radiated light waves will be exactly in phase with the incident waves, resulting in an enhanced beam of coherent light. Hence, the familiar laser effect.

#### **Helium-Neon Laser**

As shown in the figure below, a helium-neon laser corresponds to transition between two levels in Ne atoms that are 1.96 eV apart. Helium is used to assist the population of the 20.66 eV energy level in the Ne atoms.



#### **Black-Body Radiation**

#### **Planck's Law of Black-Body Radiation Equation**

$$I(v,T)dv = \frac{2hv^{3}}{c^{2}} \frac{1}{e^{\frac{hv}{kT}} - 1} dv$$

 $I(\nu,T)d\nu$  is the amount of energy per unit surface area per unit time per unit solid angle emitted in the frequency range between  $\nu$  and  $\nu + d\nu$  by a black body at temperature *T*.

h = Planck's constant

- c = the speed of light
- k =Boltzmann's constant

Wien's displacement law: If T is the absolute temperature of a black body and  $\lambda_{max}$  is the wavelength at which the radiation has maximum intensity, then



#### **ATOMIC PHYSICS**

$$\lambda_{\rm max} T = 2.898 \times 10^{-3} \,\mathrm{m \, K}$$

**Stefan–Boltzmann law:** For a body at absolute temperature *T*, the rate of radiation dQ/dt from its surface is proportional to  $T^4$ . Hence,

$$\frac{dQ}{dT}=\sigma\epsilon T^4$$

where  $\sigma =$  Stefan-Boltzmann constant

 $\varepsilon$  = the emissivity of the surface,  $\varepsilon$  = 1 for black body

#### CHAPTER

### Thermodynamics

## 4

#### A. TEMPERATURE

The average kinetic energy possessed by the molecules of a body is referred to as its temperature. The zeroth law of thermodynamics states this more formally as:

There exists a scalar quantity called temperature, which is a property of all thermodynamic systems (in equilibrium states), such that temperature equality is a necessary and sufficient condition for thermodynamic equilibrium.

The two main scales in use for measuring temperature are the Celsius (formerly known as Centigrade) and the Kelvin scales. On the Celsius scale, water boils at  $100^{\circ}$  (1 Atm) and freezes at  $0^{\circ}$ . The 100 equal divisions between these two points determine the size of the Celsius degree, and from this, the points above 100 and below 0 can be determined. When a temperature is shown followed by a degree sign, but without an abbreviation to show the scale used (e.g.,  $25^{\circ}$  instead of  $25^{\circ}$ C.), it is presumed to be a Celsius temperature. The degrees on the Kelvin scale are the same size as those on the Celsius scale. Kelvin, however, starts at absolute zero; hence, it has no negative temperature. A temperature in Kelvin is never reported with the degree sign (e.g., 300 K). To convert Kelvin to Celsius, simply subtract 273.15 from the Kelvin temperature. Put mathematically,

$$C = K - 273.15$$

For example, 300 K would be

$$(300 - 273.15) = 26.85^{\circ} \text{ C}$$

Converting from Celsius to Kelvin is just the opposite of this. Two older scales of temperature measurement, Fahrenheit and Rankine, are not often used, and hence need not be learned.

#### **B. COEFFICIENT OF THERMAL EXPANSION**

As a substance is heated, its molecules vibrate faster and move further apart, or the material is said to expand. This phenomenon is much more pronounced in gases than in liquids and solids. If the material is a solid, and one of its dimensions is much more pronounced than the other two (e.g., a rod), a coefficient of linear expansion,  $\alpha_l$ , can be determined, i.e., a value to determine the increase of the material in that one dimension only. For all other materials, a coefficient of volume expansion,  $\alpha_v$ , is used.

For solids, the  $\alpha_v$  is equal to three times the  $\alpha_l$ . For liquids and solids, the  $\alpha_v$  will vary somewhat with the temperature; for gasses (at constant pressure), it is usually equal to 0.00367/°C.

#### **Example:**

The  $\alpha_v$  for glycerin is 5.1 × 10<sup>-4</sup>/°C. If 25.62 liters of glycerine are heated from 25° to 125°, what will be its new volume?

$$V = V_0 + (V_0 \cdot \alpha_v \cdot \Delta T)$$

where

V = the new volume  $V_0 =$  the initial volume  $\Delta T =$  the change in temperature

 $\Delta T$  can be calculated as 125 - 25 = 100. Putting everything into an equation, we get

$$V = V_0 + (V_0 \cdot \alpha_v \cdot \Delta T)$$
  
= 25.62 + (25 \cdot 62 \cdot 5.1 \times 10^{-4} \cdot 100)  
= 26.93 liters

The same method is used for the linear coefficient of expansion.

#### **Example:**

or

The  $\alpha_l$  for aluminum is 2.4 × 10<sup>-5</sup>/°C. If an aluminum needle is 20 cm long at 350 K, how long will it be at 100 K?

The formula, much like the above, is

$$L = L_0 + (L_0 \cdot \alpha_l \cdot \Delta T)$$
  

$$L = 20 + (20)(2.4 \times 10^{-5})(-250)$$
  
= 19.88 cm

Note that in this instance, the  $\Delta T$  is negative; hence, the value decreases.

#### C. HEAT CAPACITY, SPECIFIC HEAT, AND C<sub>n</sub>

#### **Specific Heat of Gases**

Specific Heat at Constant Pressure ( $C_p$ ): The amount of heat required to raise the temperature of 1 mole of a gas through 1 K (or 1°C) *at a constant pressure*. Hence,

$$Q = n C_n \Delta T$$

Specific Heat at Constant Volume ( $C_V$ ): The amount of heat required to raise the temperature of 1 mole of a gas through 1 K (or 1°C) *at a constant volume*. Hence,

$$Q = n C_V \Delta T$$

For a monatomic gas (such as Ar, He, Ne, H, O etc.),

$$C_p = 20.78 \text{ J/(mol.K)}, C_V = 12.47 \text{ J/(mol.K)}$$

Internal Energy of Ideal Gas (U): The internal energy change ( $\Delta U$ ) of a given amount (n moles) of an ideal gas depends only on the temperature change ( $\Delta T$ ). It is independent of the path between its initial and final states.

$$\Delta U = (3/2) nR\Delta T$$

For an *isothermal process*,  $\Delta U = 0$  because  $\Delta T = 0$ .

#### **Dulong Petit Law**

For an atom in a crystal, three translational degrees of freedom and additionally three degrees of freedom for atomic vibrations give the total energy per atom of 3kT. Thus,

 $3kTN_A =$  the energy per mole k = Boltzmann's constant T = temperature in Kelvin  $N_A =$  Avogadro's number

The specific heat at constant volume is

 $C_v =$  the derivative with respect to T of  $3kTN_A = 3kN_A$  or 3R This is the Dulong-Petit Law.  $C_v =$  measured in J/kg/K R = universal gas constant M = molecular mass in kg/mol Chapter



#### THERMODYNAMICS

#### **Example:**

A statue made of a metal alloy has a heat capacity of 55.8 Joules/°C. How many Joules of energy will have to be absorbed to raise its temperature from 25° to 100°?

	$T = 100 - 25 = 75^{\circ}$
	$C = \Delta Q / \Delta T$
or	$55.8 = \Delta Q/75$
or	$\Delta Q = 55.8 \times 75 = 4,185 \text{ Joules}$
or	= 4.185  kJ

A more useful value is the specific heat, or c. Specific heat is defined as heat capacity per unit mass of body, or

$$c = \Delta Q / (m \Delta T)$$

where m = the mass of the object

At ordinary temperatures and ordinary temperature ranges, specific heats can be considered constant. However, it must be specified if the specific heat is measured with constant pressure  $(c_p)$  or constant volume  $(c_p)$ . For gasses,

$$c_p - c_v = R/M$$

where R = the universal gas law constant (8.314 J/mole °C) M = the molecular weight of the gas

Sometimes specific heats are given in J/mole °C, rather than in the standard J/gram °C; hence, the units need to be inspected carefully.

#### **Example:**

The  $c_p$  for lead is 0.1277 J/gm °C, and for water it is 4.186 J/gm °C. If a lead block of 75 grams is placed in 200 grams of water at 25°, and the final temperature of the water and lead is 28.11°, what was the initial temperature of the lead (assuming no heat is lost to the surroundings)?

The fact that both  $c_p$ 's are given in J/gm °C saves us the trouble of doing any conversions. The Joules lost by the lead will be equal to the number of Joules gained by the water, or

$$J_{\rm Pb} = J_{\rm H_2O}$$

This can be restated as

$$(\Delta T \cdot c_p \cdot m)_{\rm Pb} = (\Delta T \cdot c_p \cdot m)_{\rm H_2O}$$

#### HEAT OF VAPORIZATION AND HEAT OF FUSION

The  $c_p$ 's and *m*'s for each are given, the  $\Delta T$ 's can be calculated. For water,  $\Delta T = 28.11 - 25 = 3.11^{\circ}$ . For lead, the initial temperature is not known, it will be assigned the value *X*; hence,  $\Delta T = X - 28.11$ . Putting these values into the above equation, we obtain

(X - 28.11)(0.1277)(75) = (3.11)(4.186)(200).

Performing the indicated operations we get

$$2,603.7 = 9.578X - 269.2,$$

solving for X yields  $X = 299.95^{\circ}$ , which was the initial temperature of the lead.

#### **D. HEAT OF VAPORIZATION AND HEAT OF FUSION**

When a substance changes state, e.g., from a liquid to a gas, energy is absorbed by the substance without change in temperature. For example, for water at 100° to change to steam at 100°, it must absorb 2,260 Joules per gram of water. This value is known as the **Heat of Vaporization**  $(L_v)$  and is unique for each substance.

The melting of a solid involves a similar process, Joules are absorbed by the substance, but rather than an increase in temperature, there is a change of physical state. Ice at 0° to melt to water at 0° must absorb 335 Joules per gram. This value, again unique for each specific substance, is known as the **Heat of Fusion** ( $L_f$ ).

#### Example:

For mercury the boiling and freezing points are 358° and  $-39^\circ$ , respectively. The  $L_v$  and  $L_f$  are 2,197 J/gm and 11.7 J/gm, respectively. The  $c_p$  for liquid mercury is 0.138 J/gm °C. How much energy would have to be lost by 1 kg of Hg vapor at 358° to become solid Hg at  $-39^\circ$ ?

To calculate this, one needs to find: (1) The Joules to convert 1 kg of Hg vapor to 1 kg of liquid Hg. (2) The Joules lost to cool 1 kg of Hg from  $358^{\circ}$  to  $-39^{\circ}$ . (3) The Joules lost to convert 1 kg of liquid Hg to 1 kg of solid Hg. Put mathematically,

 $[m \cdot L_{v}] + [m \cdot c_{p} \cdot \Delta T] + [m \cdot L_{f}]$ = [1,000 \cdot 297] + [1,000 \cdot 0.138 \cdot 398] + [1,000 \cdot 11.7] = [297,000] + [54,924] + [11,700] = 406,848 Joules = 407 kJ lost

Had the mercury been heated to change it from a solid to a liquid and then to a gas, the procedure would have been just the opposite, and we would have been adding Joules. If a material changes directly from a solid to a gas (e.g., carbon dioxide), a similar term, **Heat of Sublimation**  $(L_s)$  is used, just as  $L_y$  and  $L_f$ .



#### E. CONDUCTION, CONVECTION, AND RADIATION

Heat can travel from one body to another by three processes: conduction, convection, and radiation. In conduction, one body is physically in contact with another, and the heat travels directly. The thermal conductivity of a material tells how easily it can transport heat. Metals are usually good at this. Steel, for example, has a thermal conductivity of  $4.60 \times 10^{-2}$  kJ/sec·meter·°C, whereas asbestos has a value of  $8.37 \times 10^{-5}$  kJ/sec·meter·°C.

#### **Example:**

Compare how much heat is transmitted per second by a steel plate 1 cm thick, with a surface area of  $5,000 \text{ cm}^2$ , and a temperature of  $300^\circ$  on one side and  $25^\circ$  on the other, to how much heat is transmitted by an asbestos plate of equal dimensions under the same conditions.

heat transfer = thermal conductivity  $\cdot$  area  $\cdot (T_1 - T_2)$ /thickness

For the steel plate this gives

 $(4.6 \times 10^{-2})(0.500)(275/0.0100) = 632.5$  kJoules/sec

Note that cm<sup>2</sup> and cm were converted to m<sup>2</sup> and m. For the asbestos plate we get

 $(8.37 \times 10^{-5})(0.500)(275/0.0100) = 1.151$  kJoules/sec

Convection simply involves heating the air molecules around an object, which then travel, via air currents, to another object where they impart their acquired energy by collisions.

Radiation involves the transfer of heat by photons, which can pass through a vacuum. A black body is one that absorbs all radiant energy that falls upon it, and emits radiation perfectly. The Stefan-Boltzmann equation states that E, the watts of power radiated by a black body, is equal to the Stefan constant ( $\delta$ ) times the fourth power of the absolute temperature, times the surface area, A, or

$$E = A\delta T^4$$

The value of  $\delta$  is 5.67 × 10<sup>-8</sup> Watts/m<sup>2</sup> · K<sup>-4</sup>. A black cubic body, 25 cm on the side, is heated to 325°C. At what rate is energy radiated from its surface? First, the surface area of the cube must be determined. Converting cm to m, we get  $0.250^2 \times 6 = 0.375 \text{ m}^2 = A$ . The absolute temperature is 325 + 273 = 598 K. Putting these into the equation,

 $E = A\delta T^4$ 

which yields  $E = (0.375)(5.67 \times 10^{-8})(598)^4 = 2,719$  Watts

#### **Ideal Gas Laws and Kinetic Theory of Gases**

#### **Boyle's Law:**

For a given quantity of gas at constant temperature

$$PV = const.$$
$$P_1V_1 = P_2V_2$$

#### **Charles' Law:**

For a given quantity of gas at constant pressure

$$\frac{V}{T} = const.$$
$$\frac{V_1}{T_1} = \frac{V_2}{T_2}$$

T,  $T_1$ , and  $T_2$  are absolute temperature (in Kelvin).

#### The Ideal Gas Law:

$$\frac{PV}{T} = const.$$
$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$$
$$PV = nRT$$
$$m = nM$$
$$N = nN_A$$

where P = pressure of the gas

- V = volume of the gas
  - n = number of moles of the substance
- m = mass of the gas in kg
- M = molecular mass of the substance
- N = number of molecules in the sample of the substance
- $N_A$  = Avogadro's number =  $6.02 \times 10^{23}$  molecules/mole R = universal gas constant = 8.314 J/(mole.K)

#### Some reversible thermodynamic processes:

1. (constant pressure): Isobaric Process

$$W = -P\Delta V = -P(V_2 - V_1)$$



2. (constant volume): Isochoric Process

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 $\Delta V = 0$ , hence W = 0

Hence,  $Q = \Delta U$ . (All the supplied heat goes to increasing the internal energy of the system.)

3. (constant temperature): Isothermal Process

 $\Delta U = 0$ , hence Q = -W

4. (thermally insulated system, Q = 0): Adiabatic Process

$$\Delta U = W$$



#### **Kinetic Theory of Gases**

According to the kinetic theory, gas consists of tiny particles called molecules. The molecules move around randomly with all possible speeds.

Average KE per Molecule: The average kinetic energy per molecule of an ideal gas depends only on its absolute temperature and is given by

$$KE_{av} = \frac{3}{2}kT$$

**Internal Energy:** The total kinetic energy of all the molecules is called its internal energy and is given by

$$U = \frac{3}{2}nRT$$

**Change in the Internal Energy:** The change in the internal energy  $\Delta U$  of a gas is independent of its pressure and volume. It depends only on the temperature change  $\Delta T$ . Thus,

$$\Delta U = \frac{3}{2} nR\Delta T$$

**The Root-Mean-Square Speed:** The average speed of the molecule that can relate to the  $KE_{av}$  is not the arithmetic mean but the root-mean-square speed (v<sub>rms</sub>). The root-mean-square speed is obtained by finding the square root of the average of the squares of the speeds of all the molecules.

For example, if there are five molecules with different speeds  $v_1$ ,  $v_2$ ,  $v_3$ ,  $v_4$ , and  $v_5$ , then  $v_{rms}$  is given by

$$v_{\rm rms} = \sqrt{\frac{v_1^2 + v_2^2 + v_3^2 + v_4^2 + v_5^2}{5}}$$

The  $v_{rms}$  for an ideal gas is given by

$$V_{\rm rms} = \sqrt{\frac{3RT}{M(kg)}} = \sqrt{\frac{3kT}{m}}$$

In the above equations:

 $KE_{av}$  = average KE for one molecule of the gas U = internal energy of the given gas = total KE of all the moleculesM = molecular mass of the gas m = mass of one molecule of the gas  $v_{rms}$  = root mean square speed of the molecules of the gas R = universal gas constant = 8.32 J/(mole.K)

T = absolute temperature of the gas

N = number of molecules of the given amount of gas

k (Boltzmann's constant) =  $R/N_A = 1.3807 \times 10^{-23} \text{ J/K}$ N<sub>A</sub> (Avogadro's number) =  $6.02 \times 10^{23}$  molecules/mole

#### F. HEAT, WORK, AND THE LAWS OF THERMODYNAMICS

#### The First Law of Thermodynamics

Recall that work is defined as a force acting through a distance, and is measured in the same units as heat. Therefore, one might correctly conclude that a transfer of heat can be made to do work. The First Law of Thermodynamics states that there is a constant amount of energy in the universe which can be

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neither created nor destroyed; it can only change its form. It is this change from one form to another that we observe and call work. Expressed mathematically, we have

$$\Delta U = Q - W$$

where  $\Delta U$  = the change in energy of a system W = the energy spent in doing useful work Q = any energy added to the system

If Q > W, the energy remains within the system, i.e., work was done on the system. If Q < W, energy was lost by the system, and work was performed. When gasses are heated at a constant volume, the added energy increases the internal energy of the molecules. However, when it is heated at constant pressure, the internal energy of the molecules is increased, but in addition, work is done by expanding the gas against the walls of its container (hence, the different values for gases of  $c_p$  and  $c_y$ ). Under isobaric conditions (constant pressure):

$$Q = \mathrm{mc}_{\mathrm{p}}(T_2 - T_1)$$

where m = the mass of the gas  $W = (\text{density}) \cdot (V_2 - V_1)$ V = the volume of the gas

Work is also defined as

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$$W = m(c_p - c_v)(T_2 - T_1)$$

in such a system. In a constant volume process (isovolumic), W = 0; hence,

$$Q = \Delta U = mc_{v}(T_2 - T_1)$$

In an isothermal process (constant temperature),  $\Delta U = 0$ ; hence, Q = W. An adiabatic process is one in which heat is not transferred to or from the system. As a result,  $Q = 0 = \Delta U + W$ . Hence,  $\Delta U = -W$ .

#### The Second Law of Thermodynamics

The Second Law of Thermodynamics states that it is impossible for heat to travel of its own accord from a colder to a hotter body. As a result of this, the maximum efficiency of any heat engine is given by the formula

$$\frac{(T_1 - T_2)}{T_1}$$

where  $T_1$  = the temperature (Kelvin) of the reservoir which supplies the working substance  $T_2$  = the temperature of the reservoir to which the working substance is exhausted

#### The Third Law of Thermodynamics

The Third Law of Thermodynamics states that it is impossible to reduce any system to absolute zero in a finite series of operations. Hence, a reservoir of absolute zero cannot be constructed, and hence (by the above formula), a heat engine of 100% efficiency cannot be built.

#### **G. ENTROPY**

Many volumes have been written about entropy. To sum it up, for the purposes of this book, entropy can be defined as the amount of disorder in a system. By the Second Law of Thermodynamics, this means that entropy will usually increase, or at the best, remain constant. It takes energy to keep things neat! By the Third Law of Thermodynamics, at absolute zero there would be no entropy. Common units of entropy are Joules/Kelvin.

#### **Statistical Mechanics**

Entropy (S) of a system is defined by the equation due to Boltzmann:

$$S = k_{_{\mathrm{B}}} \ln \Omega$$

where  $k_B = Boltzmann's constant$  $\Omega = the total number of microstates available to the system$ 

#### **Maxwell-Boltzmann Distribution**

The Maxwell-Boltzmann distribution for number of molecules in the range v and v + dv:

$$dN = 4\pi N \left[\frac{m}{2\pi kT}\right]^{\frac{3}{2}} exp\left[-\frac{\left(\frac{1}{2}mv^{2}\right)}{kT}\right] v^{2} dv$$

The Maxwell-Boltzmann distribution of speeds is written as

$$f(v) = 4\pi \left[\frac{m}{2\pi kT}\right]^{\frac{3}{2}} exp\left[-\frac{\left(\frac{1}{2}mv^{2}\right)}{kT}\right]v^{2}$$

where speed, v, is defined as

$$v = \sqrt{v_x^2 + v_y^2 + v_z^2}$$



#### **THERMODYNAMICS**

This equation gives the number of molecules with velocities in the range v + dv.

- where T = the absolute temperature
  - N = the number of molecules,
  - m = the mass of a molecule
  - v = the velocity of a molecule
  - k = the Boltzmann constant

#### **Miscellaneous Thermodynamic Functions**

#### Helmholtz free energy:

$$F = -\frac{\ln Z}{\beta}$$

**Internal energy:** 

$$U = -\left(\frac{\partial \ln Z}{\partial \beta}\right)_{N,V}$$

**Pressure:** 

$$P = -\left(\frac{\partial F}{\partial V}\right)_{N,T} = \frac{1}{\beta} \left(\frac{\partial \ln Z}{\partial V}\right)_{N,T}$$

**Entropy**:

$$S = k(\ln Z + \beta U)$$

**Gibbs free energy:** 

$$G = F + PV = -\frac{\ln Z}{\beta} + \frac{V}{\beta} \left(\frac{\partial \ln Z}{\partial V}\right)_{N,T}$$

**Enthalpy**:

$$H = U + PV$$

**Constant volume heat capacity:** 

$$C_V = \left(\frac{\partial U}{\partial T}\right)_{N,V}$$

Constant pressure heat capacity:

$$C_P = \left(\frac{\partial H}{\partial T}\right)_{N,F}$$

**Chemical potential:** 

$$\mu_i = -\frac{1}{\beta} \left( \frac{\partial \ln Z}{\partial N_i} \right)_{T,V,N}$$

### CHAPTER

## Quantum Mechanics

## 5

#### **A. PHOTOELECTRIC EFFECT**



Electrons will be emitted only if the wavelength of the incident radiation is sufficiently small.

The emission of electrons from the surface of a material when light (or UV) is incident on it is called *photoelectric effect*.

An experiment on photoelectric effect performed using a vacuum tube with cathode as the photoelectric surface and variable voltage between the anode and the cathode reveals a number of interesting characteristics for this phenomenon, namely:

- 1. The emission of photoelectron is instantaneous.
- 2. The photoelectrons have a velocity distribution from zero to certain maximum.
- 3. The maximum kinetic energy (KE<sub>max</sub>) of the photoelectron is independent of the intensity of the incident radiation.
- 4.  $KE_{max}$  depends on the frequency of the radiation.
- 5. If the frequency of the radiation is changed, there exists a threshold frequency below which there is no emission of photoelectron however intense the radiation.
- 6. The number of photoelectrons (hence the photoelectric current) is proportional to the intensity of the radiation.

The wave nature of light failed to explain all these characteristics of the photoelectric effect!!!

#### **Einstein's Theory of Photoelectric Effect**

The energy  $(\mathbf{E})$  of the photon, according to Einstein, is proportional to the frequency  $(\mathbf{f})$  of radiation.

 $\mathbf{E} = \mathbf{h}\mathbf{f}$ 

where  $\mathbf{h} = \text{Planck's Constant}$ 

This minimum energy required to detach the electron from the surface is called the *work function* ( $\phi$ ) of the material and depends on the nature of the material. The photoelectrons coming from the surface thus have the maximum kinetic energy ( $KE_{max}$ ) among the photoelectrons emitted from the surface. This leads to Einstein's photoelectric equation:

$$KE_{max} = hf - \phi$$

The threshold frequency can be obtained from the equation

$$hf_0 = \phi$$

The maximum kinetic energy  $(KE_{max})$  can be determined by measuring the stopping voltage  $(V_0)$  that would stop the photoelectric current. Thus,

$$KE_{max} = eV_s$$

#### Relationship Between Momentum (p) and Wavelength ( $\lambda$ ) of a Photon

$$\mathbf{p}=\frac{\mathbf{h}}{\lambda}=\frac{\mathbf{h}\mathbf{f}}{\mathbf{c}}$$

#### **Rate of Emission of Photons from a Given Source of Light**

The number of photons/second (N/s) emitted by the light source is proportional to the power ( $\mathbf{P}$ ) of the source

Number of photons emitted per second = P / (hf)

#### **Compton Scattering**

In *Compton scattering* an x-ray photon collides with a free electron of a material just like two balls colliding on a pool table. The scattered x-ray photon has its wavelengths increased due to the loss of some energy to the electron. The change in wavelength is a function of the scattering angle from the initial direction.

$$\lambda' - \lambda = \left(\frac{h}{m_o c}\right) (1 - \cos \theta)$$

where  $\lambda'$  = the wavelength of the scattered photon

- $\lambda$  = the wavelength of the incident photon
- $\boldsymbol{\theta}$  = scattering angle
- $\mathbf{m}_{\mathbf{o}} =$  the rest mass of electron

 $\mathbf{c}$  = the speed of light

The quantity  $h/m_o c = 0.00243$  nm is called the Compton wavelength ( $\lambda_c$ ) of electrons. The Compton wavelength is indicative of the scale of the wavelength changes in the Compton scattering.

The recoil energy of the scattered electron is equal to the loss of energy of the photon. Therefore,

$$\mathrm{KE}_{\mathrm{e}} = \frac{\mathrm{hc}}{\lambda} - \frac{\mathrm{hc}}{\lambda'}$$

#### **Wave Particle Duality**

The momentum of the photon is given by

$$\mathbf{P} = \frac{\mathbf{h}}{\boldsymbol{\lambda}}$$

#### de Broglie Wavelength

de Broglie proposed that the wavelength of a particle having a momentum p would be

 $\lambda \frac{h}{p}$ 

#### The de Broglie's Wavelength for an Accelerated Electron

If an electron (charge -e and mass m) is accelerated from rest, through a potential difference V,

then e.

$$V = \frac{1}{2}mv^2 \rightarrow mv = \sqrt{2meV}$$
$$p = \sqrt{2meV}$$

Hence,



Thus, using the de Broglie equation, we get

$$\lambda = rac{h}{\sqrt{2meV}}$$

**X-ray production:** In x-ray production with a Coolidge tube, electrons are accelerated through a voltage of  $V_a$  and are suddenly stopped by a metal anode. Their energy  $eV_a$  converts to x-rays and heat in varying proportion. The shortest wavelength (most energetic) corresponds to the electrons for which the heat produced is zero. The shortest wavelength  $\lambda_{min}$  for the x-rays produced is given by

$$hc/\lambda_{min} = eV_{e}$$

where h = Planck's constant

c = speed of light

e = elementary charge

#### **B. WAVE FUNCTIONS AND EQUATIONS**

The wave function  $\psi$  has no physical interpretation, but  $|\psi|^2$  for a body at a given place and time is proportional to the location of the body at that given time.

Once  $\boldsymbol{\psi}$  is determined, the momentum, angular momentum, and energy of the body can be determined.

#### Wave Function

$$\Psi = A + iB$$

where A, B = real functions

#### **Complex Conjugate**

$$\Psi^* = A - iB$$

If dealing with complex wave functions:

$$|\Psi|^2 = \Psi * \Psi$$

Conditions for a wave function:

1) 
$$\int_{-\infty}^{\infty} |\psi|^2 dV \neq 0 (V = \text{Volume})$$

2)  $\Psi$  must be single valued.

3) 
$$\frac{\partial \Psi}{\partial x}, \frac{\partial \Psi}{\partial y}$$
 and  $\frac{\partial \Psi}{\partial z}$ 

must be finite, single valued, and continuous.

Normalization: A wave function is normalized when  $|\Psi|^2$  is equal to the probability density, not merely proportional to it.

$$\int_{-\infty}^{\infty} |\psi| dV = 1$$

#### **C. SCHRÖDINGER'S EQUATION**

 $\Psi$  is not measurable, and therefore may be complex.

Specify  $\Psi$  in the *x* direction:

$$\Psi = Ae^{-i\omega\left(t-\frac{x}{v}\right)}$$

where  $\omega = 2\pi v$  $v = \lambda v$ 

then

$$\Psi = A e^{-2\pi i \left(vt - \frac{x}{\lambda}\right)}$$

$$E = 2\pi h$$
$$\lambda = \frac{2\pi h}{p}$$

For a free particle:

$$\Psi = Ae^{\frac{i(Et-px)}{h}}$$

To obtain Schrödinger's equation,



#### **QUANTUM MECHANICS**

$$\frac{\partial^2 \Psi}{\partial x^2} = -\frac{p^2}{\hbar^2} \Psi \text{ and } \frac{\partial \Psi}{\partial t} = \frac{iE}{\hbar} \Psi$$
$$E = \frac{p^2}{2m} + V$$

therefore:

$$E\Psi = \frac{p^{2}\Psi}{2m} + V\Psi$$
$$E\Psi = \frac{h}{i} = \frac{\partial\Psi}{\partial t} \text{ and } p^{2}\Psi = -h^{2}\frac{\partial^{2}\Psi}{\partial x^{2}}$$

#### Time Dependent One-Dimensional Schrödinger's Equation

$$i\hbar\frac{\partial\Psi}{\partial t} = \frac{\hbar^2}{2m}\left(\frac{\partial^2\Psi}{\partial x^2}\right) + V\Psi$$

In three dimensions:

$$i\mathbf{h} = -\frac{\mathbf{h}^2}{2m} \left( \frac{\partial^2 \Psi}{\partial x^2} + \frac{\partial^2 \Psi}{\partial y^2} + \frac{\partial^2 \Psi}{\partial z^2} \right) + V(x, y, z, t) \Psi$$

Steady-State Form (3-D)

$$\frac{\partial^2 \Psi}{\partial x^2} + \frac{\partial^2 \Psi}{\partial y^2} + \frac{\partial^2 \Psi}{\partial z^2} + \frac{2m}{\hbar^2} (E - V) \Psi = 0$$

#### **D. POTENTIAL WELLS AND ENERGY LEVELS**

#### Particle in a Box

A particle trapped in a box with infinitely hard walls is the simplest quantum-mechanical problem.

Equation (Schrödinger's) in the box:

$$\frac{\partial^2 \Psi}{\partial x^2} + \frac{2m}{\hbar^2} E \Psi = 0$$

As V = 0,

$$\Psi = A \sin \frac{\sqrt{2mE}}{h} x + B \cos \frac{\sqrt{2mE}}{h} x$$

As  $\Psi = 0$  at x = 0 and x = L,

$$\frac{\sqrt{2mE}}{h}L = n\pi \qquad n = 1, 2, 3, \dots$$
$$\therefore E_n = \frac{n^2 \pi^2 h^2}{2mL^2} \qquad n = 1, 2, 3, \dots$$



Normalized wave function of particle in box:

$$\Psi_n = \sqrt{\frac{2}{L}} \sin \frac{n\pi x}{L} \qquad n = 1, 2, 3, \dots$$

#### **E. HARMONIC OSCILLATOR**

There are three differences from a classical oscillator:

- 1. Allowed energies a **discrete** spectrum.
- 2. Lowest energy not E = 0, but  $E = E_0$ .
- 3. May go beyond  $\pm A$  (tunneling)



The Schrödinger equation for harmonic oscillator:

$$\frac{d^2\Psi}{dx^2} + \frac{2m}{\hbar} \left( E - \frac{1}{2}kx^2 \right) \Psi = 0$$

This leads to energy levels



Evenly Spaced Energy Levels

 $E_0$ 

#### F. REFLECTION AND TRANSMISSION BY A BARRIER

#### **Tunnel Effect**

According to classical mechanics, when a particle of energy *E* approaches a potential barrier *V* when V > E, then the particle must bounce back. In quantum mechanics, there is a chance, though usually very small, that the particle could penetrate the barrier.

According to Heisenberg's uncertainty principle,

$$\Delta x \, \Delta p \ge \frac{h}{2}$$

If we 100% certain that the principle is not inside the barrier, then we are saying that  $\Delta x = 0$ . Therefore,  $\Delta P$  and *E* would be infinite, which is impossible. Therefore, there must be some positive finite change of the particle penetrating the barrier.

A particle energy E > V approaches potential barrier:



some reflect, others go on.

Wave functions and probability densities of particles in finite potential wells. (Particle can be found in well.)



**Barrier Penetration** 



Schrödinger's Equations

$$\frac{\partial^2 \Psi_{\rm I}}{\partial x^2} + \frac{2m}{\hbar} E \Psi_{\rm I} = 0$$

$$\frac{\partial^2 \Psi_{\rm III}}{\partial x^2} + \frac{2m}{\hbar} E \Psi_{\rm III} = 0$$



with solutions

$$\Psi_{\rm I} = Ae^{ik_{\rm I}x} + Be^{-ik_{\rm I}x}$$
$$\Psi_{\rm III} = Fe^{ik_{\rm I}x} + Ge^{-ik_{\rm I}x}$$

**Outside barrier:** 

$$k_1 = \frac{\sqrt{2mE}}{h} = \frac{2\pi}{\lambda}$$

**Incoming Wave:** 

 $\Psi_{I+} = A e^{ik_1 x}$ 

**Reflected Wave:** 

 $\Psi_{I-} = Be^{-ik_1x}$ 

**Transmitted Wave:** 

 $\Psi_{\text{III}+} = Fe^{ik_1x}$ 

**Transmission Probability:** 

$$T = \frac{|\Psi_{\rm III+}|^2 v}{|\Psi_{\rm I+}|^2 v} = \frac{FF^*}{AA^*}$$

**Approximation:** 

$$T \approx e^{-2k_2L}$$
$$k_2 \frac{\sqrt{2m(V-E)}}{\hbar}$$

#### **Expectation Values**

As Schrödinger's equation yields probabilities, an expectation value can be described by  $\Psi(x, t)$ ,  $\langle x \rangle$  is the value of x that would be obtained by measuring the position of a large quantity of particles, described by a given wave function, at a given time, and averaging the results.

For position:

$$\langle x \rangle = \int_{-\infty}^{\infty} x |\Psi|^2 dx$$

$$\langle f(x) \rangle = \int_{-\infty}^{\infty} f(x) |\Psi|^2 dx$$

#### **The Franck-Hertz Experiment**

The Franck and Hertz experiment confirms the quantum theory by revealing the excited states in mercury atoms.

As shown in the figure below, a glass tube is filled with mercury vapors. Electrons are accelerated toward a positively charged grid and are collected by a collection plate that has small negative potential. The flow of the electrons in the outer circuit can be measured as a current by an ammeter. The current vs. accelerating voltage graph shows peak as shown below. The peaks are 4.9 V apart which corresponds to the excited states of mercury atoms. The first peak appears when the 4.9 V energy of electron is lost to the mercury atoms and the current drops. The successive peaks appear when the electrons that have lost energy to mercury atoms are reaccelerated again to 4.9 V energy.



Chapter

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#### CHAPTER

### **Special Relativity**

## 6

#### A. TIME DILATION AND LENGTH CONTRACTION

#### **Time Dilation**

$$f = \frac{f_0}{\sqrt{1 - \frac{\nu^2}{c^2}}}$$

where  $f_0$  = time passed on clock at rest relative to observer f = time interval on clock in motion relative to observer v = speed of motion relative to observer (relative motion) c = speed of light (3 × 10<sup>8</sup> m/s)

For an object or observer in motion, less time passes in a given interval (to the observer in motion) than for objects or observers relatively at rest.

#### Length Contraction

$$L = L_0 \sqrt{1 - \frac{v^2}{c^2}}$$

where  $L_0 =$  Length of object when at rest relative to observer

L = Length when approaching speed of light relative to observer

To an observer relatively at rest, as an object approaches the speed of light, its length contracts.

#### **B. DYNAMICS**

#### **Relativistic Mass**

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

**Relativistic Momentum** 

$$P = mv = \frac{m_0 v}{\sqrt{1 - \frac{v^2}{c^2}}}$$

#### **Relativistic Newton's Second Law**

$$F = \frac{d}{dt} \left( \frac{m_0 v}{\sqrt{1 - \frac{v^2}{c^2}}} \right)$$

#### **Mass and Energy**

Total Energy

$$mc^2 = m_0 c^2 + k$$
 (kinetic energy)

Rest Energy

 $E = m_0 c^2$ 

Total Energy

$$E = mc^{2} = \frac{m_{0}c^{2}}{\sqrt{1 - \frac{v^{2}}{c^{2}}}}$$
  
$$\therefore E^{2} = m_{0}^{2}c^{4} + p^{2}c^{2}$$

#### **C. LORENTZ TRANSFORMATIONS**

Given two reference frames (x, y, z, t) and (x', y', z', t'), when there is motion of the second frame in the x direction.
Not considering relativity:

Galilean Transformation 
$$\begin{cases} x' = x - vt \\ y' = y \\ z' = z \\ t' = t \end{cases}$$

To convert velocities in the "rest" frame to the moving frame, differentiate x', y', and z' with respect to time:

$$v'_x = v_x - v, \quad v'_y = v_y, \quad v'_z = v_z$$

If we take  $v_x$  to be c, then according to the intuitively correct Galilean transformations, c' = c - v, which is impossible since c is an absolute speed.

An alternate transformation can be proposed:

$$x' = k(x - vt)$$

which *can* reduce to x - vt.

X in terms of x':

$$x = k(x' + vt'), \quad y' = y, \quad z' = z$$

Therefore,

$$x = k^{2}(x - vt) + kvt'$$
$$t' = kt + \left(\frac{1 - k^{2}}{kv}\right)x$$

and

As light has the same speed for all inertial observers:

$$x = ct$$
 and  $x = ct'$ 

we can evaluate *k* and solve for *x*.

$$k(x - vt) = ckt + \left(\frac{1 - k^2}{kv}\right)cx$$

so

$$x = \frac{ckt + vkt}{k - \left(\frac{1 - k^2}{kv}\right)c}$$
$$= ct \left[\frac{1 + \frac{v}{c}}{1 - \left(\frac{1}{k^2} - 1\right)\frac{c}{v}}\right]$$

for x to = ct,

$$\frac{1+\frac{v}{c}}{1-\left(\frac{1}{k^2-1}\right)\frac{c}{v}} = 1$$
$$k = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

Lorentz Transformation:

$$x' = k(x - vt)$$

# **Velocity Addition in Special Relativity**

Non-relativistic Velocity Addition (Galilean Transformation)

Let a train be moving with velocity v with respect to the ground in the x direction. A ball is thrown in the train with velocity u along the x direction. Then the velocity of the ball u' with respect to the ground is (for u and v << c)

Relativistic Velocity Addition:  $u' = v \pm u$ 

If u and v are high enough that relativistic effect cannot be ignored, then the equation above takes the form

$$u' = \frac{v \pm u}{1 \pm \frac{uv}{c^2}}$$

# CHAPTER

# **Optics and Wave Phenomena**

# **A. MECHANICAL WAVES**

# **Basic Properties**

One way of transporting energy from one point to another is through wave motion.

There are two main types of wave motion.

**Transverse waves:** the disturbance in the medium is transverse, or at right angles, to the line of motion of the wave.

The shape of a transverse wave is a sine wave function, like the one shown below.





# **OPTICS AND WAVE PHENOMENA**

where  $\lambda$  = wavelength (distance between adjacent peaks or valleys) A = amplitude (distance between the central position and an extreme position)

**Longitudinal waves:** the disturbance in the medium acts in the same direction that the wave is travelling.

A longitudinal wave appears in a spring as shown below.



Sound waves are an example of longitudinal waves.

The period, T, is the amount of time to complete one oscillation in all types of waves.

The frequency,  $\nu$ , is the number of oscillations per second. In other words, frequency is the inverse of *T* for all types of waves.

$$v = \frac{1}{T}$$

For all waves, the waveform travels a distance  $\lambda$  in time T. Therefore, the speed of the wave, c, is

$$c = \frac{\lambda}{T} = v\lambda$$

# **Superposition Principle**

When two wave-forms are travelling in the opposite direction along the same line of motion, and the waveforms collide, the resultant displacement at a point along the line of motion is simply the algebraic sum of the separate wave disturbances. This is referred to as the principle of superposition.

However, the two waves will leave the collision exactly as they were before the meeting.

The superposition of separate waveforms is known as interference.

**Constructive interference:** The resultant displacement is greater than the individual displacements.

Destructive interference: The resultant displacement is less than the individual displacements.



# Reflection

Reflection occurs when a wave meets a boundary or a place where the medium propagating the wave changes.





### **Beats**

Chapter

When two tones of nearly the same pitch are sounded simultaneously, the human ear encounters a sensation known as beats.

For two sinusoidal waves with frequencies  $\nu_1$  and  $\nu_2$  and the same amplitude, the equation for displacement (the beat equation) becomes

$$y = \left[2A \cos 2\pi \left(\frac{v_1 - v_2}{2}\right)t\right] \cos 2\pi \left(\frac{v_1 + v_2}{2}\right)t$$

The beat frequency,  $\nu_B$ , is

 $\nu_B = \nu_1 - \nu_2$ 

# **B. RAY OPTICS**

# **Reciprocity Principle**

If a ray travels from 1 to 2, a ray will also travel from 2 to 1 by the same route. (See the following figure.)





# **Reflection and Refraction**

# Law of Reflection

$$\theta'_1 = \theta_1$$

# Law of Refraction

$$\frac{\sin\theta_1}{\sin\theta_2} = n_{21}$$

where  $\theta'_1$  = Angle of Reflection  $\theta_1$  = Angle of Incidence  $\theta_2$  = Angle of Refraction

 $n_{21}^2$  = Index of Refraction of medium 2 with respect to medium 1



**Reflection and Refraction** 

# **Total Internal Reflection**

Critical Angle,  $\theta_{c}$ 

$$\sin \theta_c = \frac{n_2}{n_1}$$

$$\theta_c \text{ is the critical angle (total internal reflection)}$$

$$\theta_c \text{ is the critical angle (total internal reflection)}$$

Internal Reflection



# **C. THIN LENSES**

Thin Lens: Any lens whose thickness is small when compared with its radius.

Focal Point: The point at which all rays intersect.

Focal Length: The distance from focal point to the center of the lens.

Real Image: Light rays actually pass through the image location.

Virtual Image: The image is formed by rays that appear to come from the location of the image.

# **Ray Tracing**

Procedure for locating image given object location, lens type, and focal length.

# **Example:**

Lens tracing using a convex lens



- Ray 1: Passes through the center of the thin lens and, therefore, its path is unchanged.
- Ray 2: Is parallel to the lens axis and, therefore, is deviated so that the ray goes through the principle focus,  $F_2$ .
- Ray 3: Passes through  $F_1$  and, therefore, is deviated parallel to the lens axis.

Result: Real image is formed at distance s' from the lens.

Other examples of ray tracing:



# **Convex Lens with Real Image**



# **Convex Lens with Virtual Image**



# **Concave Lens with Virtual Image**



Graphical locations of images for three different object distances and three thin lenses

The other lens equation may be used to find f, s, or s':

$$\frac{1}{f} = \frac{1}{s} + \frac{1}{s'}$$

**NOTE:** For concave lens f < 0; for virtual images s' < 0,

The magnification of the image is

$$m = \frac{s'}{s}$$

# **D. INTERFERENCE**

# Young's Experiment



Diffraction Wave Patterns for Young's Double-Slit Experiment

# **Interference Maxima and Minima**

# Maxima

$$d\sin\theta = m\lambda$$

where m = 0, 1, 2, ... (maxima)

# Minima

$$d\sin\theta = (m - \frac{1}{2})\lambda$$

where  $m = 1, 2, \ldots$  (minima)

- d = distance between slits or point sources
- $\theta$  = angle between the midpoint of the two slits or point sources and a point on the screen
- $\lambda$  = wavelength

# **E. DIFFRACTION**

# Single Slit Diffraction

 $a\sin\theta = m\lambda$ 

where m = 1, 2, 3, ... (minima) a = width of slit  $\theta =$  angle of diffraction  $\lambda =$  wavelength



# **Diffracted Intensity**

$$I_{\theta} = I_m \left(\frac{\sin\alpha}{\alpha}\right)^2$$

where

- $\alpha = \frac{\pi a}{\lambda} \sin \theta$
- $I_{\theta}$  = diffracted intensity  $I_{m}$  = maximum intensity a = width of slit
- $\lambda$  = wavelength
- $\theta$  = angle of diffraction

# **The Rayleigh Criterion**

Rayleigh's Criterion provides the minimum angle  $\theta_R$  that two objects subtend at the lens or slit of an optical system at which the two images formed with their diffraction pattern are just visible as two separate images.

If  $\theta < \theta_R$  the two images will be unresolved.

If  $\theta > \theta_{\rm R}$  the two images are well resolved.



Rayleigh's Criterion for Single Slit:  $\sin \theta_R = \frac{\lambda}{d}$  (in spectrometer)

Rayleigh's Criterion for circular aperture:  $\sin \theta_R = 1.22 \frac{\lambda}{d}$  (in telescope and microscope)

# **Double Slit Diffraction**

$$I_{\theta} = I_m(\cos\beta)^2 \left(\frac{\sin\alpha}{\alpha}\right)^2$$

# **OPTICS AND WAVE PHENOMENA**

where

$$\beta = \frac{\pi d}{\lambda} \sin \theta$$
  

$$\alpha = \frac{\pi a}{\lambda} = \sin \theta$$
  

$$I_{\theta} = \text{diffracted intensity}$$
  

$$I_{m} = \text{maximum intensity}$$
  

$$d = \text{distance between centers of slits}$$
  

$$a = \text{width of slit}$$

 $\lambda$  = wavelength

 $\theta$  = angle of diffraction

# **Multiple Slit Diffraction**

# Maxima

 $d\sin\theta = m\lambda$ 

where m = 0, 1, 2, ...

d = distance between centers of slits

 $\theta$  = angle of diffraction

 $\lambda$  = wavelength

# **Angular Width**

$$\Delta \theta_m = \frac{\lambda}{Nd\cos\theta_m}$$

 $\Delta \theta_m = \text{angular width of the maxima} \\ \lambda = \text{wavelength}$ where

N = number of slits

d = distance between the centers of slits

 $\theta_m$  = angle of diffraction for the principal maximum

# **Diffraction Gratings**

# Dispersion

$$D = \frac{m}{d\cos\theta}$$

D = dispersion (angular separation) where

m =order of maxima

 $\theta$  = angle of diffraction



# **Resolving Power**

$$R=\frac{\lambda}{\Delta\lambda}=Nm$$

where R = resolving power

 $\lambda = \text{mean wavelength}$ 

 $\Delta \lambda$  = wavelength difference

N = number of rulings in the grating

m =order of maxima

# Polarization



Law of Malus gives the intensity of polarized light transmitted through a polarizer. If  $I_1$  is the intensity of the incident polarized light (see figure above) and  $\theta$  is the angle between the polarizer axis and the plane of vibration of the incident polarized light, then the intensity of the transmitted light is given by

$$I_2 = I_1 \cos^2\theta$$

# **Doppler Effect for Sound**

The frequency of sound as heard by an observer may not be the same as that generated by the source. The frequency of sound may be affected by the motion of the source and/or the observer. This phenomenon is called the *Doppler effect*.

Here we consider the cases in which the medium is stationary (no wind) and the source (S) and observer (O) are constrained to move along the line joining them.



Let *v* be the speed of sound in the medium (not affected by the speed of the source or the observer).

- $v_o =$  the speed of the observer
- $v_s$  = the speed of the source
- f = the actual frequency generated by the source
- f' = the frequency as heard by the observer

Then the Doppler frequency f' is given by

$$f' = f \frac{V \pm V_o}{V \pm V_s}$$



**Change in Wavelength:** The wavelength of sound is changed due to the motion of the source only. Motion of the observer does not affect the wavelength of sound. If the source is moving toward the observer, the wavelength received by the observer is

$$\lambda' = rac{V-V_s}{f} = \lambda \left(1 - rac{V_s}{V}
ight)$$

If the source is moving away from the observer, the wavelength received by the observer is

$$\lambda' = \frac{V + V_s}{f} = \lambda \left( 1 + \frac{V_s}{V} \right)$$

# **Standing Waves**

Standing sound waves in pipe organs:



Standing Waves of Sound in Open Tube

**Closed Pipe:** A pipe of length L closed at one end. For the standing sound waves created in the closed pipe, there is always a node at the closed end and an antinode at the open end. Let the speed of sound be v.

For the closed pipe only the odd harmonics are present. Even harmonics are missing.

(Note: The speed of sound does not depend upon what mode the organ pipe is resonating.)

1ST MODE (Fundamental harmonic or 1st harmonic):

$$\lambda_1 = 4L, f_1 = \frac{v}{\lambda_1}$$

2ND MODE (3rd harmonic):

$$\lambda_3=\frac{4}{3}(L),\,=f_3\frac{v}{\lambda_3}=3\;f_1$$

3RD MODE (5th harmonic):

$$\lambda_5=\frac{4}{5}(L),\;f_5=\frac{v}{\lambda_5}=5\;f_1\ldots$$

In general, the resonant frequencies for the closed pipe are given by

$$f_n = \frac{nv}{4L}, n = 1, 3, 5, 7, 9, \ldots$$

**Open Pipe:** For the open pipes, both the ends are open. When a standing wave is set up in the open pipe, there are antinodes at both the open ends. Unlike the closed pipe, all the harmonics are present in the open pipe.



Speed of sound (v) is independant of the modes of vibration  $f = v/\lambda$ 

1ST MODE (Fundamental harmonic or 1st harmonic):

$$\lambda_1=2L,\;f_1=\frac{v}{\lambda_1}$$

2ND MODE (2nd harmonic):

$$\lambda_2=\frac{2}{2}(L),\;f_2=\frac{v}{\lambda_2}=2\;f$$

3RD MODE (3rd harmonic):

$$\lambda_3 = \frac{2}{3}(L), f_3 = \frac{v}{\lambda_3} = 3 f_1 \dots$$

In general, the resonant frequencies for the open pipe are given by

$$f_n = \frac{nv}{2L}, \ n = 1, \, 2, \, 3, \, 4, \, 5, \, \ldots$$

# Note:

- 1. For any standing wave,  $\lambda = 2 \times \text{length of one loop.}$
- 2. Speed of a wave does not depend on modes of vibrations.



# Standing Waves on a Stretched String

**Speed of a wave on a string:** The speed (v) of a transverse wave on a string depends on the tension (T) in the string and the linear mass density  $(\mu)$  of the string and is given by

$$v = \sqrt{\frac{T}{\mu}}$$

#### Standing waves on a string stretched between two points:

When a standing wave is set up on the stretched string, it vibrates to form one or more loops. The points of zero displacements are called **nodes** and those of maximum displacement are called **antinodes**. The length of each loop equals  $2\lambda$  for the wave travelling on the string.

In the first mode the string vibrates with one loop; hence, the wave on the string has maximum wavelength and minimum frequency in the first mode. This frequency is called first harmonic or fundamental frequency. The higher modes have multiples of the fundamental frequency.



Note:

• The wavelength  $\lambda = 2 \times \text{length of one loop.}$ 

• The speed of the wave on a string is  $v = \sqrt{\frac{T}{\mu}}$ .

## **Brewster's Law**

**Brewster's angle:** Let an unpolarized beam of light be incident on the plane surface of a nonconducting transparent medium (such as glass, water) of index of refraction *n*. The angle at which the reflected light is plane polarized is called Brewster's angle  $\theta_B$ . At this angle,



- The reflected and refracted beams are at right angles.
- $\tan \theta_B = n$



Brewster's Law

Table of Information						
Rest mass of the electron	$m_e = 9.11 \times 10^{-31}$ kilogram = 9.11 × 10 <sup>-28</sup> gram					
Magnitude of the electron charge	$e = 1.60 \times 10^{-19}$ coulomb = $4.80 \times 10^{-10}$ statcoulomb (esu)					
Avogadro's number	$N_0 = 6.02 \times 10^{23} \text{ per mole}$					
Universal gas constant	R = 8.314 joules/(mole · K)					
Boltzmann's constant	$k = 1.38 \times 10^{-23}$ joule/K = 1.38 × 10^{-16} erg/K					
Speed of light	$c = 3.00 \times 10^8 \text{ m/s}$ = 3.00 × 10 <sup>10</sup> cm/s					
Planck's constant	$h = 6.63 \times 10^{-34} \text{ joule} \cdot \text{ second}$ = 4.14 × 10 <sup>-15</sup> eV · second					
Vacuum permittivity	$\varepsilon_0 = 8.85 \times 10^{-12} \text{ coulomb}^2/(\text{newton} \cdot \text{meter}^2)$					
Vacuum permeability	$\mu_0 = 4\pi \times 10^{-7}$ weber/(ampere $\cdot$ meter)					
Universal gravitational constant	$G = 6.67 \times 10^{-11} \text{ meter}^{3}/(\text{kilogram} \cdot \text{second}^{2})$					
Acceleration due to gravity	$g = 9.80 \text{ m/s}^2 = 980 \text{ cm/s}^2$					
1 atmosphere pressure	1 atm = $1.0 \times 10^5$ newton/meter <sup>2</sup> = $1.0 \times 10^5$ pascals (Pa)					
1 angstrom	$1 \text{ Å} = 1 \times 10^{-10} \text{ meter}$ 1 weber/m <sup>2</sup> = 1 tesla = 10 <sup>4</sup> gauss					
Rydberg Constant	$R = 1.097 \times 10^7 \mathrm{m}^{-1}$					



# Notations

Vectors—bold letter	F, v
Unit vectors—usually bold letters	x, y, z
Planck's constant	$\frac{h}{2\pi} = \mathbf{h}$
Derivatives-indicated by primed symbols	<i>x'</i> , <i>x</i> "
	d d

(Whether the derivative is with respect to time  $\frac{a}{dt}$  or position  $\frac{a}{dx}$  is apparent from the context of the problem.)

# CHAPTER

# Laboratory Methods

# 8

# **A. DIMENSIONAL ANALYSIS**

Most quantities in mechanics are measured in units that involve mass (*M*), length (*L*), and time (*T*). Hence the dimensions of physical quantities can be expressed in terms of  $M^x L^x T^x$ .

**Examples:** mass  $\rightarrow M^1 L^0 T^0$ , length or distance  $\rightarrow M^0 L^1 T^0$ , time  $\rightarrow M^0 L^0 T^1$ Volume  $\rightarrow M^0 L^3 T^0$ , speed or velocity  $\rightarrow M^0 L^1 T^{-1}$ , force  $= ma \rightarrow M^1 L^1 T^{-2}$ 

A real number is dimensionless or its dimension can be written as  $M^0 L^0 T^0$ . For example, the number  $\frac{1}{2}$  in K =  $\frac{1}{2} mv^2$  and  $\pi$  in area of a circle =  $\pi r^2$ .

A dimensional analysis will reveal that an equation is not correct if the dimensions of the quantities on the two sides of the equations are not equal.

Example: A student writes a formula for the surface of the sphere as

$$A = \frac{4}{3}\pi r^3$$

On the left-hand side, the area A has the dimension of  $M^0L^3T^0$ . On the right-hand side  $\frac{4}{3}\pi$  is dimensionless and  $r^2$  has the dimensions of  $M^0L^2T^0$ , which is not the same as the dimensions of A. Hence, the equation is wrong.

Dimensional analysis can also be used to determine the power of quantities that another quantity depends on.

**Example:** In circular motion, the centripetal force F acting on a particle depends only on its mass m, radius r, and speed v. Determine the formula for F in terms of m, r, and v.



# LABORATORY METHODS

Dimensional analysis will not be able to reveal the constant factor in the equation.

Let  $F = \text{constant} (m^x v^y r^z)$ 

Dimension (F) = Dimension ( $m^x v^y r^z$ )

 $M^{1}L^{1}T^{-2} = (M^{1}L^{0}T^{0})^{x} (M^{0}L^{1}T^{-1})^{y} (M^{0}L^{1}T^{0})^{z} = M^{(x + 0 + 0)}L^{(0 + y + z)}T^{(0 - y + 0)} = M^{x}L^{y + z}T^{-y}$  $\rightarrow x = 1; y + z = 1; -y = -2$ 

Thus, F = (constant)  $m^1 v^2 r^{-1}$  = (constant)  $\frac{mv^2}{r}$ 

# **Data and Error Analysis**

# Averages and Deviations

For *n* measurement of a discrete quantity *x*, the  $\overline{x}$  average is

$$x = \frac{\sum_{i=1}^{n} x_i}{n}$$

Then the standard deviation is given by

$$\sigma = \sqrt{\frac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{n}}$$

The standard deviation of the mean is a measure of precision of the measurements and is given by

$$\sigma_m = \frac{\sigma}{\sqrt{n}}$$

This is the uncertainty  $\Delta x$ . Thus,

$$\Delta x = \frac{\sqrt{\sum_{i=1}^{n} (x_1 - \bar{x})^2}}{n}$$

## **General Formula for Propagation of Errors:**

If a calculated variable z depends on the measured variable x and y with uncertainties  $\Delta x$  and  $\Delta y$ , respectively, then the uncertainty  $\Delta z$  is given by

$$\Delta z = \sqrt{\left[\frac{\partial z}{\partial x}\right]^2 (\Delta x)^2 + \left[\frac{\partial z}{\partial y}\right]^2 (\Delta y)^2}$$

# **Counting Statistics**

 $\sqrt{N}$  **Rule:** In an experiment, if the number of counts recorded is *N*, then the uncertainty in the number of counts is  $\sqrt{N}$ . If an experiment is repeated many times, the number of counts will form a Gaussian spread with average at  $N_{av}$  and standard deviation of  $\sqrt{N_{av}}$ .

If there is only one trial to count N, then the best estimate for N is  $N \pm \sqrt{N}$ .

If count N occurs in time T (as in radioactivity), then the count rate is given by

$$R = \frac{N}{T} \pm \frac{\sqrt{N}}{T}$$

Thus, the uncertainty in R is

$$\delta R = \frac{\sqrt{N}}{T}$$

and uncertainty in N is

$$\delta N = \sqrt{N}$$

Thus, fractional uncertainty in R is

$$rac{\delta R}{R} = rac{rac{\delta N}{T}}{rac{N}{T}} = rac{\delta N}{N} = rac{\sqrt{N}}{N} = rac{1}{\sqrt{N}}$$

# **Michelson Interferometer**





# LABORATORY METHODS

When Mirror 1 moves a distance x, number of circular fringes m move in. This can be used to find the wavelength of the unknown light from the source by the equation:



A monochromatic light of known vacuum wavelength  $\lambda_{vac}$  is used. The gas cell of length L is filled with a gas (or air) at a known pressure. The gas is then gradually evacuated and the number of fringes m moving in is counted. Using the following equation, the index of refraction of the gas can be calculated.

$$2x(n-1) = m\lambda_{\rm vac}$$

# CHAPTER

# **Specialized Topics**

# 9

# A. NEUTRON STARS AND CONSERVATION OF ANGULAR MOMENTUM

Stars eventually run out of their nuclear fusion fuel in a few stages and become neutron stars or black holes. A neutron star is a very compact body of density millions of times greater than the original star. Hence, the radius of the neutron star is very small compared to the star. Therefore, the moment of inertia of a neutron star is tremendously less than the parent star. Since angular momentum is conserved, the angular speed of the neutron is proportionally larger than the parent star. The equation for the conservation of angular momentum is:

 $I_{\text{neutron star}} \omega_{\text{neutron star}} = I_{\text{star}} \omega_{\text{star}}$ 

# **B. RELATIVISTIC DOPPLER SHIFT**

For light and other electromagnetic waves, the relationship is different from that of sound due to application of special relativity.

For observer and source moving toward each other with speed v, the observed frequency and wavelengths are given by:

$$f_o = \sqrt{\frac{1 + \frac{v}{c}}{1 - \frac{v}{c}}} f_e,$$
$$\lambda_o = \sqrt{\frac{1 - \frac{v}{c}}{1 + \frac{v}{c}}} \lambda_e$$



For observer and source moving away from each other with speed v the observed frequency and wavelengths are given by:

$$f_o = \sqrt{rac{1-rac{v}{c}}{1+rac{v}{c}}}f_e, \qquad \lambda_o = \sqrt{rac{1+rac{v}{c}}{1-rac{v}{c}}}\lambda_e$$

The redshift z can be written

Chapter

$$z+1 = \frac{\lambda_o}{\lambda_e} = \sqrt{\frac{1+\frac{\nu}{c}}{1-\frac{\nu}{c}}}$$

In the non-relativistic limit—i.e., when  $v \ll c$ —the approximate expressions are

$$rac{\Delta f}{f} \simeq -rac{v}{c}; \qquad rac{\Delta \lambda}{\lambda} \simeq rac{v}{c}; \qquad z \simeq rac{v}{c}$$

# **C. HUBBLE'S LAW**

Hubble's Law: The redshift in the light from a distant galaxy is proportional to its distance.

The light coming to us from distant galaxies has its wavelength increased due to the Doppler Effect. This phenomenon is called redshift. Virtually all the galaxies (barring a few very close to us) show redshift. One can calculate the speed of a galaxy from the redshift z from the equation

$$z+1 = \frac{\lambda_o}{\lambda_e} = \sqrt{\frac{1+\frac{\nu}{c}}{1-\frac{\nu}{c}}}$$

Hubble analyzed this observation and concluded that those galaxies are moving away from us and also moving away from each other, i.e., the universe is expanding. He derived the following relationship between the velocity of the galaxy and its distance from us:

 $v = H_0 D$ ,  $H_0 = 70.1 \pm 1.3$  (km/s)/Mpc is called the Hubble's constant

# **D. ELECTRON THEORY OF METALS AND SUPERCONDUCTIVITY**

The electron theory of metals describes physical and chemical properties of metals, including bonding, flow of heat and electric current, and superconductivity through metals.

**Meissner effect in superconductors:** A small magnet placed over a superconductor will float over the magnet. This phenomenon is called the Meissner effect.

## NUCLEAR STRUCTURE



The Meissner effect was explained by Fritz and Heinz London. According to their theory, a weak magnetic field B is able to penetrate only a small depth  $\lambda$  below the surface of a superconductor;  $\lambda$  is called London penetration depth. The field decays exponentially to zero inside the bulk of the superconductor and is given by the **London Equation**:

$$\nabla^2 B = \lambda^{-2} B$$

The value of  $\lambda$  for most superconductors is of the order of 100 nm.

**Type I Superconductors:** In these superconductors, the superconductivity is destroyed when the magnetic field rises above a certain critical value  $B_c$ .

**Type II Superconductors:** With these superconductors, there are two critical values for the applied magnetic field viz.  $B_{C1}$  and  $B_{C2}$ . When the field rises above  $B_{C1}$ , an increasing amount of magnetic field penetrates the superconductors which retains zero resistance for currents not too large. When the field rises above  $B_{C2}$ , the superconductivity is lost.

**Fluxon:** The magnetic flux bounded by a supercurrent is quantized. This is a quantum of magnetic flux and is a fundamental physical constant given by

$$\Phi_o = \frac{h}{2e}$$

Its value is 2.067 833 636  $\times$  10<sup>-15</sup> Wb.

The physical, chemical, transport, and bonding of metals can be explained by the Electron Theory of Metals.

# **E. NUCLEAR STRUCTURE**

An atom consists of a **nucleus** and electrons in various shells around it. The nucleus contains all the positive charge of the atom and most of its mass. Each of the electrons carries a negative charge and very little mass.

## **Inside the Nucleus**

The total number of protons in the nucleus is called the *atomic* number (Z). Different elements have different atomic numbers. The neutrons and protons are together called a nucleon. The total number of nucleon is called the mass number of the nucleus (A). Thus, the number of neutrons in the nucleus is N = A - Z. There are no electrons in the nucleus, contrary to what was assumed before the discovery of the neutrons.



Nucleus of Lithium



# What Holds the Nucleons Together?

However, a problem immediately arises when it is assumed that there are protons in the nucleus. Since protons repel each other, the nucleus will break apart. Then how are the protons held together against the repulsive forces in the nucleus. This led to the discovery of the new fundamental force called nuclear strong force.

The *nuclear strong force* is the strongest of all the known fundamental forces. It is a short-range force. This acts between the nucleons only when they are closer than certain distance. This distance is of the order of the size of the nucleus. The nuclear force does not distinguish between the protons and neutrons.

Another nuclear force is called *nuclear weak force*. This force is responsible for the beta decay in which a neutron gives rise to a proton and an electron (beta particle).

# Radioactivity

Some of the elements such as radium, uranium, etc., naturally emit radiation, which are classified as **alpha rays**, **beta rays**, and **gamma-rays**.

**Alpha rays** consist of particles that are same as the nucleus of the helium atom, that is, they consist of two protons and two neutrons. Hence, alpha particles have twice the positive elementary charge and nearly four times the mass of a hydrogen atom.

Beta rays consist of highly energetic electrons.

Gamma rays are short-wavelength, electrically neutral, EM waves.

The rate of disintegration is called activity and is proportional to the number (N) of atom present. Thus, the rate of integration is given by

$$\frac{\mathrm{dN}}{\mathrm{dt}} = -\lambda N$$

 $\lambda$  is called the decay constant of the element.

The SI unit for  $\lambda$  is s<sup>-1</sup>.

The SI unit for the activity A = dN/dt is becguerel (Bq). 1 Bq = 1 disintegration/s.

Solving the above equation we get

$$N = N_0 e^{-\lambda t}$$

where  $N_0$  is the number of nuclei present at time t = 0.

This equation represents an exponential decay relationship.

# Half-Life (T<sub>1/2</sub>)

The time in which half the number of nuclei present initially decays is called **half-life**  $(T_{1/2})$ . Half-life is related to decay constant by

$$T_{1/2} = 0.693/\lambda$$

The equations for the activity (A) and amount of sample (m) left after time t are given by:

$$A = A_0 e^{-\lambda t}$$
$$m = m_0 e^{-\lambda t}$$

The half-life of the radioactive elements can vary from billions of years to a millionth or billionth of a second.

**Secular Equilibrium:** If a radioactive nuclide A produces a radioactive daughter nuclide B, then the number of atoms of B will be constant (equilibrium) when

$$dN_{A}/dt = dN_{B}/dt$$
$$-\lambda_{A}N_{A} = -\lambda_{B}N_{B} \rightarrow \lambda_{A}N_{A} = \lambda_{B}N_{B}$$

or

and

# **Nuclear Fission and Fusion**

In the case of fission and fusion reactions, the total mass of the product is less than the total mass of the reactant. The missing mass is called *mass defect*,  $\Delta \mathbf{m}$ . This missing mass has been converted into energy in the fission or fusion reaction. The mass defect can be calculated by subtracting the total mass of the product particles from the total mass of reacting particles. Thus,  $\Delta \mathbf{m} = \text{mass}$  (reacting particles) – mass (product particles)

The energy thus evolved from an event is called Q-value of the reaction. Thus, the amount of energy per event can be calculated from the equation:

$$E = \Delta mc^2$$

Atomic mass unit (u): This unit is defined as  $(1/12)^{\text{th}}$  the mass of a <sup>12</sup>C atom. Its energy equivalence is:

$$1 u = 931.49 MeV$$

# **Nuclear Fission**

For some elements the nuclei are not stable. One such nucleus breaks apart into two large fragments, one or more neutrons, and a large amount of energy. This phenomenon is called *nuclear fission*. The



kinds of fragments produced are not unique but do follow a certain pattern. Nuclear fission can be spontaneous or induced.

For example, <sup>235</sup>U when bombarded by slow neutrons absorbs the neutron and may break up into a nucleus of <sup>140</sup>La, <sup>97</sup>Br with the emission of two neutrons and a large amount of energy.



These neutrons if slowed down may be absorbed by other <sup>235</sup>U nuclei and produce more fission. When this process becomes *self-sustaining*, it is called *chain reaction*. The *controlled* chain reaction is used in the nuclear power plants to produce electricity.

#### Some fission reactions:

Chapter

Average energy output for  $^{235}$ U fission = 200 MeV per event

# **Nuclear Fusion**

In *nuclear fusion* two nuclei when brought close together may combine to form a larger nucleus and release large amounts of energy. For example, two nuclei of deuterium (heavy hydrogen) may fuse together to form an isotope of He and emission of a neutron. To initiate the nuclear fusion reaction the nuclei have to be brought very close together so that the short-range nuclear forces could overtake. It is,

however, very difficult to bring the two nuclei together because of the mutual repulsion due to their like (positive) charges. In order for them to come close enough, they should be made to move toward each



Two nuclei are close enough to be within the range of nuclear strong force. Strong nuclear attraction fuses the nuclei with emission of energy.

other with very high speeds equivalent to the temperatures of millions of degrees Kelvin. The sun gets its energy from the nuclear fusion reactions, which was initiated at the time of evolution of the solar system. Inside the sun's core, the temperature is 6 million Kelvin, which is sufficient to sustain the fusion reaction. However, on the earth the fusion reaction created so far is of only an uncontrolled nature (hydrogen bomb). This is triggered by creating high temperatures with an atomic bomb (fission bomb) and allowing fusion materials such as heavy hydrogen, to rise to a temperature of millions of Kelvin for the fusion reactions to start. But once started, the fusion reaction is out of control and results in an explosion. Efforts are underway to produce a controlled fusion reaction to obtain clean energy (almost free of radioactive waste) for peaceful purposes.

Some fusion reactions:

$${}^{1}_{1}H \rightarrow \text{proton}, {}^{2}_{1}H \rightarrow \text{deuteron}, {}^{3}_{1}H \rightarrow \text{triton}$$

$${}^{1}_{1}H + {}^{1}_{1}H \rightarrow {}^{2}_{1}H + {}^{0}_{+1}e + \nu + \mathbf{1.44} \text{ MeV}$$

$${}^{1}_{1}H + {}^{2}_{1}H \rightarrow {}^{3}_{2}\text{He} + \gamma + 5.49 \text{ MeV}$$

$${}^{1}_{1}H + {}^{3}_{2}\text{He} \rightarrow {}^{4}_{2}\text{He} + {}^{0}_{+1}e + \nu$$

$${}^{3}_{2}\text{He} + {}^{3}_{2}\text{He} \rightarrow {}^{4}_{2}\text{He} + {}^{1}_{1}\text{H} + \gamma + \mathbf{12.86} \text{ MeV}$$

$${}^{2}_{1}\text{He} + {}^{2}_{1}\text{H} \rightarrow {}^{3}_{2}\text{He} + {}^{0}_{0}\text{n} + 3.3 \text{ MeV}$$

$${}^{2}_{1}\text{H} + {}^{2}_{1}\text{H} \rightarrow {}^{3}_{1}\text{H} + {}^{1}_{1}\text{H} + 4.0 \text{ MeV}$$

$${}^{2}_{1}\text{H} + {}^{3}_{1}\text{H} \rightarrow {}^{4}_{2}\text{He} + {}^{0}_{0}\text{n} + 17.6 \text{ MeV}$$



# **SPECIALIZED TOPICS**

# **Elementary Particles**



#### **Model of Elementary Particles**



Quarks							
Name	Symbol	Antiparticle	Charge (e)	Mass (MeV/c2)			
Up	U	$\overline{u}$	+2/3	1.5–3.3			
Down	d	$\overline{d}$	-1/3	3.5-6.0			
Charm	с	$\overline{c}$	+2/3	1160–1340			
Strange	S	$\overline{S}$	-1/3	70–130			
Тор	t	$\overline{t}$	+2/3	169,100–173,300			
Bottom	b	$\overline{b}$	-1/3	4,130–4,370			

Electron, tauon, and muons are charged leptons, while neutrinos are leptons.

# NUCLEAR STRUCTURE

Leptons							
Name	Symbol	Antiparticle	Charge e	Mass (MeV/c <sup>2</sup> )			
Electron	e <sup>-</sup>	$E^+$	-1	~ 0.511			
Electron neutrino	$ u_{\rm e} $	$ u_{\rm e}^{-} $	0	$< 2.2 \text{ eV/c}^2$			
Muon	$\mu^-$	$M^+$	-1	~ 105.6			
Muon neutrino	$ u_{\mu} $	$ u_{\mu}^{-} $	0	< 0.170			
Tauon	$ au^-$	$T^+$	-1	~ 1,776.8			
Tauon neutrino	$\nu_{ au}$	$ u_{ au}^{-}$	0	< 15.5			

			Charge		Mass		
Name	Symbol	Antiparticle	(e)	Spin	(GeV/c <sup>2</sup> )	Force mediated	Existence
Photon	γ	Self	0	1	0	Electromagnetism	Confirmed
W boson	$W^-$	$W^+$	-1	1	80.4	Weak	Confirmed
Z boson	Z	Self	0	1	91.2	Weak	Confirmed
Gluon	g	Self	0	1	0	Strong	Confirmed
Graviton	G	Self	0	2	0	Gravity	Unconfirmed
Higgs boson	H <sup>0</sup>	Self?	0	0	> 112	See below	Unconfirmed

*Hadrons* are defined as strongly interacting composite particles. All hadrons are composed of quarks which are fermions.

Organization of Fermions								
	First Charge generation		Second generation		Third generation			
Oracla	+2/3	Up	u	Charm	c	Тор	Т	
Quarks	-1/3	Down	d	Strange	s	Bottom	В	
Leptons	-1	Electron	e-	Muon	$\mu^{-}$	Tauon	$\tau^{-}$	
	0	Electron neutrino	ν <sub>e</sub>	Muon neutrino	$\nu_{\mu}$	Tauon neutrino	$\nu_{\tau}$	

# X-ray Crystallography

X-ray diffraction can be used to study crystal structures. As shown in the figure that follows, a beam of X-rays is incident at a grazing angle  $\theta$  to the surface of the crystal. The lattice atoms act as scattering centers for the beam and a diffraction pattern can be detected. The Bragg's equation helps find the distance between the lattice planes.

# SPECIALIZED TOPICS



X-ray Diffraction from a Crystal Surface

Bragg's Law d sin  $\theta = m\lambda$ 

d = spacing between lattice planes  $\theta =$  angle between X-rays and the crystal surface m = order for intensity maximum  $\lambda =$  X-ray wavelength

# **Specific Heat of Gases**

(Questions present; no questions added)

Specific Heat at Constant Pressure  $(C_p)$ : It is defined as the amount of heat required to raise the temperature of 1 mole of a gas through 1 K (or 1°C) *at a constant pressure*. Hence,

$$Q = n C_{p} \Delta T$$

Specific Heat at Constant Volume ( $C_V$ ): It is defined as the amount of heat required to raise the temperature of 1 mole of a gas through 1 K (or 1°C) *at a constant volume*. Hence,

$$Q = n C_v \Delta T$$

For a monatomic gas (such as Ar, He, Ne, H, O, etc.),

$$C_p = 20.78 \text{ J/(mol.K)}, C_V = 12.47 \text{ J/(mol.K)}$$

Internal Energy of Ideal Gas (U): The internal energy change ( $\Delta U$ ) of a given amount (*n* moles) of an ideal gas depends only on the temperature change ( $\Delta T$ ). It is independent of the path between its initial and final states.

$$\Delta \mathbf{U} = \left(\frac{3}{2}\right) \mathbf{n} \mathbf{R} \Delta \mathbf{T}$$

For an *isothermal process*,  $\Delta U = 0$  because  $\Delta T = 0$ .

# GRE Physics Exam 1

Practice Exam 1 is also on CD-ROM in our special interactive GRE Physics TestWare<sup>®</sup>. It is highly recommended that you first take this exam on computer. You will then have the additional study features and benefits of enforced timed conditions and instant, accurate scoring. See page v for guidance on how to get the most out of our GRE Physics software.
# **Answer Sheet: Practice Exam 1**

$1. \land \bigcirc \bigcirc \bigcirc \bigcirc \bigcirc \bigcirc $	34. A B C D E	68.	A	В	$\odot$	D	E
2. A B C D E	35. A B C D E	69.	A	В	©	D	E
3. A B C D E	36. A B C D E	70.	A	В	C	D	E
4. A B C D E	37. A B C D E	71.	A	В	©	D	E
5. A B C D E	38. A B C D E	72.	A	В	C	D	E
6. A B C D E	39. (A) (B) (C) (D) (E)	73.	A	В	C	D	E
7. A B C D E	40. A B C D E	74.	A	В	©	D	E
8. A B C D E	41. A B C D E	75.	A	В	©	D	E
9. A B C D E	42. A B C D E	76.	A	В	$\odot$	D	E
10. A B C D E	43. A B C D E	77.	A	В	©	D	E
11. A B C D E	44. A B C D E	78.	A	В	C	D	E
12. A B C D E	45. A B C D E	79.	A	В	C	D	E
13. A B C D E	46. A B C D E	80.	A	В	C	D	E
14. A B C D E	47. A B C D E	81.	A	В	C	D	E
15. A B C D E	48. A B C D E	82.	A	В	C	D	E
16. A B C D E	49. A B C D E	83.	A	В	C	D	E
17. A B C D E	50. A B C D E	84.	A	В	C	D	E
18. A B C D E	51. A B C D E	85.	A	В	C	D	E
19. A B C D E	52. A B C D E	86.	A	В	C	D	E
20. A B C D E	53. A B C D E	87.	A	B	C	D	E
21. A B C D E	54. A B C D E	88.	A	В	C	D	E
22. A B C D E	55. A B C D E	89.	A	В	C	D	E
23. A B C D E	56. A B C D E	90.	A	В	C	D	E
24. A B C D E	57. A B C D E	91.	A	B	C	D	E
25. A B C D E	58. A B C D E	92.	A	В	C	D	E
26. A B C D E	59. A B C D E	93.	A	В	C	D	E
27. A B C D E	60. A B C D E	94.	A	В	©	D	E
28. A B C D E	61. A B C D E	95.	A	В	C	D	E
29. A B C D E	62. A B C D E	96.	A	В	C	D	E
30. A B C D E	63. A B C D E	97.	A	В	C	D	E
31. A B C D E	64. A B C D E	98.	A	В	C	D	E
32. A B C D E	65. A B C D E	99.	A	В	C	D	E
33. A B C D E	66. A B C D E	100.	A	В	C	D	E
	67. A B C D E						

# **Practice Exam 1**

#### TIME: 170 Minutes 100 Questions

**DIRECTIONS:** Each of the questions or incomplete statements below is followed by five answer choices or completions. Choose the best answer to each question.

1. It is possible that the Newtonian theory of gravitation may need to be modified at short range. Suppose that the potential energy between two masses m and m' is given by

$$V(r) = \frac{Gmm'}{r} = (1 - ae^{-r/\lambda})$$

For short distances  $r \ll \lambda$ , calculate the force between *m* and *m'*.

(A) 
$$F = -\frac{Gmm'}{r^2}$$
  
(B)  $F = -\frac{Gmm'(1-a)}{r^2}$   
(C)  $F = -\frac{Gmm'(1+a)}{r^2}$   
(D)  $F = -\frac{Gmm'(1-a)}{r}$   
(E)  $F = -\frac{Gmm'(1-a)}{r}$ 

2. The Stern-Gerlach experiment in quantum physics demonstrates the quantization of spin. Sample data is shown in the figure. The conclusion is that



- (A) the electron is a fermion and can have spin up or down
- (B) the electron has no spin
- (C) the electron can only have spin up
- (D) the electron can only have spin down
- (E) the electron is a fermion and can have spin 3/2, 1/2, -1/2, or -3/2

- 3. By the early 1900s the two major theories of physics were Maxwell's equations and Newton's laws. The transformation from a lab reference frame to a moving reference frame was achieved by the Galilean transformation (x' = x vt and t' = t). Which of the two major theories was initially thought to be invariant under these transformation equations?
  - (A) Maxwell's equations
  - (B) Neither Maxwell's equations nor Newton's laws
  - (C) Both Maxwell's equations and Newton's laws
  - (D) Newton's laws
  - (E) The invariance is not important.
- 4. A coin placed on a turntable revolves with the turntable without slipping. Which of the following forces is providing the centripetal acceleration to the coin?
  - (A) Static friction

(D) Its weight

- (B) Kinetic friction
- (C) Normal force

- (E) Nuclear strong
- 5. Consider that a coin is dropped into a wishing well. You want to determine the depth of the well from the time *T* between releasing the coin and hearing it hit the bottom. If the speed of sound is 330 m/s, and T = 2.059 s, what is the depth *h* of the well?
  - (A) 20.77 m
  - (B) 19.60 m
  - (C) 23,564 m
  - (D) 18.43 m
  - (E) 39.20 m



- 6. A 10 g bullet (m) is fired into a 2 kg ballistic pendulum (M) as shown in the figure. The bullet remains in the block after the collision, and the system rises to a maximum height of 20 cm. Find the initial speed of the bullet.
  - (A) 28.0 m/s
  - (B) 23.8 m/s
  - (C) 3.98 m/s
  - (D) 719 m/s
  - (E) 398 m/s



### **PRACTICE EXAM 1**

- 7. A horizontal beam of length 10 m and weight 200 N is attached to a wall as shown. The far end is supported by a cable which makes an angle of 60° with respect to the beam. A 500 N person stands 2 m from the wall. Determine the tension in the cable.
  - (A) 0 N
  - (B) 700 N
  - (C) 500 N
  - (D) 231 N
  - (E) 808 N



8. When a 4 kg mass is hung vertically on a light spring that obeys Hooke's law, the spring stretches 2 cm. How much work must an external agent do to stretch the spring 4 cm from its equilibrium position.

(A)	1.57 J	(B)	0.39 J	(C)	0.20 J
(D)	3.14 J	(E)	0.78 J		

- 9. A cylinder with a moment of inertia  $I_0$  rotates with angular velocity  $\omega_0$ . A second cylinder with a moment of inertia  $I_1$  initially not rotating drops onto the first cylinder and the two reach the same final angular velocity  $\omega_f$ . Find  $\omega_f$ .
  - $\begin{array}{ll} ({\rm A}) & \omega_f = \omega_0 \\ ({\rm B}) & \omega_f = \omega_0 I_0/I_1 \\ ({\rm C}) & \omega_f = I_0 \omega_0/(I_0 + I_1) \\ ({\rm D}) & \omega_f = \omega_0 I_1/I_0 \\ ({\rm E}) & \omega_f = \omega_0 (I_0 + I_1)/I_0 \end{array}$



- 10. Three moles of an ideal diatomic gas occupy a volume of 20 m<sup>3</sup> at 300 K. If the gas expands adiabatically to 40 m<sup>3</sup>, then find the final pressure.
  - (A) 162 Pa
    (B) 142 Pa
    (C) 284 Pa
    (D) 182 Pa
    (E) 374 Pa

- 11. Calculate the specific heat of a copper coin using the law of Dulong and Petit, which states that  $C_V = 3R$ .
  - (A) 0.047 cal/g.K
  - (B) 1.0 cal/g.K
  - (C) 0.54 cal/g.K

- (D) 0.27 cal/g.K
- (E) 0.094 cal/g.K
- 12. The tire of an automobile is filled with air to a gauge pressure of 35 psi at 20°C in the summer time. What is the gauge pressure in the tire when the temperature falls to 0°C in the winter time? Assume that the volume does not change and that the atmospheric pressure is a constant 14.70 psi.
  - (A) 49.7 psi (D) 31.6 psi
  - (B) 35 psi (E) 46.3 psi
  - (C) 14.7 psi
- 13. Polarized light of intensity I is incident on a polarizer with a plane at an angle of 45° to the optic axis of the polarizer. The intensity of light I' emerging from the polarizer is
  - (A) 0 (B)  $\frac{1}{4}I$  (C)  $\frac{1}{2}I$ (D)  $(1\sqrt{2})I$  (E)  $\sqrt{2}I$
- 14. One of the following lists the four fundamental forces of nature in order of increasing strength. Choose the correct letter.
  - (A) Gravitational, weak, electromagnetic, nuclear
  - (B) Weak, electromagnetic, nuclear, gravitational
  - (C) Electromagnetic, weak, gravitational, nuclear
  - (D) Weak, gravitational, electromagnetic, nuclear
  - (E) Nuclear, electromagnetic, weak, gravitational
- 15. A metallic chain of length L and mass M is vertically hung above a surface with one end in contact with it. The chain is then released to fall freely. If x is the distance covered by the end of the chain, how much force (exerted by the bottom surface) will the chain experience at any instant during the process?
  - (A) N = Mg Mx''(B) N = 3Mg(C) N = Mg - 2Mx''(D) N = (3M/L)gx(E) N = Mg



16. A thin-shelled sphere of radius  $\rho$  and mass *m* is constrained to roll without slipping on the lower half of the inner surface of a hollow, stationary cylinder of inside radius *R*.



If  $I = \frac{2}{3}m\rho^2$ , the Langrangian function is

(A)  $L = \frac{1}{2}m(R-\rho)^2 \frac{R^2}{\rho^2} (\theta')^2 + \frac{1}{6}mR^2(\theta')^2 + mg(R-\rho)\cos\theta$ 

(B) 
$$L = \frac{1}{6}mR^2(\theta')^2 + mg(R-\rho)\cos\theta$$

(C) 
$$L = \frac{1}{2}m(R-\rho)^2 \frac{R^2}{\rho^2} (\theta')^2 - \frac{1}{6}mR^2(\theta')^2 + mg(R-\rho)\cos\theta$$

(D) 
$$L = \frac{1}{2}m(R-\rho)^2 \frac{R^2}{\rho^2} (\theta')^2 + mg(R-\rho)\cos\theta$$

(E) 
$$L = \frac{1}{2}m(R-\rho)^2 \frac{R^2}{\rho^2} (\theta')^2 + \frac{1}{6}mR^2(\theta')^2 - mg(R-\rho)\cos\theta$$

17. Two equal masses  $m_1 = m_2 = m$  are connected by a spring having Hooke's constant k. If the equilibrium separation is  $l_0$  and the spring rests on a frictionless horizontal surface, then derive  $\omega_0$  the angular frequency.



- (A)  $\sqrt{k/m}$  (D)  $2\sqrt{k/m}$ (B)  $\sqrt{2k/m}$  (E)  $\sqrt{g/l_0}$
- (C)  $\sqrt{3k/m}$
- 18. Which of the following defines a conservative force?
  - (A)  $\oint \mathbf{F} \cdot d\mathbf{A} = 0 \text{ or } \nabla \cdot \mathbf{F} = 0.$  (D) The force must be electromagnetic.
  - (B) The force must be frictional. (E)  $\oint \mathbf{F} \cdot d\mathbf{r} = 0 \text{ or } \nabla \times \mathbf{F} = 0.$
  - (C) The force must be nuclear.
- 19. Consider a particle of mass m at temperature T which follows classical Maxwell-Boltzmann statistics. Find the average speed (v).
  - (A)  $\sqrt{3kT/m}$  (D)  $\sqrt{8kT/\pi m}$
  - (B)  $\sqrt{kT/m}$  (E)  $\sqrt{2kT/m}$

(C) 
$$\sqrt{2kT/\pi m}$$

m T

### **GRE PHYSICS**

- 20. A 0.1 kg mass attached to a spring moves on a horizontal frictionless table in simple harmonic motion with amplitude 0.16 m and period 2 s. Assuming that the mass is released from rest at t = 0 s and x = -0.16 m, find the displacement as a function of time.
  - (A)  $x = 0.16 \cos{(\pi t)}$
  - (B)  $x = -0.16 \cos(\pi t + \pi)$
  - (C)  $x = 0.16 \cos(\pi t + \pi)$
  - (D)  $x = -0.16 \cos(2\pi t + \pi)$
  - (E)  $x = -0.16 \cos\left(\frac{\pi t}{2}\right)$



(C)  $0.035 \text{ cm}^2$ 

- 21. Consider a simple laboratory experiment where the length and width of a rectangle are measured  $l = 5.45 \pm .05$  cm and  $w = 3.86 \pm .02$  cm. Find the uncertainty in the area  $\Delta A$ .
  - (A)  $0.05 \text{ cm}^2$  (B)  $0.02 \text{ cm}^2$ (D)  $0.27 \text{ cm}^2$  (E)  $0.12 \text{ cm}^2$
- 22. Suppose that a man jumps off a building 202 m high onto cushions having a total thickness of 2 m. If the cushions are crushed to a thickness of 0.5 m, what is the man's average acceleration as he slows down?
  - (A) g
  - (B) 133 g
  - (C) 5 g
  - (D) 2 g
  - (E) 266 g



- 23. A ball is thrown horizontally from the top of a tower 40 m high. The ball strikes the ground at a point 80 m from the bottom of the tower. Find the angle that the velocity vector makes with the horizontal just before the ball hits the ground.
  - (A)  $315^{\circ}$  (B)  $41^{\circ}$  (C)  $0^{\circ}$ (D)  $90^{\circ}$  (E)  $82^{\circ}$
- 24. A wheel 4 m in diameter rotates with a constant angular acceleration  $\alpha = 4 \text{ rad/s}^2$ . The wheel starts from rest at t = 0 s where the radius vector to point P on the rim makes an angle of 45° with the x-axis. Find the angular position of point P at arbitrary time t.
  - (A) 45°
  - (B)  $45 + 2t^2$  degrees
  - (C)  $45 + 114.6t^2$  degrees
  - (D)  $4t^2$  degrees
  - (E)  $229.2t^2$  degrees



# **PRACTICE EXAM 1**

25. A common laboratory experiment involves the thermionic emission of electrons from metal surfaces. Use the Richardson-Dushman law,  $J = A_0 T^2 e^{-\phi/kT}$ , to estimate the thermionic emission current density for a tungsten filament at 2000 K. Take  $\phi = 4.55$  eV as the work function and  $A_0 =$  $120 A/cm^2 K^2$  as the Richardson constant.

- (A)  $0.00083 A/cm^2$ (B)  $0.00104 \, A/cm^2$ (C)  $0.00125 A/cm^2$
- (D)  $0.00146 A/cm^2$ (E)  $0.00166 A/cm^2$
- 26. A spherically symmetric point source emits sound waves with a uniform power of 200 W. At what distance will the intensity be just below the threshold of pain? (Assume  $I = 1 W/m^2$ .)
  - (A) 15.92 m (B)  $\infty$  m (C) 7.98 m (D) 7.07 m (E) 3.99 m
- 27. The electric field of a plane EM wave travelling along the z-axis is

$$\mathbf{E} = (E_{0x}\mathbf{x} + E_{0y}\mathbf{y})\sin(\omega t - kz + \phi).$$

Find the magnetic field **B**.

- (A)  $(-E_{0y}\mathbf{x} + E_{0x}\mathbf{y})\cos(\omega t kz + \phi)/c$
- (B)  $(E_{0x} \mathbf{\ddot{x}} + E_{0y} \mathbf{\ddot{y}}) \sin (\omega t kz + \phi)/c$
- (C)  $(-E_{0y}\mathbf{x} + E_{0x}\mathbf{y})\sin(\omega t kz + \phi)/c$ (D)  $(E_{0x}\mathbf{x} + E_{0y}\mathbf{y})\cos(\omega t kz + \phi)/c$
- (E)  $(-E_{0y}\mathbf{x} E_{0x}\mathbf{y})\sin(\omega t kz + \phi)/c$



- (A)  $\alpha$ (E)  $(n + 0.5)\alpha$ (B)  $(n-1)\alpha$
- (C)  $(n + 1)\alpha$



- 29. Find the amount of horizontal deflection of a particle falling freely from height h in the Earth's gravitational field. Let  $\lambda$  be the latitude and  $\omega$  be the Earth's rotational frequency.
  - (A)  $1/3\omega \cos(\lambda) \sqrt{8h^3/g}$ (D)  $\omega R_E \cos(\lambda) \sqrt{2h/g}$ (E)  $\omega \cos(\lambda) \sqrt{8h^3/g}$ (B)  $2\omega \cos(\lambda)h$
  - (C)  $\omega \cos(\lambda) \sqrt{8hg}$
- 30. Calculate the centripetal force required to keep a 4 kg mass moving in a horizontal circle of radius 0.8 m at a speed of 6 m/s. (r is the radial vector with respect to the center.)
  - (A) 39.2 N tangent to the circle (D) -180 N r
  - (B) -30.0 N tangent to the circle (E) 180 N r
  - (C) 144.0 N r

# **GRE PHYSICS**

- 31. Find the tension  $T_2$  in cord 2 for the system drawn below. The system is in equilibrium.
  - (A) 19.6 N
  - (B) 39.2 N
  - $(C) \quad 0 N$
  - (D) 17.0 N
  - (E) 33.9 N



- 32. A simple pendulum of length 50 cm is suspended from the ceiling of a train compartment. If the train accelerates at three-fourths the acceleration of gravity, the equilibrium position of the bob is \_\_\_\_\_ cm vertically below the ceiling.
  - (A) 25 cm (B) 30 cm (D) 50 cm (E) 0 cm
- 33. A wheel 4 m in diameter rotates on a fixed frictionless horizontal axis, about which its moment of inertia is 10 kg m<sup>2</sup>. A constant tension of 40 N is maintained on a rope wrapped around the rim of the wheel. If the wheel starts from rest at t = 0 s, find the



(C) 40 cm

(A) 36 m

length of rope unwound at t = 3 s.

- (B) 72 m
- (C) 18 m
- 34. In Kepler's problem of planetary motion, various values of the eccentricity and hence the energy E classify the orbits according to conic sections. What value of the eccentricity  $\varepsilon$  and the energy E belongs to a parabolic orbit?

(D) 720 m

(E) 180 m

- $\begin{array}{ll} \text{(A)} \quad \varepsilon > 1 \text{ and } E > 0 & \text{(D)} \quad \varepsilon < 0 \text{ and } E < V_{\min} \\ \text{(B)} \quad 0 < \varepsilon < 1 \text{ and } V_{\min} < E < 0 & \text{(E)} \quad \varepsilon = 1 \text{ and } E = 0 \\ \text{(C)} \quad \varepsilon = 0 \text{ and } E = V_{\min} & \text{(E)} & \text{(E)} \quad \varepsilon = 1 \text{ and } E = 0 \\ \end{array}$
- 35. A scientist wants to take a picture of a distant yellow object using a pinhole camera such that the picture is of maximum sharpness. Let  $\lambda$  = wavelength of yellow light, d = diameter of the pinhole, and D = the distance from the pinhole to the film. Find d.
  - (A)  $\sqrt{2.44\lambda D}$  (D)  $\sqrt{\lambda D}$
  - (B)  $1.22\lambda^2/D$  (E)  $\sqrt{1.22\lambda D}$
  - (C) 2.44 $\lambda^2/D$
- 36. A 55-year-old woman has a near point of 100 cm. What lens should be used to see clearly an object at the normal near point of 25 cm? (Find the focal length of the required lens.)
  - (A) 20 cm (B) -33.3 cm (C) 33.3 cm(D) -25 cm (E) 100 cm

## **PRACTICE EXAM 1**

- 37. Consider a Young double-slit experiment where the two slits are spaced d = 0.1 mm apart. If when the screen is at a distance l = 1 m, the first bright maximum is displaced y = 0.5 cm from the central maximum, then find the wavelength of the light.
  - (A) 400 nm (B) 800 nm (C) 1000 nm (D) 500 nm (E)  $5 \times 10^{-8}$  nm
- 38. In general, waves have two velocities, the group velocity and the phase velocity. What is the phase velocity of a relativistic particle?
  - (A) Its physical speed v (D)  $c^2 p/E$
  - (B)  $c\sqrt{1+(mc^2/\hbar k)^2}$  (E) It is not defined.
  - (C)  $\partial \omega / \partial k$
- 39. Two infinite nonconducting sheets of charge are parallel to each other. Each sheet has a positive uniform charge density  $\sigma$ . Calculate the value of the electric field to the right of the two sheets.



40. The dimensions for the universal gravitational constant G is

- (A)  $M^{-1}L^{3}T^{-2}$ (B)  $M^{1}L^{-3}T^{2}$ (C)  $M^{3}L^{-1}T^{-2}$ (D)  $M^{1}L^{1}T^{1}$ (E) None; it is dimensionless.
- 41. A thin rod stretches along the z-axis from z = -d to z = d as shown. Let  $\lambda$  be the linear charge density or charge per unit length on the rod and the points  $P_1 = (0, 0, 2d)$  and  $P_2 = (x, 0, 0)$ . Find the coordinate x such that the potential at  $P_1$  is equal to that at  $P_2$ .
  - (A) 0 (D)  $\sqrt{2}d$
  - (B) *d* (E) 2*d*
  - (C)  $\sqrt{3}d$

×

d

d

- 42. A capacitor is constructed from two square metal plates of area  $L^2$  separated by a distance d. Onehalf of the space between the plates is filled with a substance of dielectric constant ( $\kappa_1$ ). The other half is filled with another substance with constant ( $\kappa_2$ ). Calculate the capacitance of the device assuming that the free space capacitance is  $C_0$ .
  - (A) 0.5  $C_0 \kappa_1 \kappa_2 / (\kappa_1 + \kappa_2)$ (B)  $(\kappa_1 + \kappa_2)C_0$ (C)  $\kappa_1\kappa_2C_0/(\kappa_1 + \kappa_2)$ (D)  $2C_0\kappa_1\kappa_2/(\kappa_1 + \kappa_2)$ (E)  $(\kappa_1 + \kappa_2)C_0/2$



43. Consider the circuit shown below. Calculate the effective resistance of the circuit and use this knowledge to find the current in the 4  $\Omega$  resistor.



44. Consider a plane wave travelling in the positive y-direction incident upon a block of glass of refractive index n = 1.6. Find the transmission coefficient.

- (A) 0.00
- (B) 1.00
- (C) 0.77



45. Which one of the following Maxwell equations implies that there are no magnetic monopoles?

(A)  $\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0}$ (B)  $\nabla \cdot \mathbf{B} = 0$ 

(C) 
$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial T}$$

(D) 
$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial T}$$

- (E) Magnetic monopoles have recently been found.
- 46. The diameter of a horizontal pipe changes along its length. As the water flows from the narrow section into the broader section of the pipe, which of the following is true about the rate of flow R and the speed of water v in the broader section?
  - (A) *R* is the same; *v* decreases. (B) *R* decreases; *v* decreases.
- (D) *R* is the same; *v* increases.
- (E) *R* increases; *v* decreases.
- (C) *R* increases; *v* increases.

47. Consider an hourglass on a scale pictured below at times t = 0, 0.001, and 1 hour. What happens to the scale's measure of weight of the hourglass plus sand combination as the sand falls?



- (A) The reading of weight is constant.
- (B) The reading of weight decreases and then increases.
- (C) The reading of weight increases.
- (D) The reading of weight increases and then decreases.
- (E) The reading of weight increases and then remains constant.
- 48. A wire in *xy*-plane 100 cm in length carries a current of 1 Amp in a region where a uniform magnetic field has a magnitude of 1 Tesla in the *x*-direction. Calculate the magnetic force (including direction) on the wire if  $\theta = 45^{\circ}$  is the angle between the wire and the *x* axis.
  - (A) $0.707 \ z \ N$ (D) $-0.707 \ z \ N$ (B) $1.414 \ z \ N$ (E)0, since I is not parallel to B(C) $-1.414 \ z \ N$
- 49. A permanent magnet alloy of samarium and cobalt has a magnetization  $M = 7.50 \times 10^5 \text{ J/Tm}^3$ . Consider two magnetized spheres of this alloy each 1 cm in radius and magnetically stuck together with unlike poles touching. What force must be applied to separate them?
  - (A) 74 N
     (B) 18.5 N
     (C) 37 N

     (D) 111 N
     (E) 9.3 N
- 50. Recall the equation for a series RLC circuit. Compare this to the parallel resonant circuit shown and find  $R_p$  if a series RLC circuit and the parallel RLC circuit are to have the same equations for the potential of capacitor while they both have the same L, C, and Q.

(A)	$R_P = R$	<i>I</i> <sub>1</sub>	$I_2$	
(B)	$R_p = L$			
(C)	$R_p = 1/C$	_ ``		à.
(D)	$R_p = L^2 / RC^2$	- <	> <sup>H</sup> p	g L
(E)	$R_P = L/RC$			Ĺ
			4	

- 51. In a simple AC circuit involving only a resistor  $R = 50 \Omega$  and a voltage source V, find the linear frequency of the generator if  $V = 0.50 V_m$  at time t = 1/720 s. (Assume V = 0 at t = 0 also.)
  - (A)  $120 \pi$  Hz (B) 21,600 Hz (C) 60 Hz
  - (D) 3400 Hz (E)  $60 \pi \text{ Hz}$

- 52. Use Ampere's law to derive for the magnetic field of a toroid (*N* turns) of inner radius *a* and outer radius *b* at a distance *r* midway between *a* and *b*.
  - (A)  $\mu_0 NI/2\pi(b-a)$ (B)  $\mu_0 NI/\pi(b-a)$
  - (C)  $\mu_0 NI/\pi b$
  - (D)  $\mu_0^0 I / \pi (b-a)$
  - (E)  $4\ddot{\mu}_0 NI/\pi(b-a)$



Cross section of toroid seen from the top



- 53. Two parallel conductors separated by a distance r carry currents  $I_1$  and  $I_2$  in the same direction as shown below. What is the force per unit length exerted on the second conductor by the first?
  - (A)  $(-\mu_0 I_1 I_2 / 2\pi x) \mathbf{x}$

(B) 
$$(\mu_0 I_1 I_2 / 2\pi x) \mathbf{x}$$

- (C)  $(\mu_0 I_1 I_2 / \pi x) \mathbf{x}$
- (D)  $(-\mu_0 I_1 I_2 / \pi x) \mathbf{x}$
- (E)  $(\mu_0 \pi I_1 I_2 / x) \mathbf{x}$
- 54. Consider a series RL circuit with  $R = 10 \Omega$ ,  $L = 10 \mu$ H, and V = 30 volts. Suppose that I = 0 at t = 0. Find the energy stored in the inductor as  $t \rightarrow \infty$ .
  - (A)  $9.0 \times 10^{-5}$  J (B) 90 J (C) 45 J (D)  $4.5 \times 10^{-5}$  J (E)  $1.5 \times 10^{-5}$  J
- 55. Consider a circuit that consists of four resistors (each with  $R = 1 M\Omega$ ), a capacitor ( $C = 1 \mu F$ ), and a battery (V = 10 V) as shown. If the capacitor is fully charged and then the battery is removed, find the current at t = 0.5 s as the capacitor discharges through the same resistors.
  - (A)  $40 \ \mu A$ (D)  $14.7 \ \mu A$ (B)  $20 \ \mu A$ (E)  $5.4 \ \mu A$ (C)  $24.3 \ \mu A$

- 56. Two wires are bent into semicircles of radius *a* as shown. If the upper half has resistance  $2R \Omega$  and the lower half has resistance  $R \Omega$ , then find the magnetic field at the center of the circle in terms of the current *I*.



# **PRACTICE EXAM 1**

- 57. What is the magnetic field at the center of a circular ring of radius r that carries current *I*?
  - (A)  $\mu_0 I/2r$
  - (B)  $\mu_0 I/2\pi r$
  - (C)  $\mu_0 I/r$

58. Consider a Fermi gas of electrons at low but non-zero temperature T. How does the thermal energy vary with the temperature?

(D)  $\mu_0 I/\pi r$ 

- (A) Proportional to T
- (B) Proportional to  $\rho T$
- (C) Proportional to  $T^2/E_{\rm F}(\rho)$

59. How fast must a 2 m stick be moving along its length if its observed length is 1 m as seen from the laboratory frame?

- (A) Length is the same in every reference frame.
- (B)  $0.866 \times 10^{-2} \text{ m/s}$
- (C)  $1.50 \times 10^8$  m/s
- (D)  $2.60 \times 10^8$  m/s
- (E)  $2.12 \times 10^8$  m/s
- 60. In the classic Fizeau experiment to verify the special theory of relativity, the speed of light in a moving liquid of refractive index n is measured. If the speed of the liquid is v, then what is the measured speed of light in the laboratory frame?
  - (A) c/n + v(D) (c/n + v)/(1 - v/cn)(B) *c/n* (E) *c*
  - (C) (c/n + v)/(1 + v/cn)
- 61. An inertial system K' is moving with a velocity 0.8 c relative to the inertial system K along the xx'axes. A particle in *K* has velocity

$$\mathbf{v} = \left(0.5, \sqrt{3}/2\right)c$$



- 62. Light of wavelength 450 nm is incident on a Na surface for which the threshold wavelength of the photoelectrons is 542 nm. Calculate the work function of sodium.
  - (A) 2.76 eV (B) 2.29 eV (C) 0.47 eV (D) 4.76 eV (E) 1.00 eV

- (D) Proportional to  $T^2$
- (E) Proportional to  $T/E_{\rm F}(\rho)$ 
  - ► V

(E) It is equal to zero.





### **GRE PHYSICS**

- 63. In a Coolidge tube, electrons are accelerated to energy of 10 keV for x-ray production. The photon for the x-ray produced have energies
  - (A) from 0-1 keV
  - (B) from 0-5 keV
  - (C) from 0–10 keV

- (D) 10 keV and higher
- (E) at 10 keV
- 64. In a Rutherford scattering experiment, 10 MeV  $\alpha$  particles are scattered by a gold foil 0.1  $\mu$ m thick into a detector whose sensitive area is 10 cm<sup>2</sup> which is placed 50 cm from the target and makes an angle of 45° with the incident beam. Calculate the differential cross section in the center of mass system in barns (b) per steradians (sr). (1 millibarn =  $10^{-28}$  square meters.)



- (A) 2.65 b/sr
- (B) 1.04 b/sr
- (C) 3.78 b/sr
- 65. What was achieved by the recent discovery of the W and Z intermediate vector bosons?
  - (A) The nuclear force was discovered.
  - (B) The gravitational and nuclear forces were interlinked.
  - (C) The proton will decay in  $10^{31}$  years.
  - (D) There are particles called rishons inside the quarks.
  - (E) The electroweak unification was verified.
- 66. Consider a mechanical model of the proton where the spin is due to its rotation. Assume the proton to be a uniform solid sphere and derive the equatorial velocity. (Assume  $m_p = 1.67 \times 10^{-27}$  kg,  $r = 10^{-15}$  m)

(A)	$1.58 imes10^8$ m/s	(D)	$1.87 imes10^8$ m/s
(B)	$3.00 imes10^8$ m/s	(E)	$7.88 imes10^7~{ m m/s}$
(C)	$3.94 imes10^7$ m/s		

- 67. In quantum mechanics, one may picture a wave function in either momentum space or configuration space. If the wave function in momentum space is  $\phi(p) = N/(p^2 + \alpha^2)$ , then calculate the wave function in configuration space (aside from a multiplicative constant).
  - (A)  $e^{-a^2x^2/\hbar^2}$ (B)  $\cos(px/\hbar)$ (C)  $\sin(px/\hbar)$ (E)  $e^{ipx/\hbar}$

68. Calculate the coefficient of reflection for a particle incident on a step potential with  $E > V_0$ . Let

$$k = \sqrt{2mE}/\hbar, \ k' = \sqrt{2m(E-V_0)}/\hbar$$

- (A) R = 0(B) R = 1(C)  $R = \left|\frac{k-k'}{k+k'}\right|^2$ (D)  $R = \left|\frac{k-k'}{k+k'}\right|$ (E)  $R = \left|\frac{k}{k'}\right|$
- 69. Consider the hydrogen molecule H<sub>2</sub> as a rigid diatomic rotor of separation  $r = 1 \times 10^{-10}$  m between two protons. Calculate the energy of the  $\ell = 3$  level in the rotational spectrum.

$$(m_p = 938.280 \times 10^6 \text{ eV}/c^2$$
  

$$h = 4.14 \times 10^{-15} \text{ eV} \cdot \text{s}$$
(A) 0.10 eV
(B) 0.05 eV
(C) 0.15 eV
(E) 0.10 eV
(C) 0.15 eV

- 70. An object of mass 0.4 kg is attached to a spring of Hooke's constant k = 200 N/m and subject to a resistive force -bx' where v = x' is the velocity of the object in m/s. If the damped frequency is 0.995 of the undamped frequency, find the value of b.
  - (A) 3.20 kg/s (B) 1.79 kg/s (C) 1.60 kg/s (D) 0.895 kg/s (E) 0.00 kg/s
- 71. The activity of a radioactive sample drops to 1% of the original activity in 5 hours. The half-life of the element is
  - (A) 0.05 hour
     (B) 500 hours
     (C) 5/ln(100) hour

     (D) 5(ln2/ln100) hours
     (E) (ln2)/5 hour
     (C) 5/ln(100) hours
- 72. According to classical mechanics, the atom will decay in a very short time. Roughly how long does it take for the electron to spiral into the nucleus as it emits electromagnetic radiation?
  - (A)  $10^{-10}$  s (B)  $10^{-8}$  s (C)  $10^{-6}$  s (D)  $10^{-4}$  s (E)  $10^{-2}$  s

73. The  $\mu$ -meson has the same charge as the electron, but a greater mass  $m_{\mu} = 207 m_e$ . Use Bohr theory to find the radius of a  $\mu$ -mesonic atom with nucleus of charge Ze orbited by the  $\mu^-$  as compared to the radius of the hydrogen-like atom.

(A)  $r_{\mu} = r_{H}$  (B)  $r_{\mu} = 207 r_{H}$  (C)  $r_{\mu} = 207^{2} r_{H}$ (D)  $r_{\mu} = r_{H}/207$  (E)  $r_{\mu} = r_{H}/207^{2}$ 

## **GRE PHYSICS**

- 74. X-rays with an energy of 50 keV undergo Compton scattering from a target. If the scattered rays are detected at 45° relative to the incident rays, find the energy of the scattered X-ray.
  - (A) 51.4 keV(B) 48.6 keV(C) 52.8 keV(D) 47.2 keV(E) 50.0 keV



76. In the Mossbauer effect, the absorption by the absorber may be destroyed by moving the source. For the case of <sup>57</sup>Fe in a lattice where the gamma ray has energy 14.4 keV and lifetime  $9.8 \times 10^{-8}$  s, find the minimum source speed necessary to destroy the resonant absorption.

(A)	0.028 cm/s	(B)	0.010 cm/s	(C)	0.100 cm/s
(D)	0.014 cm/s	(E)	0.007 cm/s		

- 77. Calculate the speed of a proton of kinetic energy 1 TeV in the Tevatron at Fermilab in Batavia, Illinois. (Use a Taylor expansion.)
  - (A) 0.999 999 6c
    (B) c
    (C) 0.999 c
    (E) 0.999 94c
- 78. Calculate the thermal energy associated with the reaction, Q, in MeV (with negative and positive referring to endothermic and exothermic, respectively) for the nuclear reaction  ${}^{27}\text{Al}(d,p){}^{28}\text{Al}$  given that  $m({}^{27}\text{Al}) = 26.98154$ ,  $m_d = 2.01473$ ,  $m_p = 1.00794$ , and  $m({}^{28}\text{Al}) = 27.98154$ , all in amu.

(A) 
$$-6.32 \text{ MeV}$$
 (B)  $0.0 \text{ MeV}$  (C)  $6.32 \text{ MeV}$   
(D)  $6.83 \text{ MeV}$  (E)  $-6.83 \text{ MeV}$ 

- 79. The nuclear charge density as found from electron scattering is given by  $\rho(r) = \rho_0/(1 + e^{(r-a)/b})$ . What is the meaning of the fit parameter *b*?
  - (A)  $b = 1.12 \text{ A}^{1/3}$  gives the half density radius.
  - (B) It is merely a fit parameter and has no physical meaning.
  - (C) It is the classical electron radius  $b = e^2/m_e c^2$ .
  - (D) Generally,  $b = \infty$  and r < a, so that  $\rho(r) = \rho_0$ .
  - (E) b = 0.6 fm is related to the surface thickness.

80. A simple wave function for the deuteron is given by  $\phi(r) = A \sin[k(r-a)]/r$  for a < r < a + band  $\phi(r) = Be^{-kr}/r$  for r > a + b. Which of the following expressions can be used to normalize the wave function  $\psi$ ?



- 81. Using the Boltzmann factor, calculate the percentage of hydrogen molecules in the first rotational level relative to the ground state at T = 300 K. Assume that  $r = 1.06 \times 10^{-10}$  m is the appropriate molecular distance.
  - (A) 75% (B) 50% (C) 25%
  - (D) 0% (E) 90%
- 82.  $^{60}_{27}$ Co is a  $\beta^-$  emitter. In addition to the  $\beta^-$  particle, there are two more particles produced one of them is  $^{60}_{28}$ Ni, the other must be
  - (A) a neutron(B) a positron(C) an anti-neutrino(D) a neutrino(E) a photon
- 83. According to Bose-Einstein statistics, there exists a Bose condensate for collections of bosons. What does this mean?
  - (A) As  $T \rightarrow \infty$ , all particles reside in excited states.
  - (B) For  $T < T_C$  (critical temp.), all particles reside in the ground state.
  - (C) Bosons are not physically meaningful particles.
  - (D) Bosons are like fermions.
  - (E) For  $T < T_C$  (critical temp.), bosons dissolve into quarks and gluons.
- 84. Which of the following most correctly describe Moseley's law (where v is the x-ray frequency)?
  - (A) v is proportional to Z for x-rays.
- (D) v is proportional to  $1/Z^2$  for x-rays.
- (B) v is proportional to 1/Z for x-rays.
- (E) v is independent of Z.
- (C) v is proportional to  $Z^2$  for x-rays.

- 85. In the Zeeman effect, the energies corresponding to spectral lines are found to be split into additional lines by an external magnetic field, *B*. The separation of these lines is proportional to
  - (A)  $eB/2m_ec$  (B)  $eB/m_ec$  (C)  $\mu_BB$ (D)  $2\mu_BB$  (E)  $\mu_BB/2$
- 86. Consider a quantum mechanical two particle system for which the wavefunctions are  $\Psi(1, 2)$  and  $\Psi(2, 1)$ . What is the symmetric eigenstate of the exchange operator  $P_{12}$ ?
  - (A)  $\frac{1}{\sqrt{2}} [\Psi(1,2) + \Psi(2,1)]$  (D)  $\Psi(2,1)$ (B)  $\frac{1}{\sqrt{2}} [\Psi(1,2 \ \Psi(2,1)]$  (E) There is none. (C)  $\Psi(1,2)$
- 87. Consider *N* noninteracting bosons in an infinite potential box of width *a*. What is the ground state energy?
  - (A)  $h^2 \pi^2 N/ma^2$ (B)  $h^2 \pi^2/2ma^2$ (C)  $h^2 \pi^2/ma^2$ (D)  $h^2 \pi^2N/4ma^2$ (E)  $h^2 \pi^2N/2ma^2$ -a/2 a/2
- 88. A block of wood is constrained to stay under water by a light string attached to the bottom of the beaker as shown in the diagram. The weight of the block is W and the buoyant force on the block is  $F_B$ . The tension in the string must be
  - (A) 0
  - (B)  $F_B$
  - (C) *W*
  - (D)  $W F_B$ (E)  $F_B - W$
- 89. The specific heat of solids is explained by the Debye theory, which is in agreement with the Law of Dulong and Petit for
  - (A) low temperature
  - (B) high temperature
  - (C) a critical temperature  $T = T_C$  only
  - (D) metals only
  - (E) insulators only



#### **PRACTICE EXAM 1**

- 90. Use the Maxwell velocity distribution, < v >= √(2kT)/m, to find the ratio of the average speed of an N₂ molecule to the escape speed from the surface of the Earth.
  (A) 1.0
  (B) 0.44
  (C) 0.038
  (D) 1.86
  (E) 0.056
  91. An unpolarized beam of light is incident on the surface of a glass plate. It is found that at a certain angle α, the reflected is plane polarized. The index of refraction is then given by
  (A) sinα
  (B) cosα
  (C) tanα
  - (D)  $\cot \alpha$  (E)  $\alpha$
- 92. The lowest nucleon resonance state is the  $\Delta$ , which has a mass of 1232 MeV/c<sup>2</sup> and a width of 120 MeV. Calculate the lifetime of this I = 3/2 nucleon state.
  - (A)  $5.5 \times 10^{-24}$  s (B)  $1.2 \times 10^{-19}$  s (C)  $3.3 \times 10^{-23}$  s (D)  $6.9 \times 10^{-9}$  s (E)  $8.4 \times 10^{-17}$  s
- 93. According to the quark model, hadrons are made up of quarks. What is the quark composition of the proton?
  - (A) The proton is not a hadron.(B) sss(C) uu(D) uu(E) uud
  - (C) udd
- 94. For the one-dimensional harmonic oscillator, the potential energy is  $U = 1/2 kx^2$  and the ground state wave function is

$$\psi_0 = Ce^{-ax^2}$$

Find the constant C. (Note  $\int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi\sigma^2}} e^{-1/2x^2/\sigma^2}$ 

- (A)  $C = \sqrt{2a/\pi}$  (B)  $C = \sqrt{a/\pi}$  (C) C = 1(D)  $C = (2a/\pi)^{1/4}$  (E) C = a
- 95. A spring-mass system is suspended from the ceiling of an elevator. When the elevator is at rest, the system oscillates with period *T*. The elevator now starts and moves upward with acceleration of a = 0.2 g. During this constant acceleration phase, the spring-mass system will have the period of
  - (A)  $\sqrt{0.2}T$  (B)  $\sqrt{(5)}T$  (C) 0.04 T
  - (D) 25 *T* (E) *T*

- 96. Time dilation may be observed in a jet airplane using an atomic clock. Suppose that the plane moves at 600 mph. If the laboratory time is  $\Delta t = 1.00$  s, by what amount is time dilated?
  - (A)  $(1 4 \times 10^{-13})s$ (B)  $(1 + 4 \times 10^{-13})s$
  - (C)  $(1 8 \times 10^{-13})s$
  - (D)  $(1 + 8 \times 10^{-13})s$
  - (E) There is no time dilation for such a low speed.
- 97. In an ideal monoatomic adiabatic expansion, if the volume of the gas doubles from  $V_0$  to  $2V_0$ , then what happens to the temperature?
  - (A) Rises to 1.59  $T_0$ (D) Falls to 0.5  $T_0$ (E) Falls to 0.63  $T_0$ (B) Remains constant
  - (C) Rises to 2  $T_0$

m

- 98. The specific gravity of lead is 11.35 g/cc and the radiation length is 0.53 cm. How much energy is lost by a beam of 10 MeV photons incident on a lead target of 3.00 g/cm<sup>2</sup> thickness?
  - (C) 6.07 MeV (A) 3.03 MeV (B) 10.0 MeV (E) 5.34 MeV (D) 9.01 MeV

99. The bulk modulus of water is  $B = 2.04 \times 10^9$  Pa. Find the wavelength of a wave with a frequency ß

of 2	62 Hz. (Note: $v = \sqrt{\frac{r}{\rho}}$ .				
(A)	10.9 m	(B)	5.45 m	(C)	8.32
(D)	4.16 m	(E)	15.3 m		

100. Which of the following is NOT true for a converging lens in optics?

- (A) A ray parallel to the optic axis passes through the focus.
- (B) A ray originating at the focus is bent parallel to the optic axis.
- (C) A ray through the center of the lens is bent parallel to the optic axis.
- (D) An object placed at the focus has image at infinity.
- (E) An object at infinity has image at the focus.

# **Answer Key**

1.	(B)	18. (E)	35. (A)	52. (B)	69. (B)	86. (A)
2.	(A)	19. (D)	36. (C)	53. (A)	70. (B)	87. (E)
3.	(D)	20. (C)	37. (D)	54. (D)	71. (D)	88. (E)
4.	(A)	21. (D)	38. (B)	55. (E)	72. (A)	89. (B)
5.	(B)	22. (B)	39. (E)	56. (B)	73. (D)	90. (C)
6.	(E)	23. (A)	40. (A)	57. (A)	74. (B)	91. (C)
7.	(D)	24. (C)	41. (C)	58. (C)	75. (E)	92. (A)
8.	(A)	25. (E)	42. (D)	59. (D)	76. (D)	93. (E)
9.	(C)	26. (E)	43. (A)	60. (C)	77. (A)	94. (D)
10.	(B)	27. (C)	44. (D)	61. (A)	78. (C)	95. (E)
11.	(E)	28. (B)	45. (B)	62. (B)	79. (E)	96. (A)
12.	(D)	29. (A)	46. (A)	63. (C)	80. (B)	97. (E)
13.	(C)	30. (D)	47. (E)	64. (D)	81. (A)	98. (C)
14.	(A)	31. (E)	48. (D)	65. (E)	82. (C)	99. (B)
15.	(D)	32. (C)	49. (C)	66. (E)	83. (B)	100. (C)
16.	(A)	33. (B)	50. (E)	67. (D)	84. (C)	
17.	(B)	34. (E)	51. (C)	68. (C)	85. (C)	

# **Practice Exam 1**

# **Detailed Explanations of Answers**

# 1. **(B)**

The force may be found from the derivative of the potential.

$$V(r) = -\frac{Gmm'}{r}(1 - ae^{-r/\lambda})$$
$$\frac{dV}{dr} = \frac{Gmm'}{r^2}(1 - ae^{-r/\lambda}) - \frac{Gmm'}{r}\frac{a}{\lambda}e^{-r/\lambda}$$
$$= \frac{Gmm'}{r^2}\left(1 - ae^{-r/\lambda}(1 + \frac{r}{\lambda})\right) \qquad \frac{r}{\lambda} <<1$$
$$F = -\frac{dV}{dr} \bigg|_{r <<\lambda} = -\frac{Gmm'}{r^2}(1 - a)$$

2. (A)

The two peaks in the intensity versus z plot verify the quantization of spin. The force exerted upon the electron in the atom is given by

$$F_z = \mu_z \frac{\partial B_z}{\partial z}$$

Since  $\mu_z = \pm \frac{1}{2} \mu_B$ , there are two peaks.

### 3. (D)

Newton's laws are invariant under the Galilean transformation equations since

$$x' = x - vt$$
 and  $t' = t$ 

$$v' = \frac{dx'}{dt'} = \frac{dx}{dt} - v$$

Thus,

and 
$$a' = \frac{d^2x'}{dt'^2} = \frac{d^2x}{dt^2} =$$

However, Maxwell's equations are not invariant. This was one of the clues that brought about the development of the special relativity theory.

а

#### 4. (A)

The centripetal force is frictional force pointing toward the center as the coin tries to slip away from the center. Since the coin is not moving relative to the surface, the frictional force must be static.

#### 5. (B)

The total time T is equal to the time t that it takes for the coin to reach the bottom and the time  $t^*$  that it takes for the sound waves to travel back to the ground level.

$$d = \frac{1}{2}gt^{2}$$

$$T = t + t^{*}, t^{*} = \frac{d}{v}$$

$$d = \frac{1}{2}g\left(T - \frac{d}{v}\right)^{2}$$

$$\frac{1}{2}gT^{2} + \frac{1}{2}g\frac{d^{2}}{v^{2}} - gT\frac{d}{v} - d = 0$$

$$d^{2} - d\left(\frac{2v^{2}}{g} + 2vT\right) + v^{2}T^{2} = 0$$

$$v = 330 \text{ m/s} \qquad g = 9.8 \text{ m/s}^{2}$$

$$d^{2} - 23,583.4d + 461,679.5 = 0$$

$$d = \frac{-b \pm \sqrt{b^{2} - 4ac}}{2a} = \frac{23,583.4 - 23,544.2}{2}$$

$$= 19.6 \text{ m}$$

#### 6. (E)

If v' is the velocity of the combined system of the pendulum and the bullet right after the collision, then according to the conservation of linear momentum

$$mv = (m + M)v'$$

	,	L .
		d
		1

From the conservation of energy



Some energy has been lost during the collision and converted to heat.

$$v = \frac{M+m}{m}v' = \frac{M+m}{m}\sqrt{2gy}$$
$$= \frac{2.010}{.010}\sqrt{2(9.8)(.20)}$$

= 398 m/s

# 7. (D)

Since the system is in equilibrium, the sum of the torques with respect to any point must be zero. By choosing the point of the beam to the wall as the reference, we can write:

$$\Sigma \tau = 0, \tau = rF \sin \alpha$$
(500)(2) sin (90°) + (200)(5) sin (90°)
$$-(10)(T) \sin (120°) = 0$$

Therefore, T = 231 N is the desired tension.



# 8. (A)

Work done by the external agent is:

$$W = \int_0^x F \cdot dx$$
$$= \int_0^x kx \, dx$$
$$= \frac{1}{2} kx^2 \begin{vmatrix} x_0 \\ 0 \end{vmatrix}$$
$$= \frac{1}{2} kx^2$$

In the free body diagram for the hanging mass, the downward gravitational force is balanced by the upward spring force.

$$F = mg = kx$$
  

$$k = \frac{mg}{x} = \frac{4(9.58)}{0.02}$$
  
= 1960 N/m  

$$W = \frac{1}{2}kx^{2}$$
  

$$= \frac{1}{2}(1960)(0.4)^{2}$$
  
= 1.57 J



# 9. (C)

From the conservation of angular momentum,

$$(\Sigma L)_0 = (\Sigma L)_f$$
$$I_0 \omega_0 = (I_0 + I_1) \omega_f$$
$$\omega_f = \frac{I_0 \omega_0}{I_0 + I_1}$$



# 10. (B)

The ideal gas law is

pV = nRT

Initially,

$$p_0 = nRT_0/V_0$$
  
= 3(8.314 × 10<sup>7</sup>)(300)/(2 × 10<sup>7</sup>)  
= 3.741 × 10<sup>3</sup> d/cm<sup>2</sup>

In an adiabatic process  $\rightarrow pV^{\gamma} = \text{constant}$ , thus

$$p_0 V_0^{\gamma} = p_f V_f^{\gamma}, \ p_f = p_0 \left(\frac{v_0}{V_f}\right)^{\gamma}$$

where the exponent is

$$\gamma = (C_V + R)/C_V = \left(\frac{5}{2}R + R\right) / \left(\frac{5}{2}R\right) = \frac{7}{5}$$

since a diatomic gas has two extra degrees of freedom.

$$p_f = (3.741 \times 10^3) \left(\frac{2 \times 10^7}{4 \times 10^7}\right)^{7/5}$$
  
Q = 1.42 Pa

11. (E)

The law of Dulong and Petit states that the molar heat capacity is  $C_V = 3R$ 

$$= 3 \left( 8.314 \times 10^7 \frac{\text{erg}}{\text{mol} \cdot \text{K}} \right) \left( \frac{\text{cal}}{4.184 \times 10^7 \text{erg}} \right)$$
$$= 5.96 \frac{\text{cal}}{\text{mol} \cdot \text{K}}$$

Basically, there are six degrees of freedom that contribute to this molar heat capacity. To get the specific heat, we must use the atomic mass of copper

$$C_V = C_V \times \frac{1(\text{mol})}{A(g)}$$
$$= 5.96 \frac{\text{cal}}{\text{mol} \cdot \text{K}} \times \frac{1}{63.5} \frac{\text{mol}}{\text{g}}$$
$$= .094 \frac{\text{cal}}{\text{gK}}$$

Note that in the above equation, A is the mass in grams of Cu.

#### 12. (D)

The absolute pressure *p* is the atmospheric pressure plus the gauge pressure.

$$p = p_A + p_G = 35 + 14.7 = 49.7 \text{ psi} = p_S$$

Now use the ideal gas law

$$pV = nRT, \ \frac{p_w}{P_s} = \frac{T_w}{T_s}$$

assuming constant volume. Thus,

$$P_w = P_s \frac{T_w}{T_s}$$
$$= 49.7 \times \frac{273}{293}$$

= 46.3 psi

$$p_{Gw} = p_w - p_G = 46.3 - 14.7 = 31.6 \text{ psi}$$

#### 13. (C)

By the Law of Malus,  $I' = I \cos^2 \theta$ . If  $\theta = 45^\circ$ ,  $\cos \theta = 1\sqrt{2}$  and  $\cos^2 \theta = \frac{1}{2}$ . Hence,  $I' = \frac{1}{2}I$ .

# 14. (A)

There are four known forces. In order of increasing strength, these are the gravitational force, the weak force, the electromagnetic force, and the nuclear or strong force.

Also, since Einstein, physicists have been trying to verify the forces in a unified field theory.

#### 15. (D)

The net total force exerted on the chain (by both the surface and gravitation) at any time is equal to its mass times the acceleration of its center of mass.



#### **DETAILED EXPLANATIONS OF ANSWERS — PRACTICE EXAM 1**

To find the equation of motion of the center of mass, according to the figure, we can write (all the distances are evaluated with respect to the hanging point):

$$x_{cm} = \frac{\Sigma mx}{\Sigma m} = \frac{(x\lambda)L + (L-x)\lambda\left(x + \frac{L-x}{2}\right)}{L\lambda}$$
$$= x + \frac{L^2 - x^2}{2L}$$
$$\Rightarrow x'_{cm} = x' - \frac{xx'}{L}$$
$$x''_{cm} = x'' - \frac{xx'' + x'^2}{L} \Rightarrow Mx''_{cm} = Mg - N = M\left(x'' - \frac{xx'' + x'^2}{L}\right)$$

N = the normal force of the surface

But x'' = g since the chain is falling freely and also we have

$$x'^2 = 2gx$$

(equation of motion with constant acceleration). So we have

$$N = \frac{M}{L}(xg + 2gx) = \frac{3M}{L}gx$$

#### 16. (A)

L = T - U, where the Lagrangian L is set equal to the difference of kinetic energy T and potential energy U.  $T = (1/2)/mV^2 + (1/2)I\omega^2$ , with the first term representing translational kinetic energy and the second rotational kinetic energy. Now, V is the velocity of the sphere's center of mass and  $\omega$  is its angular velocity. They are related by the equation  $V = (R - \rho)\omega$ . Since  $I = (2/3)m\rho^2$ , we can write the kinetic energy expression as

$$T = (1/2)/m(R - \rho)^2 \omega^2 + (1/2)(2/3)m\rho^2 \omega^2$$

The potential energy can be written as  $U = -mg(R - \rho) \cos\theta$ . We wish to find  $\omega$  in terms of  $\theta$ , as the Lagrangian involves a single generalized coordinate. To do this we note that when the sphere rolls through a given angle  $\beta$ , it traverses a distance of  $(\rho\beta)$  along the circumference of the cylinder. Thus, the corresponding  $\theta$  for that traversal is  $\theta = (\rho\beta)/R$ . Taking the derivative of both sides and solving for  $\omega$ , we obtain  $\omega = (R/\rho)\theta'$ , the prime on theta indicating the first derivative. Substituting, we obtain for the Lagrangian:

$$L = \frac{1}{2}m(R-\rho)^2 \frac{R^2}{\rho^2} (\theta')^2 + \frac{1}{6}mR^2(\theta')^2 + mg(R-\rho)\cos\theta$$

17. **(B)** 

In general

$$x' = \frac{dy}{dx}$$
 and  $y' = \frac{dy}{dt}$ 

 $x_1$  and  $x_2$  are the separation of the two masses from their equilibrium positions along the respective axis parallel to their path of motion.

$$m_1 x_1'' = -kx$$
 where  $x = x_1 - x_2$   
 $m_2 x_2'' = kx$ 



**Note:** x, the total compression or stretching of the spring, is equal to the algebraic difference of  $x_1$  and  $x_2$ . Subtract the two equations to get

$$m(x_1'' - x_2'') = -2kx$$
  

$$m_1 = m_2 = m \text{ and } x_1'' - x_2'' = x''$$
  

$$mx'' + 2kx = 0, \ x'' + \frac{2k}{m} x = 0$$

$$\omega_0^2 = \frac{2k}{m} \Rightarrow \omega_0 = \sqrt{\frac{2k}{m}}$$

since

#### 18. (E)

A conservative force is a force such that

$$\oint \mathbf{F} \cdot d\mathbf{r} = 0 \text{ or } \nabla \times \mathbf{F} = 0$$

these are equivalent conditions since

$$\nabla \times \mathbf{F} = \mathbf{0}$$
$$\int \nabla \times \mathbf{F} \cdot d\mathbf{a} = \int \mathbf{F} \cdot d\mathbf{r}$$

by Stokes' theorem.

$$\oint \mathbf{F} \cdot d\mathbf{r} = 0$$

Stokes' theorem relates the surface integral of the curl to a line integral of the original vector field.

# 19. (D)

The Maxwell-Boltzmann differential probability for speed is

$$f(v)dv = 4\pi cv^2 e^{-mv^2/2kT} \, dV$$

then

$$\langle v \rangle = \int_0^\infty v f(v) dv / \int_0^\infty f(v) dv$$
$$= \int_0^\infty v^3 e^{-\alpha v^2} dv / \int_0^\infty v^2 e^{-\alpha v^2} dv \text{ where } \alpha \equiv m/2kt$$

 $=I_3/I_2$ 

$$I_0 = \int_0^\infty e^{-\alpha v^2} dv = \frac{1}{2} \sqrt{\frac{\pi}{2}}$$
$$I_1 = \int_0^\infty v e^{-\alpha v^2} dv = \frac{1}{2\alpha}$$

and

Recall

We also have:

$$I_n = \int_0^\infty v^n e^{-\alpha v^2} \, dv$$

Using integration by parts, we can write:

$$\begin{aligned} u &= v^{n-1} \\ dy &= v e^{-\alpha v^2} dv \end{aligned} \right\} \Rightarrow \begin{aligned} du &= (n-1)v^{n-2} dv \\ y &= -\frac{1}{2\alpha} e^{-\alpha v^2} \end{aligned} \\ \Rightarrow & I_n = \frac{-v^{n-1}}{2\alpha} e^{-\alpha v^2} \bigg|_{0}^{\infty} -\int_{0}^{\infty} -\frac{1}{2\alpha} e^{-\alpha v^2} (n-1)v^{n-2} dv \\ & I_n = [0-0] + \frac{(n-1)}{2\alpha} \int_{0}^{\infty} e^{-\alpha \mu^2} v^{n-2} dv \\ & I_n = \frac{n-1}{2\alpha} I_{n-2} \end{aligned}$$

Hence, we can write:

$$I_2 = \frac{1}{2\alpha} I_0$$
$$I_3 = \frac{2}{2\alpha} I_1$$

Finally,

$$\langle v \rangle = \frac{\frac{2I_1}{2\alpha}}{\frac{I_0}{2\alpha}} = \frac{2I_1}{I_0} = 2\frac{\frac{1}{2\alpha}}{\frac{1}{2}\sqrt{\frac{\pi}{\alpha}}} = 2\frac{1}{\sqrt{\pi\alpha}} = \sqrt{\frac{8kT}{\pi m}}$$

# 20. (C)

The basic simple harmonic motion equation is

$$x = A \cos (\omega t + \delta)$$
$$A = 16 \text{ cm}, T = 2 \text{ s}$$

The linear frequency is then

$$v = 1/T$$
 or  $v = \frac{1}{2}$  Hz

Hence,  $\omega = 2\pi v = \pi$  rad/s is the angular frequency. Hence, at t = 0:

$$-16 = 16 \cos(+\delta)$$
  
 $\delta = \pi \operatorname{rad}$ 

Therefore,

$$x = 16\cos\left(\pi t + \pi\right)$$

# 21. (D)

From the theory of propagation of error:

$$A = lw$$
  

$$\Delta A = \sqrt{\left(\frac{\partial A}{\partial l}\Delta l\right)^2 + \left(\frac{\partial A}{\partial w}\Delta w\right)^2}$$
  

$$= \sqrt{(w\Delta l)^2 + (l\Delta w)^2}$$
  

$$= lw\sqrt{\left(\frac{\Delta l}{l}\right)^2 + \left(\frac{\Delta w}{w}\right)^2}$$
  

$$= (5.45)(3.86)\sqrt{\left(\frac{0.05}{4.35}\right)^2 + \left(\frac{0.02}{3.86}\right)^2}$$
  

$$= 0.27 \text{ cm}^2$$

# 22. (B)

Use basic kinematics.

$$v^{2} - v_{0} = 2a(x - x_{0})$$

$$v = \sqrt{2ax}$$

$$= \sqrt{2(9.8)200}$$

$$= 62.61 \text{ m/s}$$

$$v^{2} - v_{0}^{2} = 2a(x - x_{0})$$

$$0^{2} - 62.61^{2} = 2a(0.5 - 2)$$

$$a = 1307 \text{ m/s}^{2}$$

$$a = 133 \text{ g}$$

$$g = 9.8 \text{ m/s}^{2}$$

using

#### 23. (A)

With standard kinematics, we get



24. (C)

$$\theta_0 = 45^\circ$$
  
 $\alpha = 4 \text{ rad/s}^2 \times \frac{180^\circ}{\pi \text{ rad}}$   
 $= 720/\pi \text{ deg/s}^2$ 

Now

$$\theta = \theta_0 + \omega_0 t + \frac{1}{2} \alpha t^2$$

is one of the basic rotational kinematics equations.

$$= 45^{\circ} + \frac{360}{\pi}t^2$$
$$= 45^{\circ} + 114.6^{\circ}t^2$$

25. (E)

$$\phi = 4.55 \text{ eV}$$
  
 $kT = (1.381 \times 10^{-16})(2000) \frac{1 \text{eV}}{1.602 \times 10^{-12} \text{erg}}$ 

$$= 0.1724 \text{ eV}$$

The Richardson-Dushman Law states that

$$J = A_0 T^2 e^{-\phi/kT}$$
  
= (120)(2000)<sup>2</sup>e^{-4.55/0.172k}  
$$Q = (E-) 0.00166 \text{ A/cm}^2$$

where  $A_0 = 120 \text{ A/cm}^2 k^2$  has been used.

# 26. (E)

Point source power remains constant as sound waves spread over the region. Since intensity is proportional to the inverse of the area which receives the power, we can write:


Taking the area as the surface area of a sphere

$$r = \sqrt{P/4\pi I}$$
$$= \sqrt{200/4\pi(1)}$$
$$= 3.99 \text{ m}$$

27. (C)

The given electric field vector is

$$\mathbf{E} = (E_{ox}\mathbf{x} + E_{oy}\mathbf{y})\sin(\omega t - kz + \phi)$$
$$\nabla \times \mathbf{E} = \left(\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z}\right)\mathbf{x} + \left(\frac{\partial E_x}{\partial z} - \frac{\partial E_2}{\partial x}\right)\mathbf{y}$$

Taking the curl

 $= E_{0y}k\cos(\omega t - kt + \phi)\mathbf{x} - E_{0x}k\cos(\omega t - kt + \phi)\mathbf{y}$  $= -\frac{\partial \mathbf{B}}{\partial t}$ 

By Faraday's Law

$$\mathbf{B} = -\int (\nabla \times \mathbf{E}) dt$$

integrating

$$= -E_{0y}\frac{k}{\omega}\sin(\omega t - kz + \phi)\mathbf{x} + E_{0x}\frac{k}{\omega}\sin(\omega t - kz + \phi)\mathbf{y}$$
$$= (-E_{0y}\mathbf{x} + E_{0x}\mathbf{y})\frac{1}{c}\sin(\omega t - kz + \phi)$$

28. (B)

Use Snell's law to get:

and

$$n\sin\theta_3 = \sin\theta_4$$

 $\sin \theta_1 = n \sin \theta_2$ 

Taking the index of refraction outside of the prism to be 1.

Using the small angle approximation, we get

$$\theta_1 \approx n \, \theta_2$$



and  $n\theta_3 \approx \theta_4$ 

From geometry

and  

$$\alpha = \theta_2 + \theta_3$$

$$\delta = (\theta_1 - \theta_2) + (\theta_4 - \theta_3)$$

$$= n\theta_2 - \theta_2 + n\theta_3 - \theta_3$$

$$= (n - 1)\alpha$$

Note that  $\delta$  is the internal angle of the small upper triangle.

# 29. (A)

Breaking  $\omega$  down into its x and z components

 $\mathbf{a} = \mathbf{g} - 2\boldsymbol{\omega} \times \mathbf{v}$ 

 $\mathbf{a}_c = 2\omega gt \cos(\lambda)\mathbf{y}$ 

 $\mathbf{v} \approx -gt \mathbf{z}$ 

 $\omega = -\omega \cos \lambda \mathbf{x} + \omega \sin \lambda \mathbf{z}$ 

Use

to find

As the acceleration

 $= \mathbf{g} - 2 \omega \cos(\lambda) gt \mathbf{x} \times \mathbf{z}$ 

and

is the Coriolis acceleration

then

$$v_y = \int a_y dt = \omega g t^2 \cos \lambda$$
$$y = \int v_y dt = \frac{1}{3} \omega g t^3 \cos \lambda$$
$$h = \frac{1}{2} g t^2 \Rightarrow t = \sqrt{2h/g}$$
$$y = \frac{1}{3} \omega \cos \lambda \sqrt{8h^3/g}$$

thus

is the eastward deflection.



# 30. (D)

Use Newton's Second Law

$$F = ma$$

and the centripetal acceleration

 $a = \frac{v^2}{r}$  $F = \frac{mv^2}{r}$ 

$$=\frac{4(6)^2}{0.8}=180 \text{ N}$$

$$\mathbf{F} = -180 \text{ N } \mathbf{r}$$

## **31. (E)**

to get

Using the free body diagram and  $\Sigma F = 0$ , we get

$$T_1 = mg = 2(9.8) = 19.6$$
 N

A second free body diagram is drawn where the strings meet.

$$T_{3y} = T_1 = 19.6 \text{ N}$$
  
 $T_3 = T_{3y}/\sin(30^\circ)$   
 $= 39.2 \text{ N}$   
 $T_2 = T_{3x} = T_3 \cos(30^\circ) = 33.9 \text{ N}$  Fr



32. (C)



 $a = g \tan \theta \rightarrow \frac{3}{4} g = g \tan \theta \rightarrow \tan \theta = \frac{3}{4} \rightarrow \sin \theta = \frac{3}{5}$ 

Also  $\cos \theta = h/50 \rightarrow h = 40$  cm.



# 33. (B)

Use the diameter to find the radius:

$$r = \frac{d}{2} = \frac{4}{2} = 2$$
 m

Now, Newton's second law for rotation gives

$$\Sigma \tau = rF = rT = I\alpha$$
$$\alpha = \frac{rT}{I} = \frac{(2)(40)}{10} = 8\frac{\text{rad}}{\text{s}^2}$$
$$\theta = \theta_0 + \omega_0 \tau + \frac{1}{2}\alpha\tau^2$$
$$s = r\theta = \frac{1}{2}\alpha r\tau^2 = \frac{1}{2}(8)(2)(3)^2$$

= 72 m

34. (E)

The various curves are shown in the figure below.



The parabolic orbit has  $\varepsilon = 1$  and total energy

$$E = T + U = 0$$

# 35. (A)

Let d be the diameter of the pinhole. When the pinhole is large, then the object is imaged as a disc of diameter d' = d. When the pinhole is small, then the Rayleigh criterion gives the image disc diameter in the following way:



 $\sin\theta = 1.22 \, \lambda/d$ 

 $\theta \rightarrow \downarrow$  between light ray to the circumference of the image and the maximum intensity of the image.

Now

 $d/D = 2\theta$  and for  $\theta << 1$  rad

 $d/D = 2 \times 1.22 \times \lambda/d$ 

$$\sin\theta \approx \theta = 1.22 \,\lambda/d$$

thus

or

# $d = \sqrt{2.44\lambda D}$

## 36. (C)

We would like a lens that brings objects from 25 cm to 100 cm. Note that s' is negative because the image is supposed to be located on the same side of the lens as the object.

$$s = +25 \text{ cm}, s' = -100 \text{ cm}$$
  
 $\frac{1}{f} = \frac{1}{s} + \frac{1}{s'} = \frac{1}{25} - \frac{1}{100}$   
 $f = 33.3 \text{ cm}$ 

37. (D)



 $\Delta y = l \sin \theta$ 

is the distance between the maxima.

$$d \sin \theta = \lambda$$
$$\Delta y = l \frac{\lambda}{d}$$
$$\lambda = \frac{d}{l} \Delta y = \frac{10^{-4}}{1} 0.5 \times 10^{-2}$$
$$= 0.5 \times 10^{-6} \text{ m} = 5000 \text{ (500 nm)}$$

38. (B)

$$\begin{split} E &= \sqrt{p^2 c^2 + m^2 c^4} \\ \mathbf{h}\omega &= \sqrt{(\mathbf{h}kc)^2 + (mc^2)^2} \\ \omega &= \sqrt{(kc)^2 + (mc^2/\mathbf{h}k)^2} \\ v_p &= \frac{\omega}{k} = \sqrt{c^2 + (mc^2/\mathbf{h})^2} \\ &= c\sqrt{1 + (mc^2/\mathbf{h}k)^2} \end{split}$$

The group velocity

$$v_g = \frac{\partial \omega}{\partial k}$$

is equal to the physical speed.

39. (E)

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0}$$

According to superposition principle, we can evaluate the electric field produced by each sheet separately and then add them up, because the presence of each sheet does not have any effect on the charge of the other. Therefore, to find the electric field of one of the sheets we can write:

$$\oint \mathbf{E} \cdot d \mathbf{A} = \frac{q}{\varepsilon_0}$$
$$2EA = \frac{\sigma S}{\varepsilon_0}$$



Using a Gaussian pillbox,

$$E=\frac{\sigma}{2\varepsilon_0}$$

 $\mathbf{E}_{\text{tot}} = \frac{\sigma}{\varepsilon_0} \mathbf{x}$ 

Thus,

to the right of the sheets.

# 40. (A)

 $F = \text{mass} \times \text{acceleration} \rightarrow \text{Dim}(F) = M^1 L^1 T^{-2}$ 

 $F = \text{Gm}_1\text{m}_2/\text{r}^2 \rightarrow \text{Dim}(\text{G}) = \text{Dim}(\text{F}) \times \text{Dim}(\text{r}^2)/\text{Dim}(\text{m}_1\text{m}_2) = (\text{M}^1\text{L}^1\text{T}^{-2})(\text{L}^2)/(\text{M}^2) = \text{M}^{-1}\text{L}^3\text{T}^{-2}$ 

41. (C)

$$\theta_{1} = \int k \frac{dq}{r} = k \int_{-d}^{d} \frac{\lambda dz}{\lambda(2d-z)}$$

$$= -k\lambda \ln(2d-z) \Big|_{-d}^{d}$$

$$= K\lambda \ln(3)$$

$$\theta_{2} = \int k \frac{dq}{r} = k \int_{-d}^{d} \frac{\lambda dz}{\sqrt{x^{2}+z^{2}}}$$

$$= k\lambda \ln \frac{\sqrt{x^{2}+d^{2}}+d}{\sqrt{x^{2}+d^{2}}-d}$$

$$\theta_{1} + \theta_{2} \Rightarrow k\lambda \ln(3) = k\lambda \ln \left(\sqrt{x^{2}+d^{2}}+d/\sqrt{x^{2}+d^{2}}-d\right)$$

$$\sqrt{x^{2}+d^{2}} + d = 3\sqrt{x^{2}+d^{2}} - 3d$$

$$2\sqrt{x^{2}+d^{2}} = 4d, x^{2}+d^{2} = 4d^{2}$$

$$\Rightarrow x = \sqrt{3}d$$

42. (D)

$$C_0 = \frac{\varepsilon_0 L^2}{d}$$

D is continuous across boundary where

$$D = \kappa_0 \kappa_1 E_1 = \kappa_0 \kappa_2 E_2 = \frac{q}{L^2}$$

$$V = E_1 \frac{d}{2} + E_2 \frac{d}{2}$$

$$V = \frac{d}{2\kappa_0} \left(\frac{1}{\kappa_1} + \frac{1}{\kappa_2}\right) \frac{q}{L^2}$$

$$C = \frac{q}{V} = \frac{2\kappa_0 L^2}{d} \left(\frac{\kappa_1 \kappa_2}{\kappa_1 + \kappa_2}\right) = \frac{2C_0 \kappa_1 \kappa_2}{\kappa_1 + \kappa_2}$$





43. (A)

$$R_{\text{eff}} = 12 \ \Omega$$
  
 $I = V/R = 12 \text{ V}/12 \ \Omega$ 

Now working backward, the I = 1 A splits first into



$$I_1 = I_2 = \frac{1}{2}A$$

and then  $I_2$  splits into

$$I_3 = I_4 = \frac{1}{4} A$$

# 44. (D)

The incident wave is

$$\mathbf{E}_0 = \mathbf{z}E_0\sin\left(ky - \omega t\right)$$
$$\mathbf{B}_0 = \mathbf{x}B_0\sin\left(ky - \omega t\right)$$

The reflected wave is

$$\mathbf{E}_r = \mathbf{z}E_r\sin\left(ky - \omega t\right)$$
$$\mathbf{B}_r = -\mathbf{x}B_r\sin\left(ky - \omega t\right)$$

The transmitted wave is

$$\mathbf{E}_{t} = \mathbf{z}E_{t}\sin\left(k'y - \omega t\right)$$
$$\mathbf{B}_{t} = \mathbf{x}B_{t}\sin\left(k'y - \omega t\right)$$

Maxwell's equations require

$$\Delta E \mid_{v=0} = 0$$

which leads to

$$E_{0} + E_{r} = E_{t}$$

and also

$$\Delta B \mid_{y=0} = 0, \mid B_0 - B_r = B_t$$

Furthermore,

$$E_0=cB_0,\,E_r=cB_r,~~{\rm and}~~B_t=nE_t/c$$
 
$$E_0\,+\,E_r=E_t,$$

substituting  $B_0$  and  $B_r$  with their values in terms of  $E_0$  and  $E_\nu$ 

$$E_0/c - E_r/c = nE_{t/c}$$

$$\begin{cases} E_0 + E_r = E_t\\ E_0 - E_r = nE_t \end{cases}$$

$$2E_0 = (n+1)E_t$$

thus

$$T = \left|\frac{E_t}{E_0}\right|^2 = \frac{4}{(n+1)^2} = \frac{4}{(2.6)^2} = 0.59$$

45. (B)

 $\nabla \cdot \mathbf{B} = 0$ 

implies that there are no magnetic monopoles. If there were, then we would have

$$\nabla \cdot \mathbf{B} = \rho_{R}$$

with  $\rho_{B}$  a positive or negative magnetic charge density.

It is identical to its integral form which is analogous to Gauss's law for magnetism:

$$\oint \mathbf{B} \cdot d\mathbf{a} = Q_B$$

but we know that the result of the integral is always zero and we cannot have a magnetic monopole.

#### 46. (A)

Rate of flow of water R = Av stays the same due to the continuity equation. Hence, as A increases v must decrease.

## 47. (E)

The amount of sand in air is  $\Delta m$ . Hence, the scale reading is short of mg by  $(\Delta m)g$ . The particles of the falling sand start from  $v_o = 0$  and attain the final velocity  $v = v_o + g\Delta t = g\Delta t$  after falling for time  $\Delta t$  through a height of h. The force applied by the falling sand in time  $\Delta t$  is rate of change of momentum:

$$F = \Delta mv / \Delta t = (\Delta m)g \Delta t / \Delta t = \Delta mg$$

which exactly equals the missing weight  $(\Delta m)g$  of the falling sand.

Hence, the reading in the scale at t = 0.001 s is less than the total weight by  $(\Delta m)g$ , but as soon as the sand reaches the bottom, the impulse applied by the falling sand exactly compensates for the missing weight.



$$F = ILB \sin \theta$$
$$= (1)(1)(1) \left(\frac{1}{\sqrt{2}}\right)$$

= 0.707 Newtons

The direction of **F** is  $-\mathbf{z}$  (right-hand rule).





# **DETAILED EXPLANATIONS OF ANSWERS — PRACTICE EXAM 1**

49. (C)

First we must find from the given magnetization the amount of the magnetic moment  $m = m_1 = m_2$ .

$$M = 7.5 \times 10^{-5} \frac{J}{T \times m^3} = \frac{m}{V}$$
$$V = \frac{4\pi}{3} r^3$$
$$m = \frac{4\pi}{3} (10^{-2})^3 (7.5 \times 10^5)$$
$$= 3.142 J/T$$

Then the magnetic field may be taken as that of a magnetic dipole,

 $F = \mathbf{m} \cdot \nabla \mathbf{B}$ 

$$B = (2 \ m/r^3)(\mu_0/4\pi)$$



The force is then

thus

$$F = m_2 \frac{\partial B_1}{\partial z} = m_2 \frac{\partial}{\partial z} \left(\frac{2m_1}{z^3}\right) \frac{\mu_0}{4\pi}$$
  
=  $\mu_0 6m_1 m_2 / z^4 4\pi, z = 2r$   
=  $\frac{3}{32} \mu_0 m^2 / r^4 \pi$   
$$F = \frac{3}{32} (12.6 \times 10^{-7}) ((3.142)^2 / (10^{-2})^4 (3.142))$$
  
= 37 N

# 50. (E)

For a series RLC circuit:

$$-L\frac{dI}{dt} - RI - V = 0$$
$$Q = CV \Rightarrow I = C\frac{dV}{dt}$$
$$-LC\frac{d^2V}{dt^2} - RC\frac{dV}{dt} - V = 0$$

 $V'' + \frac{R}{L}V' + \frac{1}{LC}V = 0$ 

or

For the parallel RLC circuit shown:

$$Q = CV, I_1 = -\frac{dQ}{dt} = -C\frac{dV}{dt}$$
$$V = R_p(I_1 + I_2) = -L\frac{dI_2}{dt}$$
$$\frac{dV}{dt} = R_p(I_1' + I_2')$$
$$= -CR_pV'' - R_p\frac{V}{L}$$
$$V'' + \frac{1}{R_pC}V' + \frac{V}{LC} = 0$$

 $C = \left[ \begin{array}{c} I_1 & I_2 \\ & &$ 

Hence, for the same L, C, Q we need

$$\frac{1}{R_pC} = \frac{R}{L} \Rightarrow R_p = \frac{L}{RC}$$

# 51. (C)

The time dependent generator voltage is

$$V = V_m \sin (\omega t)$$

$$\frac{1}{2} V_m = V_m \sin (\omega/720)$$

$$\omega/720 = A \sin (\frac{1}{2}) = \pi/6$$

$$w = 120 \ \pi \text{ rad/s}$$



Hence, the linear frquuency is

$$w = w/2\pi = 60$$
 Hz

52. (B)

We use Ampere's law

$$\oint \mathbf{B} \cdot dl = \mu_0 \mathbf{I}_{in}$$

and take the Amperean path as a circle of radius r. Hence,

$$B(2\pi r) = \mu_0 NI$$
$$B = \frac{\mu_0}{2\pi} \frac{NI}{r} \quad r = \frac{b-a}{2}$$
$$B = \frac{\mu_0 NI}{\pi (b-a)}$$

53. (A)



is the magnetic field produced by  $I_1$  at distance r = x. The force that  $I_1$  exerts on  $I_2$  is given by:

$$F_2 = I_2 L B_1$$

Force per unit length =

$$\frac{F_2}{L} = \frac{\mu_0 I_1 I_2}{2\pi x}$$

in the -x direction.

The force is attractive between like currents.

## 54. (D)

Kirchhoff's law tells us that

$$V - RI - LI' = 0$$





Differentiating, we get

$$I'' + \frac{I'}{\tau} = 0$$
 where  $\tau = \frac{L}{R}$ 

is the time constant of the circuit. The solution is

$$I = rac{V}{R}(1-e^{-t/ au}), \ au = rac{L}{R}$$

Hence,

$$I_{\infty} = \frac{V}{R} = \frac{30}{10} = 3 A$$
$$U_{\infty} = \frac{1}{2} L I_{\infty}^{2}$$
$$= \frac{1}{2} (10 \times 10^{-6} H) (3a)^{2}$$
$$= 4.5 \times 10^{-5} J$$



# 55. (E)

The law for parallel resistors is

$$\frac{1}{R_t} = \Sigma \frac{1}{R}$$

Hence,

$$\frac{1}{R_T} = \frac{4}{R}$$
$$R_T = \frac{1}{4}R = \frac{1}{4}M\Omega$$



The reduced circuit is a basic RC circuit. From Kirchhoff's law

$$-RI - \frac{Q}{C} = 0 \text{ or } Q' + \frac{Q}{\tau} = 0$$

## **DETAILED EXPLANATIONS OF ANSWERS — PRACTICE EXAM 1**

,

with  $\tau = RC$  as the time constant. The solution is

$$Q = Q_0 E^{-t/\tau}$$
  
 $\tau = RC = \frac{1}{4} \times 10^6 \times 1 \times 10^{-6} = \frac{1}{4}s$   
 $I = I_0 e^{-t/\tau}$   
 $I_0 = \frac{V}{R} = 10 \times 10^6 / \frac{1}{4} \times 10^6 = 40$  A  
 $= 40e^{-.5/.25}$   
 $= 5.4$  uA

# 56. (B)



Now reduce the series resistor:

$$\frac{1}{R_T} = \frac{1}{R} + \frac{1}{2R} = \frac{3}{2R}, R_T = \frac{2}{3}R$$

$$V = R_T I = \frac{2}{3}RI$$

$$I_1 = \frac{V}{R_1}, I_1 = \frac{V}{2R} = \frac{1}{3}I, I_2 = \frac{V}{R} = \frac{2}{3}I$$

$$\mathbf{B} = \mathbf{B_1} + \mathbf{B_2} = \frac{\mu_0}{4a}z(I_2 - I_1) = \frac{\mu_0I}{12a}\mathbf{z}$$

**57. (A)** Use

$$d\mathbf{B} = \frac{\mu_0}{4\pi} \cdot \frac{Id\ell \cdot \mathbf{r}}{r^2}$$

the Biot-Savart Law where  $\mathbf{r}$  is the unit radial vector. Only the *z* component of *B* is non-zero at the origin.



## 58. (C)

For a low temperature Fermi gas, it is true that

$$E_T = \frac{5\pi^2}{12} \frac{T_2}{E_F(\rho)}$$

is the thermal energy where

$$E_F(\rho) = \frac{\hbar^2}{2 \text{ m}} \left(\frac{3\pi^2}{2}\rho\right)^{2/3}$$

is the electron Fermi energy.

## 59. (D)

The Lorentz transformation equation is

$$x' = \gamma(x - vt)$$
$$\Delta x' = \gamma \Delta x \text{ with } \Delta t = 0$$

or

The Lorentz contraction formula is

$$l = 2\gamma l_0$$

$$2 = \gamma l$$

$$\gamma = 2 = \frac{1}{\sqrt{1 - \beta^2}}$$

$$\beta = \sqrt{1 - \frac{1}{\gamma^2}} = \sqrt{\frac{3}{4}}$$

$$\beta = \frac{v}{c}$$

$$v = \frac{\sqrt{3}}{2}c = 2.60 \times 10^8 \text{ m/s}$$

# 60. (C)

The liquid is at rest in K' where the speed of light is v' = c/n. In the lab frame we must add the velocity v' to the liquid speed v, relativistically:

$$V_3 = \frac{V_1 + V_2}{1 + V_1 V_2 / c^2}$$
$$= \frac{c/n + v}{1 + \frac{c}{n} v/c^2}$$
$$= \frac{c/n + v}{1 + c/vn}$$

61. (A)

$$v_{x} = 0.5c, v_{y} = \frac{\sqrt{3}}{2}c, \beta = 0.8 \Rightarrow v = 0.8c$$

$$v_{x}' = \frac{v_{x} - v}{1 - \frac{v_{y}}{c^{2}}} = \frac{0.5 - 0.8}{1 - (0.8)(0.5)}c = -0.5c$$

$$v_{y}' = \frac{v_{y}}{\gamma(1 - \frac{v_{y}}{c^{2}})}$$

$$\sqrt{3/2}$$

$$\sqrt{3/2}$$

$$\sqrt{3/2}$$

$$\sqrt{3/2}$$

$$\gamma = \frac{1}{\sqrt{1 - B^2}} = 1.67 = \frac{5}{3}$$
$$V'_y = \frac{\frac{\sqrt{3}}{2}}{\frac{5}{3}(1 - (0.8)(0.5))}c = \frac{\sqrt{3}}{2}c$$
$$= 180^\circ - 60^\circ = 120^\circ$$

Note that

$$v = \sqrt{v_x'^2 + v_y'^2} = c$$

since the speed of light is invariant.

## 62. (B)

By conservation of energy

$$hv = \phi + T$$

At threshold

$$hv_0 = \phi$$
 and  $v_0 = c/\lambda_0$   
 $\phi = hc/\lambda_0$ 



may be used to get

= 12,400 eV.Å/5420 Å = 2.29 eV

## 63. (C)

Most energetic x-ray photons have energy  $h\nu = \text{KE}$  of the incident electron which is 10 keV. For electrons that lose some of their energy to heat, the x-ray photons have less energy all the way up to zero.

## 64. (D)

$$\frac{d\sigma}{d\Omega} = \left(\frac{k}{4T_0'}\right)^2 \left(\sin\left(\frac{\theta}{2}\right)\right)^{-4}$$
$$k = \frac{1}{4\pi\varepsilon_0} Z_1 Z_2 e^2$$

 $T'_0$  is kinetic energy in C.M. system.

 $T_0$  is kinetic energy in Lab system.

 $\theta$  is angle in C.M. system.

 $\psi$  is angle in Lab system.

Given:  $\psi = 45^{\circ}$  and  $T_0 = 10$  MeV

Since the mass of an  $\alpha$  particle is much smaller than the mass of a gold nucleus,

$$\begin{split} \Psi &\approx \theta \text{ and } t_0 \approx t'_0 \\ \frac{d\sigma}{d\Omega} &\approx \left(\frac{k}{4T_0}\right)^2 \left(\sin\left(\frac{\Psi}{2}\right)\right)^{-4} \\ &\approx \left(\frac{79(2)1.44}{4(1)}\right)^2 \sin^{-4}(22.5^\circ) \\ &\approx 1,509 \frac{\mathrm{fm}^2}{\mathrm{sr}} \times \frac{10 \ \mathrm{mb}}{1 \ \mathrm{fm}^2} \\ &\approx 15,090 \ \frac{\mathrm{mb}}{\mathrm{sr}} \\ &\approx 15.09 \ \frac{\mathrm{b}}{\mathrm{sr}} \end{split}$$

## 65. (E)

Carlo Rubbia and his collaborators at CERN verified the theory which unifies electricity, magnetism, and the weak force. The mass of the  $W^{\pm}$  or  $Z^0$  is about 90 GeV/c<sup>2</sup>. Electricity and magnetism were connected by James Maxwell into electromagnetism. The new unification is called the electroweak theory.

## 66. (E)

The moment of inertia of a sphere is

$$I = \frac{2}{5}mr^2$$

Hence,

$$L = I\omega = \frac{2}{5}mr^2\omega$$

But also

 $S = \frac{1}{2}h$ 

Since the proton is a fermion

$$\frac{2}{5}mr^2\omega = \frac{1}{2}h$$

$$v = r\omega = \frac{1}{2}h\frac{5}{2}\frac{1}{mr} = \frac{5}{4}\frac{h}{mr}$$

$$= \frac{5}{4}(1.055 \times 10^{34})/(1.673 \times 10^{-27})(10^{-15})$$

 $= 7.88 \times 10^7$  m/s

# 67. (D)

 $\phi(p) = N/(p^2 + \alpha^2)$ 

Take the Fourier transform to get

$$\Psi(x) = \frac{1}{\sqrt{2\pi\hbar}} \int_{-\infty}^{\infty} dp \theta(p) e^{z'px/\hbar}$$
$$= \frac{N}{\sqrt{2\pi\hbar}} \int_{-\infty}^{0} \frac{e^{z'px/\hbar}}{p^2 + \alpha^2} dp + \int_{0}^{\infty} \frac{e^{x'px/\hbar}}{p^2 + \alpha^2} dp$$
$$= \frac{N}{\sqrt{2\pi\hbar}} \int_{0}^{\infty} \frac{e^{z'px/\hbar} + e^{-z'px/\hbar}}{p^2 + \alpha^2} dp$$

Use Euler's theorem

$$e^{i\theta} = \cos \theta + i \sin \theta$$

$$\Psi(x) = N \sqrt{\frac{2}{\pi h}} \int_0^\infty \frac{\cos(px/h)}{p^2 \alpha^2} dp$$

$$= \sqrt{\frac{2N^2}{\pi h^3}} \int_0^\infty \frac{\cos kx}{k^2 + \frac{d^2}{h}} dk$$

$$= \frac{N}{h} \sqrt{\frac{2}{\pi h}} \frac{h\pi}{2\alpha} e^{-\alpha |x|/h} = \frac{N}{\alpha} \sqrt{\frac{\pi}{2h}} e^{-\alpha |x|/h}$$

where we have used some knowledge of the Fourier cosine transform. Thus,

$$\psi(x) \propto e^{-\alpha 1 x 1/\hbar}$$

68. (C)



The Schrödinger equation is

 $H\Psi = E\Psi$ 

where H = T + U is the Hamiltonian operator and *E* is the energy eigenvalue. The solution to this equation is

$$\Psi(x) = \begin{cases} e^{ikx} + re^{-ikx}, & x < 0 \quad k = \sqrt{2mE}/\hbar\\ te^{ik'x}, & x > 0 \quad k'\sqrt{2m(E - V_0)}/\hbar \end{cases}$$
  
$$\Psi(0) \text{ continuous} \rightarrow 1 + r = t \text{ or } r = t - 1$$
  
$$\Psi'(0) \text{ continuous} \rightarrow ik - ikr = ik't,$$
  
$$t = \frac{k}{k'} - \frac{k}{k'}r$$
  
$$r = \frac{k}{k'} - \frac{k}{k'}r - 1$$

Now

$$r\left(1+\frac{k}{k'}\right) = \frac{k}{k'} - 1, \quad r = (k-k')/(k+k')$$

is the amplitude of the reflected wave and

$$R = rr^* = \left|\frac{(k-k')}{(k+k)}\right|^2$$

is the reflection coefficient.

69. (B)

The rotational energy eigenvalue is

$$E_l = \frac{\hbar^2 \ell (\ell + 1)}{2\mu r^2}$$

where l is the angular momentum quantum number and r is the relative distance.

$$\mathbf{h} = 4.14 \times 10^{-15} \text{ eV.s}$$

$$m_p = 938.280 \times 10^6 \text{ eV}/c^2 = m_1 = m_2$$

$$H \qquad H$$

The reduced mass is

$$\mu = m_1 m_2 / (m_1 + m_2)$$
  
=  $m_p / 2 = 469.140 \times 10^6 \text{ eV} / c^2$   
 $E_3 = (4.14 \times 10^{-15})^2 3(4) / 2(469.140 \times 10^6)(1)^2$   
= 0.05 eV

70. (B)

By Newton's second law,

$$\Sigma F = -kx - bx' = mx''$$

The equation of motion is then

$$x'' + \frac{b}{m}x' + \frac{k}{m}x = 0 \text{ where } \omega_0^2 = \frac{k}{m}$$

gives the square of the rational frequency

$$x = Ae^{-bt/2m}\cos(\omega t + \phi)$$

is a solution.

$$\boldsymbol{\omega} = \sqrt{\omega_0^2 - \frac{1}{4} \frac{b^2}{m^2}}$$

is the damped frequency

$$0.995\omega_0 = \sqrt{\omega_0^2 - \frac{1}{4}\frac{b^2}{m^2}}$$
$$0.010\omega_0^2 = \frac{1}{4}\frac{b^2}{m^2} = 0.010\frac{k}{m} \text{ since } \omega_0^2 = \frac{k}{m}$$

Hence,

$$b = \sqrt{0.040 \text{ km}} = \sqrt{0.040(200)(0.4)}$$
  
= 1.789 kg/s

#### 71. (D)

Activity A is given by  $A = A_o e^{-\lambda t} \rightarrow 0.01 A_o = A_o e^{-5\lambda} \rightarrow 100 = e^{5\lambda} \rightarrow 5\lambda = \ln(100)$ But  $\lambda = \ln 2/T$ , hence,  $5(\ln 2/T) = \ln(100) \rightarrow T = 5(\ln 2/\ln 100)$  hours.

#### 72. (A)

Larmor's formula states that an electron circling a nucleus with centripetal acceleration a emits energy with rate

$$p = \frac{1}{4\pi\varepsilon_0} \times \frac{2}{3} \times \frac{e^2 a^2}{c^3} = \frac{dU}{dt}$$

From this it may be shown that the electron will spiral into the nucleus in about  $10^{-10}$  s.

#### 73. (D)

Using F = ma with  $a = v^2/r$  as the centripetal acceleration and  $F = kq_1q_2/r^2$  as the Couloumb force, we have

$$F = \frac{mv^2}{r} = \frac{kZe^2}{r^2}$$

From Bohr theory

 $L = mvr = n\mathbf{h}$   $v = \frac{n\mathbf{h}}{mr}$   $\frac{m\left(\frac{n\mathbf{h}}{mr}\right)^2}{r} = \frac{kZe^2}{r^2}$   $r_{\mu} = \frac{n^2\mathbf{h}^2}{kZe^2m_{\mu}}$   $r_{H} = \frac{n^2\mathbf{h}^2}{kZe^2m_{e}}$   $r_{\mu} = \frac{m_{e}}{r_{\mu}} = \frac{1}{r_{\mu}}$ 

$$r_{\mu} = \frac{m_e}{m_{\mu}}r_H = \frac{1}{207}r_H$$

where  $r_H$  is the radius of the Hydrogen-like atom in Bohr theory.

thus

е

initial

φ

final

# 74. (B)

The Compton shift is

$$\Delta \lambda = \lambda' - \lambda$$
$$\lambda' - \lambda = \lambda_c (1 - \cos \theta)$$

 $\lambda_c = hc/m_e c^2$ 

where

is the Compton wavelength



Thus

is the scattered photon energy. Note that the photon loses energy to the electron.

#### 75. (E)

Consider the initial and final states in the two frames:



Now  $\mathbf{u}_1 = \mathbf{v}_1/2 = \mathbf{v}_1 - \mathbf{v}_1/2$ hence,  $\mathbf{u}_1' = \mathbf{v}_1' - \mathbf{v}_1/2$ as in the figure at the right. by geometry and also  $\alpha = \psi$ 

$$\dot{\theta} = 2\psi$$

since  $|u'_1| = \frac{v_1}{2}$  making the triangle isosceles.

# 76. (D)

The Mossbauer effect is the recoilless resonance emission/absorption of nuclear radiation. For the  $^{57}\mathrm{Fe}$  case

$$E_0 = 14.4 \text{ keV}, \quad t = 9.8 \times 10^{-8} \text{ s}$$

is given. The energy width is

 $\Gamma = \frac{h}{t} = \frac{(1.055 \times 10^{-27})}{(9.8 \times 10^{-8})(1.602 \times 10^{-12} \text{ erg/eV})}$  $= 6.72 \times 10^{-9} \text{ eV}$ 

One destroys resonant absorption in the lattice via the Doppler shift

$$\Delta E \ge \Gamma, \quad \frac{v}{c} E_0 \ge \Gamma$$
$$v \ge \Gamma c/E_0 = \frac{(6.72 \times 10^{-9})(2.998 \times 10^{10})}{14.4 \times 10^3}$$
$$v_{\min} = 0.014 \text{ cm/s}$$

77. (A)

$$T = 1 \text{ TeV} = 10^3 \text{ GeV}$$
  
 $m_p = 0.938 \text{ GeV}$   
 $E = m_p + T = 10^3 + 0.938 \text{ GeV} = 1000.938 \text{ GeV}$   
 $\gamma = E/m = 1000.938/0.938 = 1067$ 

$$= \frac{1}{\sqrt{1 - \beta^2}}$$
$$1 - \beta^2 = \frac{1}{\gamma^2}$$
$$\beta = \sqrt{1 - \frac{1}{\gamma^2}}$$

However,

$$\sqrt{1+x} = f(0) + f'(0)x = 1 + \frac{1}{2}x$$

using a Taylor expansion. Hence,

$$\beta = 1 - \frac{1}{2\gamma^2} = 1 - 4 \times 10^{-7}$$

$$= 0.9999996,$$

so the answer is 0.9999996c. It is incredible how close to the speed of light one can get!

78. (C)

Consider the reaction

 $I + T \rightarrow E + R$ 

where I = incident particle, T = target, E = emitted particle, and R = residue which is also written

T(I, E) R

The *Q*-value is

$$Q = m_I + m_T - m_E - m_R$$
  
= (2.01473 + 26.98154 - 1.00794 - 27.98154) amu  
× 931.502 MeV/amu  
= 6.32 MeV

# 79. (E)

The parameter b is a measure of the surface thickness. The falloff distance is approximately the same for all nuclei. R. Hofstadter of Stanford University was awarded the Nobel Prize for these experiments with electron scattering.

#### 80. (B)

In general, probability P of finding the particle is  $P = \int_V \psi \psi^* dV$ , where the product of the wave function and its conjugate takes place over volume V. If the integration is carried out over all space, P = 1.

Therefore, we can write

$$P = 1 = \int_{V} \Psi \Psi^* \, dV = \int \phi \phi^* \, r^2 dr \int Y_{00} Y_{00}^* \, d\Omega$$

The first integral on the right represents the radial part of the wave function and the second, its spherical harmonic angular part.

Thus, integrating over all space and substituting, we obtain the expression

$$1 = \int_{a}^{a+b} A^{2} \sin^{2} k(r-a)dr + \int_{a+b}^{\infty} B^{2} \frac{e^{-2kr}}{r^{2}} dr$$

Note that the normalization constant will be a function of A and B after integration is carried out.

#### 81. (A)

The rotational energy eigenvalues of the Schrödinger equation are:

$$E_i = j(j+1)\hbar^2/2\mu r^2$$

The hydrogen atom consists of a proton and an electron:

$$m_H = 938.280 + 0.511 = 938.791 \text{ MeV}$$

The reduced mass is therefore

$$\mu = m_1 m_2 / (m_1 + m_2) = m_H / 2 = 469.396 \text{ MeV}$$

and

$$r = 1.06 \text{ Å}$$

anu

is the given relative distance 
$$r = |\mathbf{r}_2 - \mathbf{r}_1|$$
.

$$kT = (1.38 \times 10^{-16})(300)/(1.602 \times 10^{-12}) = 0.0259 \text{ eV}$$
$$E_0 = 0$$
$$E_1 = \frac{\hbar^2}{\mu r^2}$$
$$= (1973.50^2/(469.396 \times 10^6)(1.06)^2$$
$$= 0.00739 \text{ eV}$$
$$N_1/N_0 = \ell^{-E_1/kT}$$
$$= 0.75$$

# 82. (C)

The reaction can be written as

$$^{60}_{27}$$
 Co  $\rightarrow^{60}_{28}$  Ni+  $^{0}_{-1}$  e+----

Applying the conservation of atomic number (charge) and mass number (# of nucleons) to the above reaction, the missing particle must have mass # = 0 and atomic number = 0. Moreover, conservation of lepton number implies the particle must be an antilepton because electron is a lepton. Hence, the particle is an anti-neutrino.

# 83. (B)

The Bose condensation phenomenon occurs for low temperatures T less than a critical temperature  $T_c$  where all particles reside in the lowest state.

#### 84. (C)

Moseley discovered that the frequency of x-ray emission v is proportional to the square of the atomic number Z of the emitting system. This follows quite naturally from quantum theory since

$$E_n = -13.6 \text{ eV } Z^2/n^2$$

is the energy eigenvalue for a hydrogen-like atom and thus the transition energy

$$E_m - E_n = h\nu \propto Z^2$$

produces a photon of frequency  $\nu$ .

## 85. (C)

In the Zeeman effect, the frequency is shifted by an angular frequency

$$\Delta \omega = \pm eB/2m_e c$$
$$= \pm \mu_B B/\hbar$$

thus the energy shift is

$$\Delta E = \mathbf{h} \Delta \omega$$
$$= \pm \mu_B B$$

#### 86. (A)

By definition of the exchange operator,

$$P_{12}\Psi(1,2) = \Psi(2,1)$$
$$\Psi^{s}(1,2) = \frac{1}{\sqrt{2}}(\Psi(1,2) + \Psi(2,1))$$

## **DETAILED EXPLANATIONS OF ANSWERS — PRACTICE EXAM 1**

then

$$P_{12}\Psi^{s}(1,2) = (1)\Psi^{s}(1,2)$$

or

$$P_{12}\Psi^{s}(1,2) = \frac{1}{\sqrt{2}}(\Psi(1,2) + \Psi(2,1))$$

#### 87. (E)

The potential energy function is

$$V(x) = \begin{cases} \infty & x < -a/2 \\ 0 & -a/2 < x < a/2 \\ \infty & x > a/2 \end{cases}$$

The Schrödinger equation

$$H\Psi_n = E\Psi_n$$

with Hamiltonian H = T + U has eigenvalues given by

$$\Psi_n(x) = \cos(n\pi x/a)$$
$$\frac{-\mathbf{h}}{2m}\frac{d^2\Psi_m}{dx^2} = \frac{\mathbf{h}^2 n^2 \pi^2}{2ma^2}\Psi_n$$

since

$$E_n = nE = \frac{\hbar^2 \pi^2}{2ma^2} n^2$$

for a single boson. In the ground state, n = 1. For N such bosons,

$$E_t = NE_1$$

#### 88. (E)

Consider all the forces acting on the block: W downward,  $F_B$  upward, and tension downward. The total force  $= 0 \rightarrow F_B - W$  - Tension  $= 0 \rightarrow$  Tension  $= F_B - W$ .

#### 89. (B)

The law of Dulong and Petit states that  $C_V = 3 nR$  where *n* is the number of moles and  $R = N_A k$  is the ideal gas constant. In the Debye theory one finds that  $C_V$  increases from zero at T = 0 K to approach 3 *nR* at high temperature. Therefore, the law of Dulong and Petit is valid only for high temperatures where  $C_V \approx 3 nR$ .

## 90. (C)

One may find  $\langle v \rangle$  from

$$\frac{\int vf(v)dv}{\int f(v)dv}$$

using the Maxwell-Boltzmann distribution f(v).

$$\langle v \rangle = \sqrt{\frac{2kT}{m}}$$
  
=  $\sqrt{2(1.381 \times 10^{-16})(300)/(28/6.022 \times 10^{23})}$   
= 4.22 × 10<sup>4</sup> cm/s

The escape speed from the surface of the earth is equal to the velocity of a particle whose kinetic energy is equal to its gravitational potential energy at the surface of the earth. Hence:

$$\frac{1}{2}mv^2 = \frac{GMm}{R}$$

$$v = \sqrt{2GM/R}$$

$$= \sqrt{2\frac{(6.672 \times 10^{-8})(5.98 \times 10^{27})}{6.38 \times 10^8}}$$

$$= 1.12 \times 10^6 \text{ cm/s}$$

Thus,  $\langle x \rangle / v = 0.038$  which explains the large presence of N<sub>2</sub> in the atmosphere.

## 91. (C)

According to Brewster's Law,  $\tan \theta_{\rm B} = n$ ; hence,  $\tan \alpha = n$ .

# 92. (A)

$$\Delta E = 120 \text{ MeV}$$
  
 $\tau = \frac{h}{\Delta E}$ 

Using the uncertainty principle

= 197.35/120  
= 
$$1.64 \frac{\text{fm}}{\text{c}} \times \frac{10^{-13} \text{cm}}{3 \times 10^{10} \text{cm/s}}$$
  
 $\tau = 5.5 \times 10^{-24} \text{ s}$ 

# **DETAILED EXPLANATIONS OF ANSWERS — PRACTICE EXAM 1**

## 93. (E)

The u quark has charge  $\frac{2}{3}$  and the d quark has charge  $-\frac{1}{3}$ . Hence, the combination

uud

has charge

 $\frac{2}{3} + \frac{2}{3} - \frac{1}{3} = 1$ 

the charge of the proton.

94. (D)

$$U = \frac{1}{2} kx^2, \Psi_0 = ce^{-ax^2}$$

are given

 $\int_{-\infty}^{\infty} \Psi_0^* \Psi_0 \, dx = 1$ 

by the normalization condition

$$\int_{-\infty}^{\infty} C^2 e^{-2ax^2} dx = 1$$

Now we know that

$$\int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2}\frac{x^2}{\sigma^2}} dx = 1$$

using the standard Gaussian probability density function. Thus,

$$\int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}\sqrt{1/4a}} e^{-\frac{1}{2}\frac{x^2}{1/4a}} dx = 1$$

Hence:

$$C^2 = \frac{1}{\sqrt{2\pi}} 2\sqrt{a} = \sqrt{2a/\pi}$$

$$C = (2a/\pi)^{1/4}$$

#### 95. (E)

The period of spring mass system is given by  $T = 2\pi \sqrt{(m/k)}$  independent of gravity. Hence, the acceleration of the elevator will not affect the period.

96. (A)

$$t = \gamma \left( t' + \frac{vx'}{c^2} \right)$$

is one equation of the Lorentz transformation. Hence

$$\Delta t' = \sqrt{1 - \beta^2} \ \Delta t = \Delta t / \gamma \text{ using } \Delta x' = 0$$

Now since  $\beta \ll 1$ , we may use a Taylor expansion.

$$\sqrt{1+x} \approx 1 + \frac{1}{2}x$$

and get

$$\Delta t' = \left(1 - \frac{1}{2}\beta^2\right)\Delta t$$

converting

$$600\frac{\text{mi}}{\text{hr}} \Rightarrow 960\frac{\text{km}}{\text{hr}}$$

$$= \left(1 - \frac{1}{2} \left(960 \frac{\text{km}}{\text{hr}} \frac{1000 \text{ m}}{\text{km}} \frac{\text{hr}}{3600 \text{ s}} \frac{1}{3 \times 10^8 \text{m/s}}\right)^2\right) (1.00 \text{ s})$$
$$= (1 - 4 \times 10^{-13}) \text{ s}$$

a small but measurable difference!

# 97. (E)

The adiabatic gas law is

$$pV^{\gamma} = p_0 V_0^{\gamma}$$
$$pV = nRT$$

is the ideal gas equation of state for the final situation

$$p_0 V_0 = nRT_0$$

and the initial situation

$$n\frac{RT}{V}V^{\gamma} = n\frac{RT_0}{V_0}V_0^{\gamma}$$
$$\frac{T}{T_0} = \left(\frac{V_0}{V}\right)^{\gamma-1} = \left(\frac{V_0}{V}\right)^{2/3}$$

## **DETAILED EXPLANATIONS OF ANSWERS — PRACTICE EXAM 1**

Taking  $\gamma = \frac{5}{3}$  for a monatomic ideal gas with 3 degrees of freedom. Thus,

$$\frac{T}{T_0} = \left(\frac{1}{2}\right)^{2/3} = 0.63$$

# 98. (C)

The particle area density is related to the normal density by

$$\rho_t = \rho_x$$

where x is the physical target thickness.

$$x = \rho_t / \rho = \frac{(3.00 \text{ g/cm}^2)}{(11.35 \text{ g/cm})^3} = .265 \text{ cm}$$

The kinetic energy loss is exponential:

$$\Delta T = T_0 e^{-x/L}$$
  
= 10e^{-0.265/0.53}  
= 6.07 MeV

99. (B)

$$B = 2.04 \times 10^{9} \text{ Pa}$$
  

$$\rho = 1 \text{ g/cm}^{3} = 10^{3} \text{ kg/m}^{3}$$
  

$$v = \sqrt{\frac{B}{\rho}} = \sqrt{\frac{2.04 \times 10^{9}}{10^{3}}} = 1430 \text{ m/s}$$
  

$$\lambda = \frac{v}{f} = \frac{1430}{262} = 5.45 \text{ m}$$

## 100. (C)

A ray through the center of the lens is not bent at all.



# GRE Physics Exam 2

Practice Exam 2 is also on CD-ROM in our special interactive GRE Physics TestWare<sup>®</sup>. It is highly recommended that you first take this exam on computer. You will then have the additional study features and benefits of enforced timed conditions and instant, accurate scoring. See page v for guidance on how to get the most out of our GRE Physics software.
# **Answer Sheet: Practice Exam 2**

1. A B C D E	34. $(A)$ $(B)$ $(C)$ $(D)$ $(E)$	68. A B C D E
2. A B C D E	35. A B C D E	69. A B C D E
3. (A) (B) (C) (D) (E)	36. A B C D E	70. A B C D E
4. (A) (B) (C) (D) (E)	37. A B C D E	71. A B C D E
5. A B C D E	38. A B C D E	72. A B C D E
6. A B C D E	39. A B C D E	73. A B C D E
7. A B C D E	40. $(A)$ $(B)$ $(C)$ $(D)$ $(E)$	74. A B C D E
8. A B C D E	41. A B C D E	75. A B C D E
9. A B C D E	42. A B C D E	76. A B C D E
10. A B C D E	43. A B C D E	77. A B C D E
11. A B C D E	44. A B C D E	78. A B C D E
12. A B C D E	45. A B C D E	79. A B C D E
13. A B C D E	46. A B C D E	80. A B C D E
14. $A \oplus C \oplus E$	47. A B C D E	81. A B C D E
15. A B C D E	48. A B C D E	82. A B C D E
16. A B C D E	49. A B C D E	83. A B C D E
17. A B C D E	50. A B C D E	84. A B C D E
18. A B C D E	51. A B C D E	85. A B C D E
19. A B C D E	52. A B C D E	86. A B C D E
20. A B C D E	53. A B C D E	87. A B C D E
21. A B C D E	54. A B C D E	88. A B C D E
22. A B C D E	55. A B C D E	89. A B C D E
23. A B C D E	56. A B C D E	90. A B C D E
24. A B C D E	57. A B C D E	91. A B C D E
25. A B C D E	58. A B C D E	92. A B C D E
26. A B C D E	59. A B C D E	93. A B C D E
27. A B C D E	60. A B C D E	94. A B C D E
28. A B C D E	61. A B C D E	95. A B C D E
29. A B C D E	62. A B C D E	96. A B C D E
$30. \land \bigcirc \bigcirc \bigcirc \bigcirc \bigcirc$	63. A B C D E	97. A B C D E
31. A B C D E	64. A B C D E	98. A B C D E
32. A B C D E	65. A B C D E	99. A B C D E
33. A B C D E	66. A B C D E	100. A B C D E
	67. A B C D E	

# **Practice Exam 2**

Time: 170 Minutes **100 Questions** 

Directions: Each of the questions or incomplete statements below is followed by five answer choices or completions. Choose the best answer to each question.

- The displacement vector **r** of a point mass may be expressed in cylindrical coordinates. In that rep-1. resentation, determine the velocity vector  $\mathbf{v} = d\mathbf{r}/dt$ .
  - (A)  $\mathbf{v} = \dot{x}\hat{x} + \dot{y}\hat{y} + \dot{z}\hat{z}$
  - (B)  $\mathbf{v} = \dot{\rho}\hat{\rho} + \rho\dot{\theta}\hat{\theta} + \dot{z}\hat{z}$
  - (C)  $\mathbf{v} = \dot{\rho}\hat{\rho} + \rho\dot{\theta}\sin\theta\hat{\theta} + \dot{z}\check{z}$
  - (D)  $\mathbf{v} = \dot{r}\hat{r} + \rho\dot{\theta}\hat{\theta} + r\dot{\phi}\sin\theta\hat{\phi}$
  - (E)  $\mathbf{v} = r\hat{r} + r\dot{\phi}\hat{\theta} + r\dot{\phi}\hat{\phi}$



- (A)  $\partial T/\partial t$
- (B)  $\mathbf{F} \cdot \mathbf{v}$
- (C)  $\nabla U$
- (D) dU/dt
- (E)  $\partial U/\partial t$
- In the realistic fall of a spherical object in fluid air, calculate the magnitude of the viscous force. 3. Given that the Reynolds number is  $R_e = 0.5$ , the kinematic viscosity is 0.149 cm<sup>2</sup>/s, the radius of the sphere is 0.005 cm, and  $\rho_{air} = 1.22 \times 10^{-3}$  g/cc.

(A)	0.128 md	(B)	5.22 md	(C)	1.66 md
(D)	3.22 md	(E)	2.15 md		





т

4. Determine the corrected value for the time of flight of a projectile near the Earth's surface (in two dimensions) subject to a resistive force  $\mathbf{F}_{R} = -b\mathbf{v}$ . Let  $\gamma = b/m$ .

(A)	$t = \frac{2v_{0y}}{g}$	(D) $t = \frac{2v_{0y}}{g} \left(1 + \frac{\gamma v_{0y}}{3g}\right)$
(B)	$t = \frac{2v_{0y}}{g} \left( 1 - \frac{\gamma v_{0y}}{3g} \right)$	(E) $t = \frac{2v_{0y}}{g} \left(1 + \frac{\gamma v_{0y}}{g}\right)$
(C)	$t = \frac{2v_{0y}}{g} \left( 1 - \frac{\gamma v_{0y}}{g} \right)$	

- 5. In the photoelectric effect, the threshold wavelength is 275.6 nm. If light of wavelength 170 nm is incident on a metal substance, determine the kinetic energy of the photoelectrons.
  - (A) 4.50 eV
    (B) 2.25 eV
    (C) 3.60 eV
    (D) 2.79 eV
    (E) 7.29 eV

- 6. X-rays of wavelength  $3 \times 10^{-10}$  m are incident on a substance. The scattered X-rays observed at 45° have a different wavelength due to the Compton effect. Find the scattered wavelength.

(D)  $2.93 \times 10^{-10}$  m (E)  $3.07 \times 10^{-10}$  m

- (A)  $3.02 \times 10^{-10} \,\text{m}$
- (B)  $2.98 \times 10^{-10} \text{ m}$
- (C)  $3.01 \times 10^{-10} \text{ m}$
- 7. In statistical physics, the counting factor N! is very important. Approximate this for large N.
  - (A)  $(N/e)^N$  (B)  $N^N$  (C) N(D)  $(N/ln 2)^N$  (E) N(N-1)
- 8. The *N*-step random walk in two dimensions (with step length 1) looks very much like the famous Brownian motion which supported the kinetic theory. What is the root mean square distance from the origin?
  - (A)  $\sqrt{N/4}$
  - (B)  $\sqrt{2N/3}$
  - (C)  $\sqrt{N}$
  - (D)  $\sqrt{N/2}$
  - (E)  $\sqrt{N/3}$



9. As shown in the figure below, a wedge-shaped block of mass 2M is placed on a horizontal surface and a small block of mass M is placed at the top of the wedge at a height h.



All the surfaces are frictionless. The block is released and it slides down the wedge on to the horizontal surface. The speed of the wedge must be

- (A) 0 (B)  $\sqrt{(gh/3)}$  (C)  $\sqrt{(gh/2)}$ (D)  $\sqrt{(2gh)}$  (E)  $\sqrt{(3gh)}$
- 10. Determine the laboratory threshold kinetic energy  $T_N$  for the reaction

$$p + p \rightarrow p + p + \pi^0$$
.

The target is at rest and the projectile is accelerated to have kinetic energy  $T_{N'}$  Let  $m_p = m_N = 0.938$  GeV and  $m_{\pi} = 0.140$  GeV.



11. What is the correct relativistic Lagrangian which yields the Lorentz force law

$$d\mathbf{p}/d\tau = \gamma q(\mathbf{E} + \mathbf{u} \times \mathbf{B})?$$

Let  $u^{\mu} = (\gamma \mathbf{u}, \gamma \mathbf{c})$  be the 4-velocity and  $A^{\mu} = (\mathbf{A}, \phi)$  be the 4-potential where  $\gamma = 1/\sqrt{1-\beta^2}$ .

(A)  $\frac{1}{2}mu^2 + \int q(E+\upsilon \times B) \cdot d\rho$  (D)  $\frac{\gamma}{2}mu^{\mu}u_{\mu} + \gamma qA^{\mu}u_{\mu}$ (B)  $\frac{1}{2}m\gamma u^2 - \int \gamma q(E+\upsilon \times B) \cdot d\rho$  (E)  $\frac{1}{2}mu^{\mu}u_{\mu} - \int \gamma q(E+\upsilon \times B) \cdot d\rho$ (C)  $\frac{1}{2}mu^{\mu}u_{\mu} + qA^{\mu}u_{\mu}$  12. Find the correct four-dimensional Lorentz transformation matrix for a boost in the y-direction  $K \rightarrow K'$ . Let  $x^{\mu} = (\mathbf{r}, ct)$  be the 4-distance.

(A)	$\left(\begin{array}{cccc} \gamma & 0 & 0 & -\beta\gamma \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\beta\gamma & 0 & 0 & 0 \end{array}\right)$	(D) $\begin{pmatrix} \gamma & 0 & 0 & \beta \gamma \\ 0 & 1 & 0 & 0\gamma \\ 0 & 0 & 1 & 0 \\ \beta \gamma & 0 & 0 & \gamma \end{pmatrix}$
(B)	$\left(\begin{array}{rrrrr} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \gamma & -\beta\gamma \\ 0 & 0 & -\beta\gamma & \gamma \end{array}\right)$	(E) $ \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \gamma & 0 & -\beta\gamma \\ 0 & 0 & 1 & 0 \\ 0 & -\beta\gamma & 0 & \gamma \end{pmatrix} $
(C)	$\left(\begin{array}{cccc} 0 & \gamma & 0 & -\beta\gamma \\ -\beta\gamma & 0 & 0 & \gamma \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right)$	

- 13. In the Rutherford scattering of  $p + \frac{238}{92}$ U, the differential cross section at angle  $\theta$  is measured to be 10 barns. The kinetic energy of the incident proton is 7.6 MeV. Find  $\theta$ .
  - (A) 30.0° (B) 60.0°
  - (C) 90.0°

  - (D) 43.6°
  - (E) 21.8°



14. Calculate the vector force due to a potential energy  $U = kr^n$ .

- (A)  $-knr^{n-2}$  **r** (B)  $- knr^{n-1}$  **r**
- (C) +  $knr^{n-2}$  **r**

- (D) +  $knr^{n-1}$  **r** (E) +  $knr^n$  r
- 15. Study the problem of a rocket in a constant gravitational field  $g = 9.8 \text{ m/s}^2$ .
  - If the initial velocity is 0.40 km/s, the burn time is 100 s, the exhaust velocity is 2.0 km/s, and the mass decreases by a factor of three, find the final velocity.
  - (A) 0.81 km/s
  - (B) 0.98 km/s
  - (C) 2.40 km/s
  - (D) 1.62 km/s
  - (E) 1.42 km/s



- 16. The spherical region a < r < b is filled with mass of uniform density  $\rho$ . Determine the magnitude of the gravitational field in this region.
  - (A)  $g = 4/3 \pi G \rho [r a^3/r^2]$ (B)  $g = 4/3 \pi G \rho b^3/r^2$ (C)  $g = 4/3 \pi G \rho [b^3 - a^3]/r^2$ (D)  $g = 4/3 \pi G \rho a^3/r^2$ (E)  $g = 4/3 \pi G \rho [r + a^3/r^2]$



- 17. Discover the gravitational field of a disk (in the *yz* plane) of uniform mass density  $\sigma$  at a point P = (x, 0, 0) along the *x* axis. Let R = disk radius.
  - (A)  $\mathbf{g} = -2\pi\sigma G \left[ 1 + x/\sqrt{x^2 + R^2} \right] \mathbf{x}$
  - (B)  $\mathbf{g} = \pi \sigma G R^2 / x^2 \mathbf{x}$
  - (C)  $\mathbf{g} = -2\pi\sigma G \left[ 1 x/\sqrt{x^2 + R^2} \right] \mathbf{x}$
  - (D)  $\mathbf{g} = -\pi\sigma G R^2 / x^2 \mathbf{x}$
  - (E)  $\mathbf{g} = +2\pi\sigma G \left[1 x/\sqrt{x^2 + R^2}\right] \mathbf{x}$
- 18. What is the integral quantity that must be minimized to determine the path of a light ray moving from point P to point Q in a medium of variable index of refraction n(x,y)? Let the speed of light = c.
  - (A)  $\int_{P}^{Q} n(x, y) dx$ (B)  $\frac{1}{c} \int_{P}^{Q} \sqrt{1 + y'^{2}} n(x, y) dx$ (C)  $\frac{1}{c} \int_{P}^{Q} \sqrt{1 + y^{2}} n(x, y) dx$ (D)  $\frac{1}{c} \int_{P}^{Q} n(x, y) dx$ (E)  $\frac{1}{c} \int_{P}^{Q} \sqrt{x^{2} + y^{2}} n(x, y) dx$
- 19. The nuclear reaction  ${}^{27}_{13}$ Al( $\alpha$ ,p)  ${}^{30}_{14}$ Si has a positive Q-value. Hence, energy is given off in the reaction. Determine the minimum kinetic energy needed in the lab system for the *reverse* reaction.

## **GRE PHYSICS**

- 20. The propagation of sound in a gas may be modeled as adiabatic in nature. For a three degree of freedom ideal gas, by what factor does the sound speed change when the pressure is doubled at constant density?
  - (A) Increase by factor  $\sqrt{2}$
  - (B) Increase by factor 2
  - (C) Decrease by factor 2

- (D) Decrease by factor  $\sqrt{2}$ (E) Stays the same
- 21. Find the mean square speed at temperature T for particles of mass *m* that follow Maxwell-Boltzmann statistics.
  - (A)  $2kT/\pi m$
  - (B) 3kT/m
  - (C)  $8kT/\pi m$
  - (D) 2kT/m
  - (E) kT/m





V

(C) 0.012

 $V_0$ 

V

- 22. Think about a box of volume  $10^8$  cm<sup>3</sup> with  $10^{27}$  particles in it. Take a small cubical region of that box of length 100 Å and determine the probability that that region has 0 particles in it.
  - (A)  $4.5 \times 10^{-5}$
  - (B) 0.0
  - (C) 1.0
  - (D)  $9.0 \times 10^{-5}$
  - (E)  $2.2 \times 10^{-5}$



p

- measured to be r = 25.0 cm  $\pm 0.1$  cm and the mass of the sphere is found to be m = 183 g  $\pm 3$  g. Use the theory of propagation of error to determine the relative error in  $\rho$  given by  $\Delta \rho / \rho$ .
  - (A) 0.016 (B) 0.004
  - (D) 0.020 (E) 0.008
- В  $T_H$  $T_C$ С  $Q_C$
- 24. For the Carnot refrigeration cycle shown, determine the efficiency, defined here as work performed divided by heat withdrawn from the cold reservoir.
  - (A)  $e = 1 T_C/T_H$ (B)  $e = 1 T_H/T_C$ (C)  $e = 1 + T_C^{''}/T_H^{''}$ (D)  $e = T_C/T_H - 1$ (E)  $e = T_H/T_C - 1$

- 25. Find the density of states for a single particle with one degree of freedom confined to a distance  $\ell$ . The Hamiltonian is  $H(x,p) = p^2/2m$ .
  - (A)  $\ell \sqrt{2mE^2/2\pi h}$  (D)  $\ell \sqrt{2mE/2\pi h}$ (B)  $\ell \sqrt{2m/E/2\pi h}$  (E)  $\ell \sqrt{2mE/2\pi h}$ (C)  $\ell \sqrt{2mE^{3/2}/2\pi h}$
- 26. Consider a two state system with degeneracies  $g_1, g_2$  and energies  $0, \in$ . Determine the average total energy for *N* particles in this system at temperature  $T = 1/\beta k$ .
  - $\begin{array}{ll} \text{(A)} & g_2 \mathcal{N} \varepsilon / [g_2 e^{\beta \varepsilon} + g_1] \\ \text{(B)} & g_1 \mathcal{N} \varepsilon / [g_2 e^{\beta \varepsilon} + g_1] \\ \text{(C)} & g_1 \mathcal{N} \varepsilon / [g_1 e^{\beta \varepsilon} + g_2] \end{array} \end{array} \\ \begin{array}{ll} \text{(D)} & (g_1 + g_2) \mathcal{N} \varepsilon / 2 \\ \text{(E)} & g_2 \mathcal{N} \varepsilon / [g_1 e^{\beta \varepsilon} + g_2] \end{array}$
- 27. It is common to represent many classical mechanics problems and/or differential equations through parametric plots or phase space orbits. Identify the figure.
  - (A) Simple harmonic motion
  - (B) Gravitational motion (g = constant)
  - (C) Damped motion
  - (D) Lissajous figure
  - (E) Bifurcation diagram



**-** ε

- 0

28. What is the solution to the damped motion problem

$$x'' + \gamma x' + \omega_0^2 x = 0$$
 where  $\gamma = b/m$ ,  $\omega_0 = \sqrt{k/m}$ ,  
and  $\omega = \sqrt{\omega_0^2 - \gamma^2/4}$ 

in the light damping case?

- (A)  $x = A \cos(\omega_0 t + \delta)$  (D)  $x = B e^{-\gamma t/2} \cos(\omega t + \delta)$
- (B)  $x = A \cos(\omega t + \delta)$
- (C)  $x = B e^{-\gamma t/2} \cos(\omega_0 t + \delta)$

(D)  $x = B e^{-\gamma t} \cos(\omega t + \delta)$ (E)  $x = (A + Bt)e^{-\gamma t/2}$ 

29. Atomic spectroscopy relates the wavelengths of the observed spectral lines of a substance to a mathematical formula. Given the Rydberg constant for hydrogen  $R = 109,677.6 \text{ cm}^{-1}$ , find the lower limit for the Paschen series.

- (B) 820.6 nm
- (C) 364.6 nm
- (D) 2280 nm
- (E) 1460 nm



### **GRE PHYSICS**

- 30. Determine the average energy for a photon gas at temperature  $T = 1/k\beta$  where the energy levels are given by  $E_j = j\mathbf{h} \,\omega, \mathbf{j} = 0, 1, 2, \dots$ 
  - (B)  $\frac{\hbar\omega}{(e^{\hbar\omega\beta} + 1)}$  (C) 2kT(E)  $\frac{\hbar\omega}{(e^{\hbar\omega\beta} 1)}$ (A) kT(D) 3/2 *kT*
- 31. Which of the following represents the electron configuration for potassium (atomic number 19)?
  - (C) [Ar]  $4s^1$ (A) [He]  $2s^2 2p^1$ (B) [Ne]  $3s^1$
  - (D) [Kr]  $5s^1$ (E) [Kr]  $4d^8 5s^1$
- 32. Determine the electric potential of a circular annulus of inner radius a and outer radius b (in the yz plane) of charge density  $\sigma$  = charge/area < 0 at a distance x along the *x*-axis.

(A) 
$$\phi = \pi k \sigma [b^2 - a^2]/x$$
  
(B)  $\phi = -2\pi k \sigma \left[ \sqrt{b^2 + x^2} - \sqrt{a^2 + x^2} \right]$   
(C)  $\phi = -2\pi k \sigma \left[ \sqrt{b^2 + x^2} - x \right]$   
(D)  $\phi = +2\pi k \sigma \left[ \sqrt{b^2 + x^2} - \sqrt{a^2 + x^2} \right]$   
(E)  $\phi = +2\pi k \sigma \left[ \sqrt{b^2 + x^2} - x \right]$ 



- 33. What is the intensity (energy per unit volume per unit frequency) distribution that produced the ultraviolet catastrophe in the early theory of black-body radiation? Given that  $\beta \equiv 1/kT$ .
  - (A)  $u(\omega) = \omega kT/\pi^2 c^3$ (B)  $u(\omega) = (\omega/c)^3 e^{-\hbar\omega/kT}$

  - (C)  $u(\omega) = \omega^2 kT/\pi^2 c^3$

- (D)  $u(\omega) = (\frac{\ln \omega^3}{\pi^2 c^3})/(e^{\frac{\ln \omega}{kT}} 1)$ (E)  $u(\omega) = \frac{\omega^3 kT}{\pi^2 c^3}$
- 34. The Thomas-Reiche-Kuhn sum rule connects the complete set of eigenfunctions and energies of a particle of mass m. What value is assigned by this quantum mechanical rule to

$$\sum_{j} |x_{j0}|^2 [E_j - E_0]?$$

- (B)  $h^2/m$ (C)  $3h^2/m$ (A)  $2 h^2/m$ (D)  $h^2/2m$ (E)  $h^2/4m$
- 35. What is the value of the commutator [H, x] for the quantum mechanical Hamiltonian  $H = p^2/2m + V(x)?$ 
  - (A) hx'/i(B) 2h x'/i(C) 3h x'/i(D) 4h x'/i(E) h x'/2i

- 36. Which of the following is NOT a true statement about the classic Franck-Hertz experiment?
  - (A) A tube is filled with vapor (e.g., Hg) at high pressure so that electrons will experience a high number of collisions.
  - (B) It showed that electrons undergo transitions from higher to lower energy levels.
  - (C) Electrons are accelerated through a potential difference such that  $eV_0 = 1/2 mv^2$ .
  - (D) A tube is fulled with vapor (e.g., Hg) at low pressure so that electrons will experience a low number of collisions.
  - (E) An electron may lose most of its kinetic energy in an inelastic collision with an atom.
- 37. The angular wave function for the rigid diatomic rotor with quantum numbers l = 1 and  $m_l = 1$  is given by  $Y_{11}(\theta, \phi) = -N \sin \theta$   $e^{i\phi}$ . Determine the normalization constant *N*. (A)  $\sqrt{3/4\pi}$  (D)  $\sqrt{3/\pi}$ (B)  $\sqrt{3/8\pi}$  (E)  $\sqrt{3/6\pi}$ (C)  $\sqrt{3/2\pi}$
- 38. Consider the hydrogen-like atom eigenvalue problem: one electron orbits a nucleus of charge Ze. The general wave function is

$$\Psi_{nlm}(r,\theta,\phi) = R_{nl}(r)Y_{lm}(\theta,\phi).$$

Determine the value of r where the radial probability is a maximum for the ground state.

- (A)  $a_0$  (B)  $a_0/Z$  (C)  $a_0/2Z$ (D)  $a_0/3Z$  (E)  $a_0/4Z$
- 39. The n = 2 and l = 1 hydrogen-like atom radial wave function is

$$R_{21}(r) = Nre^{-Zr/2a_0}$$

What is the correct normalization factor N?

- (A)  $Z/\sqrt{3}a_0$  (D)  $(Z/2a_0)^3 Z^2/3a_0^2$ (B)  $(Z/2a_0)^{3/2}$  (E)  $(Z/2a_0)^3$
- (C)  $(Z/2a_0)^{3/2}Z/\sqrt{3}a_0$
- 40. Which of the following is NOT a true statement about the Bohr theory of the hydrogen-like atom with nuclear charge Ze and reduced mass  $\mu$ ?

n is the principal quantum number and l is the angular momentum quantum number.

- (A) The energy eigenvalue is proportional to  $1/n^2$ .
- (B) The energy eigenvalue depends on  $\ell(\ell + 1)$ .
- (C) The energy eigenvalue is proportional to  $\mu$ .
- (D) For large values of Z, the energy can have magnitude keV.
- (E) The radius of the electron orbit is proportional to  $n^2$ .

- 41. Which of the following IS a true statement about the nuclear binding energy in the semi-empirical mass formula model?
  - (A) The volume term is proportional to  $A^2$ .
  - (B) The Coulomb term is proportional to  $A^{-2/3}$ .
  - (C) The symmetry term is proportional to  $(A 2Z)^2/A$ .
  - (D) The area term is proportional to  $A^{1/3}$ .
  - (E) The symmetry term is proportional to  $(A 2Z)^2$ .
- 42. A slow neutron is absorbed by  ${}^{235}_{92}U$  and results in a fission event:

1	2	35		139	9	4	
n	+	U	$\longrightarrow$	Ва	+	Kr	+
0	9	92		56	3	6	

How many neutrons were produced in this event?

- (A) 0 (B) 1 (C) 2 (D) 3 (E) 4
- 43. Consider the coupled inductor-capacitor circuit shown. Determine the ratio of the frequency of the anti-symmetric mode to that of the symmetric mode  $\omega_2/\omega_s$ . Let k = 1/LC and  $\kappa = 1/L\gamma$ .
  - (A) -1(B)  $\sqrt{1+k/\kappa}$ (C)  $\sqrt{1+2k/\kappa}$ (D)  $\sqrt{1+2\kappa/k}$ (E)  $\sqrt{1+\kappa/k}$



- 44. A spaceship traveling at  $1.50 \times 10^8$  m/s leaves the Earth in the year 2050 with John on board. John leaves his twin brother James behind on Earth and goes off to a star 25 light-years away. Upon arrival, he immediately returns. On return, what is the difference in their ages?
  - (A) 3.3 years (B) 25 years (C) 12.5 years (D) 10.0 years (E) 25 /+yr 25 light yr Star
  - (E) 13.4 years
- 45. Discover for special relativity (and in reality) that it is more difficult to accelerate a particle in a linear accelerator than in a circular accelerator, i.e., compute the ratio  $F_{\parallel}/F_{\perp}$  of the parallel force to the perpendicular force given that the desired acceleration is the same  $a_{\parallel} = a_{\perp}$ .
  - (A)  $\gamma^3$  (B)  $\gamma^2$  (C)  $\beta$ (D)  $\gamma\beta$  (E)  $\gamma$
- 46. Find the magnetic field that the electron exerts at the nucleus of the hydrogen atom according to Bohr theory. Assume a size  $r = a_0 = 0.529$  Å.
  - (A) 1.25 T (B) 125 T (C) 12.5 T (D) 1250 T (E) 12,500 T

- 47. Consider Maxwell's equations in differential form in media. Now Е suppose  $j = \rho = 0$ . If further  $\in = \in {}_{0}e^{\alpha t}$  and  $\mu = \mu_{0}e^{\alpha t}$ , then find the relevant wave equation for a plane wave propagated in the x-direction such that E = Ey and H = Hz. (A)  $\partial^2 E / \partial x^2 = \mu \in \partial^2 E / \partial t^2$ (B)  $\partial^2 D / \partial x^2 = \mu \in \partial^2 D / \partial t^2$ (C)  $\partial^2 D / \partial x^2 = \mu \ \partial^2 D / \partial t^2 + \mu \alpha \partial D / \partial t$ (D)  $\partial^2 E / \partial x^2 = \mu \partial^2 D / \partial t^2 + \mu \alpha \partial D / \partial t$ 
  - (E)  $\partial^2 E / \partial x^2 = \mu \partial^2 E / \partial t^2 + \mu \alpha \partial E / \partial t$



- 48. As observed from the earth, a spaceship is passing by with a speed of v = 0.5c (c is the speed of light). The spaceship fires a missile in the forward direction with speed u = 0.5c measured in its own frame of reference. The speed v' of the missile as observed from the earth must be
  - (A) 0 (B) 0.25c (C) 0.5c (E) c
  - (D) 0.8c

(C) 3

49. A neutron of kinetic energy T = 1876 MeV is incident on a neutron at rest. The neutron scatters elastically at angle  $\theta$ . Given that  $m_n = 938 \text{ MeV/c}^2$ . Find  $\theta$ .



- 50. A wide range of temperatures is currently accessible in the laboratory and through observation. Which of the following is NOT a true statement about temperature?
  - (A) 0 K is the absolute zero of temperature.
  - (B) 20 K is the vaporization point of hydrogen.
  - (C) 144 K is the vaporization point of nitrogen.
  - (D) 1234 K is the fusion temperature of silver.
  - (E) 6000 K is the sun's surface temperature.
- 51. In an interference experiment a slide with a double slit is used with monochromatic light of wavelength  $\lambda$ . The slit's separation is  $d = 6\lambda$  and each slit has width  $w = 2\lambda$ . An overlapping double-slit interference and single-slit diffraction pattern is obtained on a screen. The smallest order for the missing interference maximums must be
  - (A) 0 (B) 2
  - (E) 12 (D) 6

## **GRE PHYSICS**

- 52. One must see the de Broglie wavelength concept to "derive" the Schrödinger equation from the one-dimensional wave equation. What de Broglie wavelength must be used to get the general time independent equation?
  - (A)  $\lambda = h/\sqrt{2mE}$ (B)  $\lambda = h/\sqrt{2mU}$ (C)  $\lambda = h/\sqrt{2m(E-U)}$ (D)  $\lambda = h/\sqrt{2m(E+U)}$ (E)  $\lambda = h/\sqrt{m(E-U)}$
- 53. Determine the time dependent total energy for the lightly damped one-dimensional harmonic oscillator in the approximation

$$\gamma << 1$$
 where  $\gamma = b/m$  and  $\omega_0 = \sqrt{k/m}$ 

given that  $x = B_0 e^{-\gamma t/2} \cos(\omega \tau + \delta)$ .

- (A)  $E(t) \cong \frac{1}{2}kB_0^2 e^{-\gamma t/2}$ (B)  $E(t) \cong \frac{1}{2}mB_0^2 e^{-\gamma t}$ (C)  $E(t) \cong \frac{1}{2}mB_0^2 e^{-\gamma t/2}$ (D)  $E(t) \cong \frac{1}{2}kB_0^2 e^{-\gamma t/2}$
- 54. A particle of energy  $E < V_0$  is incident on a step potential of height  $V_0$ . k and k' are wave numbers outside and inside the barrier, respectively, where

$$k = \sqrt{2mE}/\hbar$$
 and  $k' = \sqrt{2m(V_0 - E)}/\hbar$ 

Find the transmission coefficient.

(A) 1 (B) 0 (C)  $k^2/k'^2$ (D)  $4k^2/(k^2 + k'^2)$ (E) k/k'



55. The particle in a box has a ground state wave function given by

$$\phi(x) = \left(1/\sqrt{a}\right)\cos \pi x/2a$$

Calculate the expectation value of  $x^2$ .

(A)  $a^{2}/3$ (B)  $a^{2}(1/3 - 2/\pi^{2})$ (C)  $a^{2}(2/3 - 4/\pi^{2})$ (D)  $2a^{2}/\pi^{2}$ (E)  $a^{2}/4$ 



- 56. A star of mass M and radius R has an average rotational period of T. When this star dies and turns into a neutron star, its radius will be reduced to 1/nth of the original radius. Assuming the mass has not changed, the period of the neutron star would be?
  - (A) T/n (B) nT
  - (D)  $n^2T$  (E) Unchanged
- 57. An RLC circuit vibrates subject to the initial conditions  $I = I_0$  and I' = 0 at t = 0. What is the time dependent current in the critical damping case? Let  $\gamma = R/L$ ,

$$\omega_0^2 = 1/LC, \ \omega = \sqrt{\omega_0^2 - \gamma^2/4}, \ \text{and} \ \tan \delta = -\gamma/2\omega.$$

- $\begin{array}{ll} \text{(A)} & I = I_0 e^{-\gamma t/2} \cos(\omega t + \delta) \\ \text{(B)} & I = I_0 e^{-\gamma t/2} \cos(\omega t + \delta) / \cos \delta \\ \text{(C)} & I = I_0 (1 + \gamma t/2) e^{-\gamma t/2} \\ \text{(D)} & I = [(\omega + \gamma/2) I_0 e^{(\omega \gamma/2)t} + (\omega \gamma/2) I_0 e^{-(\omega + \gamma/2)t}] / 2\omega \\ \text{(E)} & I = I_0 (1 + \gamma t) e^{-\gamma t} \end{array}$
- 58. Consider the physical possibility of a shot fired around the world. An object is propelled horizontally at radius *r* around an assumed spherical Earth. What is the period?

(A) 
$$\frac{1}{2\pi}\sqrt{GM_E/r^3}$$
  
(B)  $\frac{1}{2\pi}\sqrt{r^3/GM_E}$   
(C)  $2\pi\sqrt{GM_E/r^3}$   
(D)  $2\pi\sqrt{r^3/GM_E}$ 

59. A pendulum bob of mass *m* is raised to a height *h* and released. After hitting a spring of non-linear force law  $F = -kx - bx^3$ , calculate the compression distance *x* of the spring.

(A) 
$$x = \sqrt{2mgh/h}$$
  
(B)  $x = (4mgh/b)^{1/4}$   
(C)  $x = \left[\sqrt{4mgh/b + k^2/b^2} - k/b\right]^{1/2}$   
(D)  $x = \left[\sqrt{4mgh/b + k^2/b^2} + k/b\right]^{1/2}$   
(E)  $x = \sqrt{2k/b}$ 



V

(C)  $T/n^2$ 



## **GRE PHYSICS**

60. A frictionless wire connects points P and Q in a constant gravitational field as shown below. What is the equation that must be solved to find the curve down which a mass will slide in the shortest possible time? Let



61. A spherical mass is dropped off of a building from rest. Determine the position y = y(t) if the mass experiences a resistive force  $F_R = -bv$ . Take the starting point to be the origin y = 0 as shown below. Let  $\gamma = b/m$ .

(A) 
$$y = \frac{gt}{\gamma} - \frac{g}{\gamma^2}(1 - e^{-\gamma t})$$
  
(B)  $y = \frac{1}{2}gt^2$   
(C)  $y = \frac{g}{\gamma}(1 - e^{-\gamma t})$   
(D)  $y = \frac{gt}{\gamma}$   
(E)  $y = -\frac{g}{\gamma^2}(1 - e^{-\gamma t})$ 



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- 62. Use Gauss' law for gravitation to determine the magnitude of the gravitational field for two infinite sheets of mass density  $\sigma$  in regions I and II.
  - (A)  $4\pi G\sigma$ (D)  $2\pi G\sigma/\varepsilon_0$ (B)  $2\pi G\sigma$ (E) 0(C)  $\pi G\sigma$
- 63. A boom is suspended by a cable as shown below. Given that angle  $\theta = 45^{\circ}$  and the weight of the boom is W = 1000 N. Find the reaction force **R**.
  - (A) (1000 N, 45°)
  - (B) (707 N, 30°)
  - (C) (500 N, 60°)
  - (D)  $(707 \text{ N}, 45^{\circ})$
  - (E) (500 N, 30°)



- 64. A right circular cylinder of radius *r* rolls down an incline from height *h*. Determine the ratio of its speed at the bottom to the speed of a point object following the same path. Assume rolling friction, but negligible sliding friction.
  - (A) 1 (D)  $\sqrt{2/3}$ 
    - (E) 2
  - (B)  $\sqrt{2}$ (C)  $\sqrt{3}$
- 65. An object orbits a star in an elliptical orbit. The distance at aphelion is 2a and the distance at perihelion is a. Determine the ratio of the objects speed at perihelion to that at aphelion.
  - (A) 2 (D)  $\sqrt{2}$
  - (B) 3 (E)  $\sqrt{3}$
  - (C) 1



- 66. A one-quarter circular line mass of total mass M exerts a gravitational force on a point mass m as shown below. Find the force.
  - (A)  $\left(2\sqrt{2}GmM/\pi R^2, 45^\circ\right)$
  - (B)  $(GmM/R^2, 45^\circ)$
  - (C)  $(GmM/\pi R^2, 45^\circ)$
  - (D) (2 *GmM*/ $\pi R^2$ , 45°)
  - (E)  $\left(\sqrt{2}GmM/\pi R^2, 45^\circ\right)$



- 67. Determine the gravitational field magnitude near the surface of a planet of radius *R* at altitude *h* to second order. Take  $g(h = 0) = g_0$ .
  - (A)  $g_0[1 2h/R]$ (B)  $g_0[1 - h/R + 0.5(h/R)^2]$ (C)  $g_0[1 - h/R + (h/R)^2]$ (D)  $g_0[1 + h/R + (h/R)^2]$ (E)  $g_0[1 - 2h/R + 3(h/R)^2]$
- 68. In the laboratory, two quantities of the same fluid are mixed. The mass of the hotter sample  $(m_1)$  is twice that of the cooler one  $(m_2)$ . The initial temperature of the hotter sample  $(T_1)$  is also twice the other initial temperature  $T = 30^{\circ}$ C. Find the equilibrium temperature.
  - (A)  $40^{\circ}$ C (B)  $45^{\circ}$ C (C)  $50^{\circ}$ C (D)  $55^{\circ}$ C (E)  $35^{\circ}$ C

69. The moon causes a tidal force on the Earth's ocean. Determine the differential tidal acceleration. Let  $a = r_{\text{earth to moon}}$ ,

$$r = r_{\text{earth}}, m = m_{\text{moon}}, \text{ and } M = m_{\text{earth}}.$$

- (A)  $Gm/a^2$
- (B)  $2Gma/r^3$
- (C)  $2GMr/a^3$
- (D)  $GM/r^2$
- (E)  $2Gmr/a^3$



- 70. One laboratory technique to determine the mass of one star of a binary star system involves measuring the distance *a* between the stars from parallax and observing the period of revolution *T*. Suppose  $m_1 = m_2 = m$ . What is the mass *m*?
  - (A)  $\pi^2 a^3 / T^2$
  - (B)  $2\pi^2 a^3/T^2$
  - (C)  $4\pi^2 a^3/T^2$
  - (D)  $8\pi^2 a^3/T^2$
  - (E)  $4\pi^2 T^2/a^3$



- 71. In the 3D harmonic oscillator version of the nuclear shell model, what are the nuclear magic numbers?
  - (A) 1, 4, 16(B) 1, 4, 9(C) 4, 12, 24(D) 4, 16, 40(E) 2, 8, 20
- 72. Monochromatic light of wavelength 6000 Å is incident on two slits of spacing 0.15 mm and the resulting intensity pattern observed 1.5 m away. Determine the location of the first maximum.
  - (A) 6 mm
  - (B) 12 mm
  - (C) 24 mm
  - (D) 3 mm
  - (E) 1.5 mm



(C) 3/2

- 73. Use the Mayer and Jensen nuclear shell model with spin-orbit interaction to figure out the spin of the  ${}^{17}_{8}$ O nuclide.
  - (A) 7/2
     (B) 5/2

     (D) 1
     (E) 1/2
- 74. A star may be modeled as a uniform spherical distribution of matter. Let *m* be the star's mass and *V* the volume. What is the dependence of the gravitational pressure on volume?
  - (A)  $P \propto V$ (B)  $P \propto V^{-1/3}$ (C)  $P \propto V^{-2/3}$ (D)  $P \propto V^{-1}$ (E)  $P \propto V^{-4/3}$



- 75. Which of the following is NOT a true statement about the modern particle picture of the structure of matter?
  - (A) There are only six leptons.
  - (B) There are only six quarks.
  - (C) Tau lepton number is not conserved.
  - (D) Electron lepton number is conserved.
  - (E) The anti-leptons have opposite charge and lepton number.
- 76. Which of the following IS a true statement about the modern QCD quark and gluon picture of the structure of matter?
  - (A) Not all six quarks have been observed in a free unconfined state.
  - (B) The six quarks are up, down, sideways, charm, bottom, and top.
  - (C) The s, c, and b quarks have special quantum numbers S, C, B.
  - (D) The d, s, b quarks have charge 2/3 e.
  - (E) The u, c, t quarks have charge -1/3 e.
- 77. Relate the half-life  $\tau$  to the decay constant  $\lambda$  in the theory of radioactive decay.
  - (A)  $\tau = 1/\lambda$  (D)  $\tau = \lambda/\ln(2)$ (B)  $\tau = \lambda$  (E)  $\tau = 1/\lambda^2$
  - (C)  $\tau = \ln(2)/\lambda$

78. A Michelson Interferometer with light of vacuum wavelength 580 nm is used to determine the index of refraction of air. A cell of length 10 cm is placed in the path of one of the beams and air is pumped out. The air is then let in slowly through a valve and the numbers of circular fringes moving in at the center are counted. When all the air is pumped out, the number of fringes counted is 100. The index of refraction of air thus determined is

- (A) 1 (B) 1.029 (C) 1.0029 (D) 1.00029 (E) 0
- 79. A typical Mossbauer effect setup is shown below. For the case of  $\frac{191}{77}$ Ir where the  $\gamma$  ray energy is 129 keV and the half-life is 0.14 ns, calculate the normal recoil energy.



80. An object sits 15 cm to the left of a convex refractive interface of curvature radius 5 cm. The substance to the left has refractive index n = 1.2, while that to the right has index 1.7. Determine the image distance s', primary focal distance f, and secondary focal distance f'. For fat lenses,



- 81. A laser operates by light amplification and the stimulated emission of optical radiation. For two molecular states j and i such that  $E_i - E_i = h\omega$  in equilibrium in a cavity, we have  $N_i/N_i =$  $[A_{ii} + B_{ii}u(\omega)]/B_{ii}u(\omega)$ . Use the Boltzmann factor to find  $u(\omega)$  the energy per unit frequency per unit volume. Given that  $\beta = 1/kT$ .
  - (A)  $A_{ji}e^{-\hbar\omega b}/B_{ij}$
  - (B)  $A_{ji}/[B_{ij}e^{\hbar\omega \vec{b}} B_{ji}]$
  - (C)  $A_{ii}/B_{ii}$

  - (D)  $A_{ji}/[B_{ij} B_{ji}]$ (E)  $A_{ji}/[B_{ji}e^{-h\omega b} B_{ij}]$
- 82. Helmholtz coils are commonly used in the laboratory to determine the charge to mass ratio of the electron. Find the magnetic field directly between the coils. Let N = number of turns and R = radius.
  - (A)  $8N\mu_0 I/5^{3/2}R$
  - (B)  $4N\mu_0 I/5^{3/2}R$
  - (C)  $2N\mu_0 I/5^{3/2}R$
  - (D)  $N\mu_0 I/2\pi R$
  - (E)  $N\mu_0 I/\pi R$



 $\mathbf{E} = \mathbf{y} E_{\alpha \nu} \cos(\omega t - kx + \alpha) + \mathbf{z} E_{\alpha \nu} \cos(\omega t - kx + \beta).$ 

Under what conditions is this light wave circularly polarized?

(A)  $\beta - \alpha = \pm \pi/2$ (B)  $\beta - \alpha = \pm \pi/2$  and  $E_{ov} = E_{oz}$ (C)  $\alpha = \beta$  and  $E_{oy} = E_{oz}$ (D)  $\beta - \alpha = \pm \pi/2$  and  $E_{oy} = 2E_{oz}$ (E)  $\alpha = \beta$  and  $E_{oy} = 2E_{oz}$ 



84. A point charge of magnitude q is located at distance h above an infinite conducting plane (say the xy plane) at coordinates (0, 0, h). Let

$$s = \sqrt{x^2 + y^2}$$

Denote the cylindrical distance from (0, 0, 0). Find the electric field in the plane.

$$(A) \quad E = 0$$

(B) 
$$E = -q/2\pi\varepsilon_0$$

- (C)  $E = -q/2\pi\varepsilon_0 h^2$
- (D)  $E = -qh/[2\pi\varepsilon_0(s^2 + h^2)^{3/2}]$
- (E)  $E = -q/2\pi\varepsilon_0 s^2$



85. The Fraunhofer single-slit diffraction intensity is given by

$$I = I_0 \sin^2(x)/x^2$$
 where  $x = \pi d_w y/\lambda l$ .

 $d_w$  is the slit width, y the detector distance, and l the distance from slit to screen. What is the value of the cumulative intensity

$$\int_{-\infty}^{+\infty} I(y) dy?$$

(A) 0 (B)  $I_0 \lambda l/d_w$ (C)  $I_0 \lambda d_w/l$ (D)  $I_0 d_w l/\lambda$ (E)  $I_0$ 



- 86. Use the fundamental concepts of electromagnetism to determine the electric field of an electric dipole **d** at distance  $\mathbf{r} = r\hat{\mathbf{r}}$ .
  - (A)  $k[3\hat{r} \cdot p\hat{r} p]/r^3$ (B)  $k\hat{r} \cdot p/r^2$ (C)  $k\hat{r} \cdot p/r^3$ (D)  $k[3\hat{r} \cdot p\hat{r} - p]/r^2$ (E)  $k[2\hat{r} \cdot p\hat{r} - p]/r^3$

87. A particle is constrained to move on the surface of a sphere of radius *R* near the Earth's surface. Find the Lagrangian in spherical coordinates.

- (A)  $1/2m(R^2 + R^2 \sin^2 \theta) mgR \cos \theta$
- (B)  $1/2m(R^2\theta'^2 + R^2\phi'^2\sin^2\theta) mgR\cos\theta$
- (C)  $1/2m(R^2\theta'^2 + R^2\phi'^2\sin^2\theta) + mgR\cos\theta$
- (D)  $1/2m(R^2 + R^2 \sin^2 \theta) mgR \cos \theta$
- (E)  $1/2(R^2\theta'^2 + R^2\phi'^2) mgR\cos\theta$

# **GRE PHYSICS**

- 88. Determine the electric potential of the infinite sheet of charge shown for x > 0. Let the charge density be  $\sigma$  and the x-direction be to the right.
  - (A)  $-\sigma x/\varepsilon_0$
  - (B)  $+\sigma x/2\varepsilon_0$
  - (C)  $-\sigma x/2\varepsilon_0$ (D)  $+\sigma x/\varepsilon_0$
  - (E)  $-2\sigma/\varepsilon_0$



- 89. The electric potential of a grounded conducting sphere of radius a in a uniform electric field is given as  $\varphi(r, \theta) = -E_0 r [1 - (a/r)^3] \cos \theta$ . Find the surface charge distribution on the sphere.
  - (A)  $\varepsilon_0 E_0 \sin \theta$
  - (B)  $\varepsilon_0 E_0 \cos \theta$
  - (C)  $2\varepsilon_0 E_0 \cos \theta$
  - (D)  $3\varepsilon_0 E_0 \cos \theta$
  - (E)  $2\varepsilon_0 E_0 \sin \theta$



- 90. What is the distance at which the eye can in principle resolve two truck headlights separated by d = 2.0 m? Take the pupil of the eye to be a circular aperture of diameter D = 3 mm and the relevant wavelength to be  $\lambda = 600$  nm.
  - (A) 4.1 km (B) 8.2 km (C) 12 km (E) 2.0 km (D) 16 km
- 91. Light is incident at near normal incidence angle on a prism of apex angle  $\alpha$ . The prism is made of a substance of refractive index  $n_{\mu}$  and the light comes from and returns to medium  $n_i$ . Find the angle of deviation using the small angle approximation.
  - (D)  $(n_i/n_r + 1)\alpha$ (E)  $(n_r 1)\alpha$ (A)  $(n_r/n_i + 1)\alpha$
  - (B)  $(n_r/n_i 1)\alpha$
  - (C)  $(n_i/n_r 1)\alpha$





- 92. Which of the following is NOT a correct statement about the optical process of fluorescence?
  - (A) It is the emission of electromagnetic radiation.
  - (B) The process takes place in about  $10^{-8}$  s.
  - (C) Often fluorescence is guenched by collisions.
  - (D) Ultraviolet photons may thereby be converted to visible photons.
  - (E) A quantum of light is scattered elastically.
- 93. According to the Orbital Rule, the change allowed for the orbital quantum number ( $\Delta l$ ) in electron transition in an atom is
  - (C)  $\Delta l = +1$ (A)  $\Delta l = 0 \text{ or } \pm 1$ (B)  $\Delta l = 0$ (D)  $\Delta l = -1$ (E)  $\Delta l = \pm 1$

- 94. For a galaxy moving away at 80% the speed of light, the redshift z must be
  - (A) 1 (B) 2 (C) 3 (D) 4 (E) 5
- 95. According to Hubble's Law, the relationship between the distance (D) and speed (v) of a galaxy is
  - (A) v is directly proportional to D
- (D) v is directly proportional to  $1/D^2$
- (B) v is directly proportional to  $D^2$
- (E) v is independent of D
- (C) v is directly proportional to 1/D
- 96. The Meisner Effect in superconductors can be explained by the interaction of magnetic field *B* with it given by the London Equation  $\nabla^2 B = \lambda^{-2} B$ . In this equation the quantity  $\lambda$  refers to
  - (A) the reflection coefficient of B at the superconductor surface
  - (B) the transmission coefficient of B at the superconductor surface
  - (C) the penetration depth of B into the superconductor
  - (D) the thickness of the superconductor
  - (E) the density of the superconductor
- 97. X-rays of wavelength 0.25 nm are used in a Bragg diffraction experiment. If the smallest angle for the Bragg diffraction is 30°, the distance between the lattice planes must be
  - (A) 0.125 nm(B) 0.250 nm(C) 0.50 nm(D) 1.00 nm(E) 2.00 nm
- 98. An electron is projected at t = 0 at an angle of 30° with respect to the *x*-axis with a speed of  $4 \times 10^5$  m/s. The electron moves in a constant electric field  $\mathbf{E} = 100 \text{ N/C} \mathbf{y}$ . At what time after t = 0 will the electron recross the *x*-axis?
  - (A) 10 ns (B) 12 ns (C) 15 ns (D) 18 ns (E) 23 ns
- 99. Which of the following is NOT a true statement about the optical device known as a retarder?
  - (A) A retarder is used to change the incident wave polarization.
  - (B) The produced phase difference is  $\alpha 1/\lambda_{\text{vacuum}}$ .
  - (C) A full wave plate has  $2\pi$  rad retardance.
  - (D) A half-wave plate has  $\pi$  rad retardance.
  - (E) A given wave plate has continuously adjustable retardance.
- 100. What must the speed of the sliding bar be when the current in the resistor is 0.5 amp? Given that B = 1 T, R = 2  $\Omega$ , and w = 0.5 m.
  - (A) 2 m/s
  - (B) 4 m/s
  - (C) 1 m/s
  - (D) 3 m/s
  - (E) 5 m/s



# **Answer Key**

1.	(B)	18. (B)	35. (A)	52. (C)	69. (E)	86. (A)
2.	(E)	19. (C)	36. (A)	53. (D)	70. (B)	87. (B)
3.	(A)	20. (A)	37. (B)	54. (D)	71. (D)	88. (C)
4.	(B)	21. (B)	38. (B)	55. (B)	72. (A)	89. (D)
5.	(D)	22. (A)	39. (C)	56. (C)	73. (B)	90. (B)
6.	(E)	23. (D)	40. (B)	57. (C)	74. (E)	91. (B)
7.	(A)	24. (E)	41. (C)	58. (D)	75. (C)	92. (E)
8.	(C)	25. (B)	42. (D)	59. (C)	76. (C)	93. (E)
9.	(B)	26. (E)	43. (D)	60. (A)	77. (A)	94. (B)
10.	(C)	27. (D)	44. (E)	61. (A)	78. (D)	95. (A)
11.	(C)	28. (D)	45. (B)	62. (A)	79. (A)	96. (C)
12.	(E)	29. (B)	46. (C)	63. (D)	80. (E)	97. (C)
13.	(D)	30. (E)	47. (D)	64. (D)	81. (B)	98. (E)
14.	(A)	31. (C)	48. (D)	65. (A)	82. (A)	99. (E)
15.	(D)	32. (D)	49. (C)	66. (A)	83. (C)	100. (A)
16.	(A)	33. (C)	50. (C)	67. (E)	84. (D)	
17.	(C)	34. (D)	51. (C)	68. (C)	85. (B)	

# **Practice Exam 2**

# **Detailed Explanations of Answers**

# 1. **(B)**

This problem involves vectors:

$$\mathbf{r} = x\hat{x} + y\hat{y} + z\hat{z} = \rho\cos\theta\hat{x} + \rho\sin\theta\hat{y} + z\hat{z}$$

The tangent vectors are:

$$\frac{\partial \mathbf{r}}{\partial z} = \hat{z}$$
$$\frac{\partial \mathbf{r}}{\partial \theta} = -\rho \sin \theta \hat{x} + \rho \cos \theta \hat{y}$$
$$\frac{\partial \mathbf{r}}{\partial \rho} = \cos \theta \hat{x} + \sin \theta \hat{y}$$

and

The unit vectors are:

 $\hat{z} = (\partial \mathbf{r}/\partial z)/|\partial \mathbf{r}/\partial z| = \hat{z}$ 

and similarly

$\hat{\theta} = -\sin\theta \hat{x} + \cos\theta \hat{y}$
$\hat{\rho} = \cos \theta \hat{x} + \sin \theta \hat{y}$

and

Inverting, one finds

$$\hat{x} = \cos \theta \hat{\rho} - \sin \theta \theta$$
$$\hat{y} = \sin \theta \hat{\rho} + \cos \theta \hat{\theta}$$

$$\therefore \mathbf{r} = \rho \hat{\rho} + z \hat{z}$$

Finally 
$$\mathbf{v} = \frac{d\mathbf{r}}{dt} = \dot{\rho}\hat{\rho} + \rho\frac{d\hat{\rho}}{dt} + \dot{z}\hat{z} = \dot{\rho}\hat{\rho} + \rho\dot{\theta}\hat{\theta} + \dot{z}\hat{z}$$

2. (E)

E = T + U is the total mechanical energy. Differentiate it to get:

$$\frac{dE}{dt} = \frac{dT}{dt} + \frac{dU}{dt}$$
$$= \mathbf{F} \cdot \mathbf{v} + \frac{\partial U}{\partial t} + \sum_{j} \frac{\partial U}{\partial x_{i}} x_{i}'$$
$$= \mathbf{F} \cdot \mathbf{v} + \frac{\partial U}{\partial t} + \nabla U \cdot \mathbf{v}$$
$$= \frac{\partial U}{\partial t} \text{ since } \mathbf{F} = -\nabla U$$

3. (A)

The viscous force is

$$\frac{1}{2} \pi r^2 \rho_f v |v| c_d(v) = F_V$$

The drag coefficien t is

$$c_d(v) = 24/R_e$$
  
= 24/0.5 = 48

For a sphere,

$$R_e = 2|v|r/v$$

$$v = R_e v/2r = (0.5)(0.149)/2/(0.005)$$

$$= 7.45 \text{ cm/s}$$

$$F_V = \frac{1}{2} \pi r^2 \rho_f v^2 c_d(v)$$

$$F_V = \frac{1}{2} \pi (0.005)^2 (1.22 \times 10^{-3})(7.45)^2 (48)$$

$$= 0.128 \text{ md}$$

Then

This problem concerns 2D motion. From Newton's second law:

$$\begin{cases} mv'_x = -bv_x \\ mv'_y = -mg - bv_y \end{cases}$$

Now define  $\gamma = b/m$ . Then the first equation has solution

and 
$$v_x = v_{ox} e^{-\gamma t}$$
  
 $x = \frac{v_{ox}}{\gamma} (1 - e^{-\gamma t})$ 

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by integration. Similarly,

$$v_y = -g/\gamma + (v_{0y} + g/\gamma)e^{-\gamma t}$$
$$y = -gt/\gamma + 1/\gamma(v_{0y} + g/\gamma)(1 - e^{-\gamma t})$$

by integrating the second equation. At y = 0 for positive t = T, we find the time of flight

$$T = \frac{2v_{oy}}{g + v_{oy}\gamma} + \frac{\gamma t^2}{3}$$
$$\approx 2v_{oy}\left(1 - \frac{v_{oy}\gamma}{g}\right)\frac{1}{g} + \frac{\gamma}{3}\left(\frac{2v_{oy}}{g}\right)^2$$
$$= \frac{2v_{oy}}{g}\left(1 - \frac{\gamma v_{oy}}{3g}\right)$$
$$\frac{1}{1 - x} \approx 1 - x$$

Where we have used

5. (D)

The work function is

$$\phi = hv_0 = hc/\lambda_0$$
  
= 12,400/2576 = 4.50 eV

and hence Cu is the substance. The light energy is

$$E = hv = hc/\lambda$$
  
= 12,400/1700 = 7.29 eV

by conservation of energy

 $t + \phi = hv$ 

Thus, the kinetic energy of the photoelectrons is

$$T = hv - \phi$$
$$= 2.79 \text{ eV}$$

#### 6. (E)

This is a standard Compton scattering problem, with

$$\lambda = 3.00 \text{ Å}$$

 $\Psi = 45^{\circ}$ 

and

given. The Compton shift is

$$\Delta \lambda = 2\lambda_c \sin^2 \frac{\Psi}{2} = \lambda' - \lambda$$

The Compton wavelength is

$$\lambda_C = \frac{h}{m_e c} = \frac{6.626 \times 10^{-34}}{(9.109 \times 10^{-31})(3 \times 10^8)}$$
$$= 2.42 \times 10^{-12} \text{ m} = 0.0242 \text{ Å}$$
$$\Delta \lambda = 2(0.0242)(\sin^2 22.5^\circ) = 0.071 \text{ Å}$$
$$\lambda' = \lambda + \Delta \lambda = 3.07 \text{ Å}$$

Hence,

Finally,

7. (A)

Stirling's theorem (1st order approximation)

$$\ln N! = \ln N + \dots + \ln 1$$
$$= \sum_{j=1}^{N} \ln j$$
$$\approx \int_{1}^{N} \ln x \, dx$$
$$= x \ln x - x \Big|_{1}^{N}$$
$$= N \ln N - N$$
$$\therefore N! = \left(\frac{N}{e}\right)^{N}$$

8. (C)

To get the total displacement, we sum up

$$x_N = \Sigma x = \Sigma \cos \theta$$
$$y_N = \Sigma y = \Sigma \sin \theta$$
$$r^2 = x_N^2 + y_N^2$$



$$< r^{2} >= \frac{\int r^{2} d\theta_{1} \dots d\theta_{N}}{\int d\theta_{1} \dots d\theta_{N}}$$

$$= \frac{1}{(2\pi)^{N}} \int [(\cos \theta_{1} + \dots + \cos \theta_{N})^{2} + (\sin \theta_{1} + \dots + \sin \theta_{N})^{2}] d\theta_{1} \dots d\theta_{N}$$

$$= \frac{1}{(2\pi)^{N}} \int [(\cos \theta_{1} + U)^{2} + (\sin \theta_{1} + V)^{2}] d\theta_{1} \dots d\theta_{N}$$

$$= \frac{1}{(2\pi)^{N}} \int [\cos^{2} \theta_{1} + \sin^{2} \theta_{1} + U^{2} + V^{2} + 2U \cos \theta_{1} + 2V \sin s\theta_{1}] d\theta_{1} \dots d\theta_{N}$$

$$= \frac{1}{(2\pi)^{N}} \int \left[ \int_{0}^{2\pi} (1 + U^{2} + V^{2}) d\theta_{1} \right] d\theta_{1} \dots d\theta_{N}$$

$$= \frac{1}{(2\pi)^{N-1}} \int [(\cos \theta_{2} + \dots + \cos \theta_{N})^{2} + (\sin \theta_{2} + \dots + \sin \theta_{N})^{+} 1] d\theta_{2} \dots d\theta_{N}$$

$$= 1 + \dots + 1$$

$$= N$$

$$\therefore R_{RMS} = \sqrt{\langle r^{2} \rangle} = \sqrt{N}$$

# 9. (B)

The linear momentum and mechanical energy are conserved. That is,

$$(2M) v = Mv' \rightarrow v' = 2v$$

and

10. (C)

 $p + p \rightarrow p + p + \pi^0$  is the given reaction.

The total *u*-vector momentum squared is

$$(\underline{p}_1 + \underline{p}_2)^2 = -\frac{E_{cm}^2}{c^2} = -m_1^2 c^2 - m_2^2 c^2 - \frac{2}{c^2} E_1 E_2$$

where each  $\underline{p} = (\mathbf{p}, iE/c)$ . Now

$$E_1 = T_1 + m_1$$

$$\sqrt{s} = E_{cm} = \sqrt{m_1^2 + m_2^2 + 2(T_1 + m_1)m_2}$$

$$2m_N + m_\pi = \sqrt{4m_N^2 + 2m_N T_N}$$

and

Using  $m_1 = m_2 = m_N$ 

$$2(0.938) + 0.140 = \sqrt{4(0.938)^2 + 2(0.938)T_N}$$

Solve for

$$T_N = 0.290 \text{ GeV} = 290 \text{ MeV}$$

as the answer.

## 11. (C)

The correct relativistic Lagrangian is

$$L = \frac{1}{2}mu^{\mu}u_{\mu} + qA^{\mu}u_{\mu}$$

To see this, use

$$\frac{d}{dt}\left(\frac{\partial L}{\partial u^{\nu}}\right) - \frac{\partial L}{\partial x^{\nu}} = 0$$

which is the relativistic Lagrange/Euler equation

$$\frac{dp^{\mu}}{dt} = q \left[ -\frac{\partial A^{\mu}}{\partial \tau} - \sum_{j} \frac{\partial A^{\mu}}{\partial x^{j}} c^{j} + \sum_{\nu} \frac{\partial A^{\mathbf{u}}}{\partial x^{\mu}} u_{\nu} \right]$$
  
e.g., 
$$\frac{dp^{1}}{dt} = q \left[ -\frac{\partial A^{1}}{\partial \tau} - \frac{\partial \phi}{\partial x^{1}} u^{4} + \gamma (\mathbf{u} \times B)^{1} \right]$$
$$\therefore \quad \frac{d\mathbf{p}}{dt} = \gamma q (E + \mathbf{u} \times B)$$

12. (E)

The 4-distance is

nd  
$$x^{\mu} = (\mathbf{r}, ct)$$
$$x^{\mu^{1}} = \sum_{\nu} \lambda_{\nu}^{\mu'} x^{\nu}$$

is the transformation equation. The  $\mu = 1$  and  $\mu = 3$  components are

$$\left.\begin{array}{c}x^{1'} = x^1\\x^{3'} = x^3\end{array}\right\}$$

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### **DETAILED EXPLANATIONS OF ANSWERS — PRACTICE EXAM 2**

since there is no change in the transverse directions. For the parallel direction and time component, we have

$$x^{2'} = \gamma(x^2 - \beta x^4)$$
$$x^{4'} = \gamma(x^4 - \beta x^2)$$

the standard forward Lorentz transformation for y and t. Hence,

$$\lambda = \left(egin{array}{cccccc} 1 & 0 & 0 & 0 \ 0 & \gamma & 0 & -eta\gamma \ 0 & 0 & 1 & 0 \ 0 & -eta\gamma & 0 & \gamma \end{array}
ight)$$



is the transformation matrix and  $\underline{x}' = \gamma \underline{x}$  is the matrix equation.

#### 13. (D)

 $p + \frac{238}{92}$ U is the elastic Rutherford scattering reaction.

$$\frac{d\sigma}{d\Omega} = \left(\frac{Z_1 Z_2 e^2}{4T}\right)^2 \sin^{-4}\frac{\theta}{2}$$

is the differential cross section.

$$10b \times \frac{1000mb}{b} \times \frac{1fm^2}{10mb} = 1000fm^2$$
$$1000 = \left[\frac{(1)(92)(1.44)}{4(7.6)}\right]^2 \sin^2\frac{q}{2}$$
$$\sin\frac{q}{2} = 0.37 \Rightarrow q = 43.6^\circ$$

14. (A)

The given central potential is

$$U = kr^n$$

Hence,

$$\mathbf{F} = -\nabla U = -k\left(\hat{r}\frac{\partial}{\partial r} + \hat{\theta}\frac{1}{r}\frac{\partial}{\partial \theta} + \hat{\phi}\frac{1}{r\sin\theta}\frac{\partial}{\partial\phi}\right)r^n$$

Writing the gradient in spherical coordinates,

$$= -k\hat{r} nr^{n-1}$$
$$= -kn r^{n-2}r\hat{r} = -kn r^{n-2}r$$

since  $\frac{\partial r}{\partial \theta}$  and  $\frac{\partial r}{\partial \phi}$ 

are zero (curvilinear orthogonal coordinates). Also,

 $\mathbf{r} = x\mathbf{x} + y\mathbf{y} + z\mathbf{z}$ 

in spherical coordinates.

### 15. (D)

A rocket in a gravitational field may be understood from

$$-mg = mv' + um'$$

by Newton's Second Law

$$-g \int_{0}^{t} dt = \int_{v^{0}}^{v} dv + u \int_{m^{0}}^{m} \frac{dm}{m}$$
$$v = v_{0} - gt + u \ln \frac{m_{0}}{m}$$
$$= 0.4 - 9.8 \times 10^{-3} (100) + 2.0 \ln \frac{3}{1}$$
$$= 1.62 \text{ km/s}$$

16. (A)

Use

$$\oint \gamma \cdot d\alpha = -4\pi G m_{in}$$

the integral form of Gauss's law for gravitation. Evaluate the integral in the different regions

$$r < a, \quad \oint \gamma \cdot d\alpha = 0 \rightarrow g = 0$$
$$r > b, \quad \oint \gamma \cdot d\alpha = -4\pi GM = -g4\pi r^2$$

thus

$$g = \frac{GM}{r^2} \text{ or } \gamma = -\frac{GM\rho}{r^2}$$
$$a < r < b, \quad \oint \gamma \cdot d\alpha = -4\pi r^2 g = -4\pi G\rho \frac{4\pi}{3} (r^3 - a^3)$$
$$g = \frac{4}{3}\pi G\rho \left(r - \frac{a^3}{r^2}\right)$$

A plot of g versus r is shown in the following figure.



## 17. (C)

The gravitational field of a disk may be found from fundamentals

$$\mathbf{g} = -G \int \frac{dm}{r^2} \mathbf{r}$$

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where  $dm = \sigma da = 2\pi\sigma s ds$ .

$$\mathbf{g} = -G \int \frac{\sigma \mathbf{r} da}{r^2}$$
  
=  $-2\pi\sigma G \int (s \, ds/r^2)(x/r) \mathbf{x}$   
$$\mathbf{g} = -2\pi\sigma G x \int_0^R \frac{s \, ds}{(x^2 + s^2)^{3/2}} \mathbf{x}$$
  
=  $-2\pi\sigma G \left[ 1 - \frac{x}{\sqrt{x^2 + R^2}} \right] \mathbf{x}$ 

# 18. (B)

Fermat's principle requires that the time be a minimum. The time is given by the action integral

$$A = \int \frac{ds}{v}$$
  
Using  $n = \frac{c}{v}$  or  $v = \tau/n$ 

we get 
$$A = \frac{1}{c} \int n(x, y) \sqrt{dx^2 + dy^2}$$

Since 
$$ds = \sqrt{dx^2 + dy^2}$$

thus 
$$A = \frac{1}{c} \int_{P}^{Q} \sqrt{1 + y'^2} n(x, y) dx$$

## 19. (C)

The nuclear reaction is

$$^{27}_{13}\text{Al}(\alpha,p) \,^{30}_{14}\text{Si or} \,^{4}_{2}\alpha \,+\,^{27}_{13}\text{Al} \rightarrow \,^{1}_{1}p \,+\,^{30}_{14}\text{Si}$$

with Q-value.

$$Q = m_I + m_T - m_R - m_E$$
  
= (4.0026 + 26.9815 - 29.9738 - 1.0078) × 931.502  
= 2.3 MeV

For the reverse reaction

$$Q = -2.3 \text{ MeV}$$

and

$$T_{\text{lab}} = \left(1 + \frac{m_I}{m_T}\right) |Q|$$
$$= 2.7 \text{ MeV}$$

is the minimum kinetic energy needed.

# 20. (A)

Sound waves in a gas may be modeled as adiabatic. Hence,  $pV^{\gamma} = \text{constant}$  where  $\gamma = \frac{5}{3}$  for three degrees of freedom. Differentiate to get

$$\frac{d}{dv}(pV^{\gamma}) = \gamma pV^{\gamma-1} + V^{\gamma}\frac{dp}{dV} = 0$$
$$\frac{dp}{dV} = -\frac{\gamma p}{V}$$

Thus,

The sound speed is

$$c_1 = \sqrt{\frac{\left(B + \frac{4}{3}S_m\right)}{\rho}} = \sqrt{\frac{B}{\rho}}$$

for an ideal fluid which has zero shear modulus and

$$B = -V\frac{dp}{dV} = \gamma p$$
Finally,

$$c_1 = \sqrt{\frac{\gamma p}{\rho}}$$

and hence doubling p changes the sound speed  $c_1$  by a factor of  $\sqrt{2}$  .

# 21. (B)

The Maxwell-Boltzman distribution is given by

$$dn = gN\left(\frac{1}{2\pi mkT}\right)^{3/2} e^{-E/kT} d^3p$$

where the degeneracy g = 1 for classical particles. One can always find the factor  $(1/2 \pi mkT)^{3/2}$  by normalization

$$\int_0^N dn = N$$

Clearly,

$$\frac{dn}{N} = p(v)dv$$

defines a probability density function p(v). Thus,

$$= \int_{0}^{\infty} v^{2} p(v) dv$$
  
=  $\frac{1}{(2\pi m kT)^{3/2}} \int_{0}^{\infty} e^{-mv^{2}/2kT} v^{2} 4\pi m^{2} v^{2} m dv$   
=  $4 \left(\frac{m}{2\pi kT}\right)^{3/2} \int_{0}^{\infty} e^{-at} t^{5/2-1} \frac{dt}{2}$ , where  $a \equiv \frac{m}{2kT}$   
=  $\frac{1}{\sqrt{2\pi}} \left(\frac{m}{kT}\right)^{3/2} \frac{3}{2} \cdot \frac{\sqrt{\pi}}{2} / \left(\frac{m}{2kT}\right)^{5/2} = 3kT/m$ 

where

$$\left(\frac{3}{2}\right)! = \frac{3}{2} \times \left(\frac{1}{2}\right)! = \frac{3}{2} \frac{\sqrt{\pi}}{2}$$

was used.

# 22. (A)

The situation is governed by the binomial distribution

$$p(n) = \binom{N}{n} = p^n (1-p)^{N-n}$$
$$\binom{N}{n} \frac{n!}{n! (N-n)!}$$

where

is the standard binomial coefficient. The probability parameter is

$$p = \frac{V_0}{V} = \frac{10^{-18} \text{ cm}^2}{1 \times 10^8 \text{ cm}^3} = 10^{-26}$$

The desired probability is p(0)

$$p(0) = \frac{N!}{O!N!} p^0 (1 - 10^{-26})^{10^{27}}$$
$$\approx e^{-10} = 4.5 \times 10^{-5}$$

where we have used

$$(1-x)^N \approx (e^{-x})^N = e^{-xN}$$

since  $x \ll 1$ .

# 23. (D)

The standard rule for the propagation of error is

$$\Delta f = \sqrt{\left(\frac{\partial f}{\partial x_1}\Delta x_1\right)^2 + \ldots + \left(\frac{\partial f}{\partial x_N}\Delta x_N\right)^2}$$

the density of a sphere is

$$\rho = \frac{m}{v} = m/\frac{4}{3}\pi r^3$$

Hence,

$$\Delta \rho = \sqrt{\left(\frac{\partial \rho}{\partial m} \Delta m\right)^2 + \left(\frac{\partial \rho}{\partial r} \Delta r\right)^2}$$

$$\frac{\Delta\rho}{\rho} = \sqrt{\left(\frac{\Delta m}{m}\right)^2 + 9\left(\frac{\Delta r}{r}\right)^2}$$
$$= \sqrt{\left(\frac{3}{183}\right)^2 + 9\left(\frac{0.1}{25}\right)^2}$$
$$= 0.020$$

for the solid A1 sphere's relative uncertainty.

# 24. (E)

For the entire cycle ADCBA,

$$\Delta U = 0 = \Delta Q - \Delta W$$
  
Hence,  
$$\Delta Q = Q_C - Q_H = \Delta W$$

since  $\mathcal{Q}_{C}$  is absorbed and  $\mathcal{Q}_{H}$  is ejected. The efficiency is then

$$e = rac{|\Delta W|}{Q_c} = rac{Q_H}{Q_c} - 1$$

since  $Q_H > Q_C$ .

Finally, one needs to use the fact that  $Q_H/Q_C = T_H/T_C$  to get

$$e = \frac{T_H}{T_C} - 1$$

where

 $T_H > T_C$ 

## 25. (B)

A single particle with one degree of freedom may be described in a space (x, p) of two dimensions. The cumulative number of states is

$$\Gamma(E) = \frac{\ell}{2\pi\hbar} \int \theta[E - H(x, p)] dx \, dp$$
$$= \frac{\ell}{2\pi\hbar} \int_{\left(\frac{p^2}{2m} - E\right)} dp = \frac{\ell}{2\pi\hbar} \int_{-\sqrt{2mE}}^{\sqrt{2mE}} dp$$
$$= \ell\sqrt{2mE}/\pi\hbar$$

The density of states is

$$\frac{d\Gamma}{dE} = \ell \sqrt{2m/E}/2\pi\hbar$$

# 26. (E)

For the two state system with degeneracies  $(g_1, g_2)$  and energies  $(E_1, E_2) = (0, \varepsilon)$  the partition function is

$$Z = \Sigma g_j e^{-\beta E_j} = g_1 + g_2 e^{-\beta \epsilon}$$

The average energy is then

$$\langle E \rangle = \frac{-\partial}{\partial \beta} \ln Z = -\frac{1}{Z} \frac{\partial Z}{\partial \beta}$$
  
=  $g_2 \varepsilon e^{-\beta \varepsilon} / (g_1 + g_2 e^{-\beta \varepsilon})$ 

and the total energy for N particles is

27. (D)

Lissajous figures are generated from the coupled harmonic equations

$$\begin{cases} x'' + \omega_x^2 x = 0 \\ y'' + \omega_y^2 y = 0 \end{cases}$$

with solution

$$x = A \cos (\omega_x t + \alpha)$$
$$y = B \cos (\omega_y t + \beta)$$

The figure is thus a parametric plot (x(t), y(t)). Let  $\delta = \beta - \alpha$  be the phase difference. Then

$$A = B, \, \omega_y = 2\omega_x, \, \delta = \pi/2$$

gives the "butterfly." Also

$$\delta = \pm \pi/2, \, \omega_v = \omega_x$$

gives an ellipse, and

 $\delta = 0$  or  $\pm \pi$  with  $\omega_{y} = \omega_{x}$ 

is a line.

## 28. (D)

The light damping or underdamped motion solution corresponds to

$$\omega_0^2 > \gamma^2/4$$

The differential equation is

$$mx'' = -kx - bx'$$
$$x'' + \gamma x' + \omega_0^2, \ \gamma \equiv b/m, \ \omega_0 \equiv \sqrt{k/m}$$

A solution would be  $x = e^{pt}e^{i\delta}$  with auxiliary equation  $p^2 + \gamma p + \omega_0^2 = 0$ . This implies

$$p_{\pm} = -\frac{\gamma}{2} \pm i\omega$$
 where  $\omega \equiv \sqrt{\omega_0^2 - \gamma^2/4}$ 

One can write

$$x = x_{+} + x_{-}$$
$$= Ae^{-\gamma t/2} (e^{i(\omega t + \delta)} + e^{-i(\omega t + \delta)})$$
$$= 2Ae^{-\gamma t/2} \cos(\omega t + \delta)$$
$$= Be^{-\gamma t/2} \cos(\omega t + \delta)$$

with  $B \equiv 2A$ .

#### 29. (B)

$$R = 109,677.6 \text{ cm}^{-1}$$

is given as the Rydberg constant for hydrogen. The Lyman, Balmer, Paschen, Brackett, and Pfund series result from electronic transitions from level m to n = 1, 2, 3, 4, 5, respectively. For the Paschen series n = 3 and so

$$\frac{1}{\lambda} = R\left(\frac{1}{n^2} - \frac{1}{m^2}\right)$$
$$= R\left(\frac{1}{9} - \frac{1}{m^2}\right)$$

Clearly  $m \to \infty$  gives  $\lambda = 820.6$  nm the lower limit. m = 4 gives the upper limit  $\lambda = 18,760$ Å.

# **GRE PHYSICS**

# 30. (E)

It is desired to find the average entry for a photon gas, i.e., black body radiation. Proceed using Boltzmann factors:

$$E_{j} = j\hbar w, j = 0, 1, 2, \dots \infty$$

$$< E > = \Sigma E_{j} e^{-\beta E j} / \Sigma e^{-\beta E j}$$

$$= \hbar \omega \sum_{j} j e^{-jx} / \sum_{j} e^{-jx}, x = \hbar \omega \beta$$

$$= \hbar \omega \sum_{j} j y^{j} / \sum_{j} y^{i} m y \equiv e^{-x}$$

$$= \hbar \omega (y / (1 - y)^{2}) / (1 / (1 - y))$$

$$= \hbar \omega / (e^{\hbar \omega \beta} - 1)$$

We have used the infinite geometric series results

$$1 + y + y^2 + \ldots = 1/(1 - y)$$

and

$$y(1+2y+3y^2+\ldots) = y\frac{d}{dy}(1-y)^{-1} = \frac{y}{(1-y)^2}$$

31. (C)

The 19 electrons of potassium are arranged as

1s<sup>2</sup>2s<sup>2</sup> 2p<sup>6</sup> 3s<sup>2</sup> 3p<sup>6</sup> 4s<sup>1</sup>

of which the first 18 electrons have the same configuration as for the argon atom, namely,

$$1s^22s^2 2p^6 3s^2 3p^6$$

32. (D)



# **DETAILED EXPLANATIONS OF ANSWERS — PRACTICE EXAM 2**

The electric field and electric potential are related by:

$$E = -\nabla \phi = -\frac{d\phi}{d}$$
 or  $\phi = -\int \mathbf{E} \cdot d\mathbf{r}$ 

Another fundamental formula is that

$$\phi = +k \int \frac{dq}{r}$$

Thus,

$$\phi = +k \int_{a}^{b} \frac{2\pi\sigma s ds}{\sqrt{s^2 + x^2}}$$

$$= +2\pi k\sigma \sqrt{s^2 + x^2} \begin{vmatrix} s = b \\ s = a \end{vmatrix}$$

$$=+2\pi k\sigma\left(\sqrt{b^2+x^2}-\sqrt{a^2+x^2}\right)$$

Graphically, this looks like

## 33. (C)

The black body distribution is given by

$$u(\boldsymbol{\omega}) = \frac{\mathbf{h}\boldsymbol{\omega}^3}{\pi^2 c^3} \frac{1}{(e^{\mathbf{h}\boldsymbol{\omega}\boldsymbol{\beta}} - 1)}$$

For high temperatures or low frequencies, one can make a Taylor expansion of  $e^x$ 

$$e^x = 1 + x + \frac{x^2}{2!} + \dots$$

Hence,

$$u(\boldsymbol{\omega}) = \frac{\hbar\omega^3}{\pi^2 c^3} \frac{1}{1 + \hbar\omega\beta - 1} = \omega^2 kT / \pi^2 c^3$$

If one tried to now calculate the total energy density

$$U=\int_0^\infty U(\omega)d\omega,$$

one gets the ultraviolet catastrophe.



# 34. (D)

The Thomas-Reiche-Kuhn sum rule states that

$$\sum_{j} |x_{j0}|^2 (E_j - E_0) = \mathbf{h}^2 / 2m$$

Since

$$[H,x] = \frac{\mathbf{h}}{i} \frac{p}{m}, \ [x,[H,x]] = \frac{\mathbf{h}}{im} [x,p] = \frac{\mathbf{h}}{im} \left(\frac{-\mathbf{h}}{i}\right) = \frac{\mathbf{h}^2}{m^2}$$

Also,  $\left\langle 0 \left| \frac{\mathbf{h}^2}{m} \right| 0 \right\rangle = \frac{\mathbf{h}^2}{m} = \left\langle 0 \left| [x, Hx - xH] \right| 0 \right\rangle$ 

using a wave function  $\Psi_{0}$  and expanding the commutator

$$\mathbf{h}^2/m = \left< 0 |2xHx - x^2H - Hx^2|0 \right>$$

Further,

$$<0|xHx|0>=\sum_{j}<0|x|j>E_{j}=\Sigma|(x_{j0})|^{2}E_{j}$$

and

$$<0|Hx^{2}|0>=<0|x^{2}H|0>$$

$$= \sum_{j} <0|x|j> < j|x|0>E_{0} = \sum |x_{j0}|^{2}E_{0}$$

Finally, we get

$$\frac{\hbar^2}{2m} = \sum_j |x_{j0}|^2 (E_j - E_0)$$

## 35. (A)

A general rule for commutators in quantum physics is

$$[A^2, B] = A[A, B] + [A, B]A.$$

This problem involves the operator

$$H = \frac{p^2}{2m} + V(x)$$

Obviously, [V(x), x] = 0. Then consider

$$\frac{1}{2m}[p^2, x] = \frac{1}{2m}(p[p, x] + [p, x]p)$$
$$= \frac{1}{2m}2p\frac{h}{i}$$
$$= \frac{hx'}{i}$$

using p = mx'. Thus,

$$[H,x] = \frac{\mathbf{h}}{i}x'$$

#### 36. (A)

A typical experimental set-up is pictured below.



The heated filament supplies electrons which are accelerated by the positive voltage of the grid. The accelerated electrons gain energy so that  $(eV_0 = \frac{1}{2}mv^2)$  and experience inelastic collisions with Hg atoms. The back voltage  $\Delta V$  deters some electrons from reaching the plate. The current-voltage curve peaks at the quantum energy levels.

## 37. (B)

For any spherical harmonic, the normalization condition is

$$\int Y_{lm_i} Y_{lm_i} * d\Omega = 1$$

where  $\Omega$  is the solid angle. For the given problem, we have

$$\ell = m_l = 1$$
$$\int Y_{\parallel} Y_{\parallel} * d\Omega = 1$$
$$\int N^2 \sin^2 \theta e^{i\phi} e^{-i\phi} d(\cos \theta) d\phi$$

where we have used the fact that

$$d\Omega = \sin \theta \, d\theta \, d\phi$$
$$N^2 \int_{-1}^{1} \sin^2 \theta \, d(\cos \theta) \int_{0}^{2\pi} d\phi$$

Use the identity  $\sin^2 = 1 - \cos^2 \theta$  to get

$$2\pi N^2 \int_{-1}^{1} (1 - \cos^2 \theta) d(\cos \theta) = 2\pi N^2 \left[2 - \frac{1}{3}2\right]$$
$$1 = \frac{8\pi}{3}N^2 \Rightarrow N = \sqrt{3/8\pi}$$

## 38. (B)

The hydrogen radial wave functions come from the associated Laguerre polynomials. The ground state wave function is found from the quantum numbers n = 1, l = 0,  $m_l = 0$ .

$$R_{10}(Y) = Ne^{-Zr/a0} = 2\left(\frac{Z}{a_0}\right)^{3/2} e^{-Zr/a0}$$

The radial probability density is  $p(r) = RR^* r^2$  and this peaks where

$$\frac{d}{dr}(r^{2}e^{-2Zr/a^{0}}) = 0$$
  

$$2re^{-2Zr/a^{0}} - r^{2}\frac{2Z}{a_{0}}e^{-Zr/a^{0}} = 0$$
  

$$r^{2}\frac{2Z}{a_{0}} = 2r$$
  

$$r\frac{2Z}{a_{0}} = 2$$

Finally,

$$r = \frac{a_0}{Z}$$

One may also show that

$$\frac{d^2p}{dr^2} < 0 \text{ at } \frac{a_0}{Z}$$

proving that the extremum is a maximum.

## 39. (C)

The n = 2 and l = 1 radial wave function is under consideration

$$R_{21}(r) = Nre^{-Zr/2a_0}$$

Use the normalization condition  $\int RR^* r^2 dr = 1$ 

$$= N^2 \int_0^\infty r^4 e^{-Zr/a_0} dr$$

$$= N^{2} \int_{0}^{\infty} r^{5-1} e^{-Ar/a_{0}} dr$$
$$= N^{2} \Gamma(5) / (Z/a_{0})^{5} = N^{2} 4! \times a_{o}^{5} / Z^{5}$$
$$N^{2} = \frac{Z^{3}}{8a_{0}^{3}} \frac{Z^{2}}{3a_{0}^{2}} \Rightarrow N = \left(\frac{Z}{2a_{0}}\right)^{3/2} \frac{Z}{\sqrt{3}a_{0}}$$

Thus,

# 40. (B)

The hydrogen-like atom energy is a function of Z,  $\mu$ , and n

$$E_n = -k^2 Z^2 \mu e^{4/2 h^2 n^2}$$
$$= -\frac{Z^2}{n^2} \frac{\mu}{m_e} 13.6 \ eV$$

This is most easily derived from Bohr theory

$$F = \frac{\mu v^2}{r} = \frac{kZe^2}{r^2}$$
$$L = \mu vr = h h$$
$$\Rightarrow r = n^2 a_0/Z$$

with the Bohr radius as  $a_0 = \frac{\hbar^2}{\mu ke^2}$ . Then

$$E = T + U = \frac{1}{2}\mu v^2 - \frac{kZe^2}{r}$$
$$= -\frac{kZe^2}{2r}$$
$$= -\frac{kZe^2}{2}\frac{Z}{n^2}\frac{\mu ke^2}{h^2}$$
$$= -\frac{k^2Z^2\mu e^4}{2h^2n^2}$$

41. (C)

Experimentally, the nuclear binding energy is

$$BE = 931.50A_{\rm el} - 938.28 Z - 939.57 N$$

neglecting the electron masses. According to the semi-empirical mass formula, we get

$$BE = -C_{v}A + C_{c}Z(Z-1)A^{1/3} + C_{A}A^{2/3} + C_{s}\frac{(A-2Z)^{2}}{A}$$

with parameters

$$C_V = 15.6 \text{ MeV}, C_C = 0.7 \text{ MeV}, C_A = 17.2 \text{ MeV}, C_S = 23.3 \text{ MeV}$$

## 42. (D)

Applying the conservation of atomic number (charge) and mass number (# of nucleons) the equation is missing three nucleons and no charge. Thus, there are three neutrons produced.

## 43. (D)

By Kirchhoff's current law

$$I_1 = I + I_2 \rightarrow I = I_1 - I_2.$$

By Kirchhoff's voltage laws:

$$-q_1/c - LI'_1 - \frac{q_1 - q_2}{\gamma} = 0$$
$$-q_2/c - LI'_2 - \frac{q_1 - q_2}{\gamma} = 0$$

Differentiate and let K = 1/LC and  $\kappa = 1/L\gamma$ . Then we obtain

$$I_1'' = -I_1 K + (I_2 - I_1) \kappa$$
$$I_2'' = -I_2 K - (I_2 - I_1) \kappa$$

Add and substract the equations letting  $y = I_1 + I_2$  and  $z = I_2 - I_1$ , respectively, to get

$$y'' = -ky$$
 and  $z'' = -kz - 2\kappa z$ .  
 $\omega_z / \omega_z = \sqrt{(k+2\kappa)/k} = \sqrt{\frac{1+k}{2\kappa}}$ 

Hence, the ratio

$$\omega_a/\omega_s = \sqrt{(k+2\kappa)/k} = \sqrt{\frac{1+2\kappa}{k}}$$

#### 44. (E)

This is the standard twin paradox problem. We are given  $v = 1.50 \times 10^8$  m/s. Hence,

$$\beta = \frac{v}{c} = \frac{1}{2}$$
 and  $\gamma = \frac{1}{\sqrt{1 - \beta^2}} = \frac{2}{\sqrt{3}}$ 

The time dilation equation is  $t = t_0 \gamma$  where  $t_0$  is spaceship time and t is Earth time. We extrapolate that  $t_0 = \frac{t}{\gamma}$ . We are given  $v = 1.50 \times 10^8$  m/s. Hence,

$$\beta = \frac{v}{c} = \frac{1.50 \times 10^8 \text{m/s}}{3.0 \times 10^8 \text{m/s}} = \frac{1}{2}$$

and  $\gamma = \frac{1}{\sqrt{1-\beta^2}} = \frac{2}{\sqrt{3}}$ . In terms of Earth years, for a round trip it will take John:

$$t = \frac{2}{v} = \frac{2 \times 25}{\frac{1}{2}c} = 100$$
 years



# **DETAILED EXPLANATIONS OF ANSWERS — PRACTICE EXAM 2**

The time dilation equation is  $t = t_0 \times \gamma$ , where  $t_0$  is spaceship time and t is Earth time.

$$t_0 = \frac{t}{\gamma} = \frac{100}{2/\sqrt{3}} = 86.6$$
 years

 $\Delta t = t - t_0 = 100 - 86.6 = 13.4$  years (light years)

**45. (B)** In relativity,  $\mathbf{F} = \frac{d\mathbf{p}}{dt}$  just as holds classically.

However, the relativistic momentum is  $\mathbf{p} = m\mathbf{v}\gamma$ . If  $\mathbf{v}$  changes only in direction, then

$$\mathbf{F} = \mathbf{F}_{\perp} = \frac{d}{dt}(m\mathbf{v}\boldsymbol{\gamma}) = m\boldsymbol{\gamma}\frac{d\mathbf{v}}{dt} = m\boldsymbol{\gamma}\mathbf{a}_{\perp}$$

If v changes only in magnitude, then

$$\mathbf{F} = \mathbf{F}_{\parallel} = \frac{d}{dt} (m\mathbf{v}\gamma)$$
$$= m\gamma \frac{d\mathbf{v}}{dt}t + m\beta^2 \gamma^3 \frac{d\mathbf{v}}{dt}$$
$$= m\frac{d\mathbf{v}}{dt}\gamma^3 \left(\frac{1}{\gamma^3} + \beta^2\right)$$
$$= m\gamma^3 \mathbf{a}_{\parallel}.$$

For 
$$|\mathbf{a}_{\perp}| = |\mathbf{a}_{\parallel}|$$
 we get  $\frac{\mathbf{F}_{\perp}}{\mathbf{F}_{\parallel}} = \gamma^2$ .

46. (C)

The Biot-Savart Law must be used

$$d\mathbf{B} = \frac{\mu_0}{4\pi} \frac{Idl \times \mathbf{r}}{r^2}$$

$$B_z = \frac{\mu_0 I}{4\pi} \int_0^{2\pi r} \frac{dl}{r^2} = \frac{\mu_0 I}{2r}$$

The current is

$$I = \frac{q}{t} = \frac{e}{2\pi r/v} = \frac{ev}{2\pi r}$$

and the velocity comes from (in Bohr theory)

$$mv^{2}/r = ke^{2}/r^{2}$$
$$v = \sqrt{ke^{2}/mr} = \sqrt{\frac{9 \times 10^{9}(1.6 \times 10^{-10})^{2}}{9.1 \times 10^{-31}(0.529 \times 10^{-10})}}$$
$$= 2.19 \times 10^{6} \text{ m/s}$$

Thus,

$$I = (1.60 \times 10^{-19})(2.19 \times 10^{6})/2\pi (0.529 \times 10^{-10}) = 0.00105 \text{ A}$$

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and finally

$$B_z = (4\pi \times 10^{-7})(.00105)/2(0.529 \times 10^{-10})$$
  
= 12.5 Tesla

# 47. (D)

Maxwell's equations are

$$\nabla \cdot \mathbf{D} = 0 \qquad \nabla \cdot \mathbf{B} = 0$$
$$\nabla \times E = -\frac{\partial B}{\partial t} \qquad \nabla \times H = +\frac{\partial \mathbf{D}}{\partial t}$$

where  $\mathbf{D} = \varepsilon \mathbf{E}$  and  $\mathbf{B} = \mu \mathbf{H}$ . Now suppose

$$\mathbf{E} = E\mathbf{y}$$
, and  $\mathbf{H} = \mathbf{H}\mathbf{z}$ ,

$$\nabla \times \mathbf{E} = \left(\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y}\right) \mathbf{z} \equiv \frac{\partial E_y}{\partial x} \mathbf{z}$$

then

since 
$$E_{y} = 0$$
 and

$$abla imes \mathbf{H} = \left(rac{\partial H_x}{\partial z} - rac{\partial H_z}{\partial x}
ight) \mathbf{y} \equiv -rac{\partial H_z}{\partial x} \mathbf{y}$$

since  $H_x = 0$ . Then we may rewrite the above

$$\frac{\partial H}{\partial x} = -\frac{\partial D}{\partial t}$$
 as  $\frac{\partial B}{\partial x} = -\mu \frac{\partial D}{\partial t}$ 

Hence, we obtain

$$\frac{\partial E}{\partial x} = -\frac{\partial B}{\partial t} \text{ and } \frac{\partial B}{\partial x} = -\mu \frac{\partial D}{\partial t}$$
$$\varepsilon = \varepsilon_0 e^{\alpha t} \text{ and } \mu = \mu_0 e^{\alpha t},$$

If

then we get

$$\frac{\partial^2 E}{\partial x^2} = \frac{-\partial^2 B}{\partial t \partial x} = \frac{\partial}{\partial t} \left( \mu \frac{\partial D}{\partial t} \right) = \mu \frac{\partial^2 D}{\partial t^2} + \mu \alpha \frac{\partial D}{\partial t}$$

# 48. (D)

$$v' = (v + u) / (1 + uv/c^2) = (0.5c + 0.5c)/(1 + 0.25c^2/c^2) = c/(1 + 0.25) = 0.8c$$

# 49. (C)

The initial picture is



and the final situation is



By symmetry, the kinetic energy is evenly split after the collision.

Now:

$$E = T + mc^2$$
 and  $p = \sqrt{E^2 - m^2 c^4}$ 

give  $p^2 = T^2 + 2mc^2T$ .

Similarly,

$$E' = \frac{1}{2}T + mc^2$$
 and  $p' = \sqrt{E'^2 - m^2c^4}$  yield  $p'^2 = T^2/4 + mc^2T$ 

By conservation of momentum  $p = 2p' \cos \theta$  or

$$\cos\theta = \sqrt{T^2 + 2mc^2T} / \sqrt{T^2 + 4mc^2T}$$

Plug in  $mc^2 = 938$  MeV and T = 1876 MeV  $= 2 mc^2$  to get  $\theta = A \cos(2/3) = 35.3 \rightarrow 2\theta = 70.6^{\circ}$ .

#### 50. (C)

There is a wide range of temperatures in the physical universe, many accessible in the laboratory.

- 0 K the absolute zero or coldest temperature
- 20 K the normal boiling point of hydrogen
- 77 K the normal boiling point of nitrogen
- 1234 K the normal melting point of silver
- 6000 K the sun's surface temperature
- $10^6$  K the sun's interior or core temperature
- $10^{12}$  K the temperature achieved in a nuclear collision

#### 51. (C)

Double-slit interference  $\rightarrow d \sin \theta = m\lambda$  for bright fringes

Single-slit diffraction  $\rightarrow w \sin \theta = n\lambda$  for dark fringes

If  $\theta$  is the same for the two patterns, that is the position at which bright fringe of interference pattern is missing due to dark fringe of the diffraction pattern formed there.

Hence,  $\sin\theta = (m\lambda/d)/(n\lambda/w) \rightarrow m/n = d/w \rightarrow m/n = 6\lambda/2\lambda = 3$ . The smallest value of *m* is when n = 1, i.e., m = 3.

## 52. (C)

In order to get from the usual wave equation to the Schrödinger equation, one uses the de Broglie wavelength concept

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2m(E-U)}}$$

Start with the 1-D wave equation

$$\frac{\partial^2 \Psi}{\partial x^2} = \frac{1}{v^2} \frac{\partial^2 \Psi}{\partial t^2}, \ \Psi(x,t) = \phi(x) e^{i\omega t}$$

Separating variables

$$\frac{d^2\phi}{dx^2} + \frac{\omega^2}{v^2}\phi(x) = 0$$

where

$$\omega^2 = 4\pi f^2 = 4\pi^2 v^2 / \lambda^2 = 4\pi^2 2m(E - U) v^2 / h^2$$

Substituting

$$\frac{d^2\phi}{dx^2} + \frac{2m(E-U)}{\hbar^2}\phi(x) = 0$$

Rearranging

$$-\frac{\hbar^2}{dm}\frac{d^2\phi}{dx^2} + U\phi = E\phi$$

or finally  $H\phi = E\phi$  in operator form.

# 53. (D)

The damped one-dimensional harmonic oscillator differential equation is

$$x'' + \gamma x'' + \omega_0^2 x = 0$$

where

$$\omega_0 = \sqrt{\frac{k}{m}} \text{ and } \gamma = \frac{b}{m}$$

In the light damping situation  $w_0^2 > \gamma^2/4$  and the solution is

$$x = B_0 e^{-\gamma t/2} \cos(\omega t + \delta)$$
 where  $\omega = \sqrt{\omega_0^2 - \gamma^2/4}$ 

If  $\gamma \ll 1$ , then

$$x' = -B_0 \omega e^{-\gamma t/2} \sin(\omega t + \delta)$$

and

$$E = \frac{1}{2}mx'^{2} + \frac{1}{2}kx^{2}$$
$$= \frac{1}{2}B_{0}^{2}e^{-\gamma r}(m\omega^{2}\sin^{2}(\omega t + \delta) + k\cos^{2}(\omega t + \delta))$$
$$= \frac{1}{2}kB_{0}^{2}e^{\gamma r} \text{ using } \omega \approx \omega_{0}$$

# 54. (D)

For  $E < V_0$ , the two needed wave numbers are

$$k = \sqrt{2mE}/\hbar$$
 and  $k' = \sqrt{2m(V_0 - E)}/\hbar$ 

The wave function is

$$\phi(x) = \begin{cases} e^{ikx} + re^{-ikx} & x < 0\\ \\ te^{-k'x} & x > 0 \end{cases}$$

The boundary conditions give

$$\phi(0) = 1 + r = t$$
$$\phi'(0) = ik - ikr = -k't$$

Solving simultaneously gives t = -2ik/(k' - ik) and the transmission coefficient is

$$T = tt^*$$
  
=  $(4k^4 + 4k^2k'^2)/(k^2 + k'^2)^2$   
=  $4k^2(k^2 + k'^2)/(k^2 + k'^2)^2$   
=  $4k^2/(k^2 + k'^2)$ 

# 55. (B)

The given wave function is

$$\phi(x) = \frac{1}{\sqrt{a}} \cos \frac{\pi x}{2a}$$

and we want to find the value of  $\langle x^2 \rangle$ .

$$\langle x^2 \rangle = \int \phi * x^2 \phi dx$$
  
 $= \frac{1}{a} \int x^2 \cos^2 \frac{\pi x}{2a} dx$ 

Use

$$\cos^2\theta = \frac{1}{2} + \frac{1}{2}\cos 2\theta$$

to get

$$\langle x^2 \rangle = \frac{1}{a} \int \left( \frac{1}{2} x^2 + \frac{1}{2} x^2 \cos \frac{\pi x}{a} \right) dx$$

Integrate by parts

$$= \frac{1}{a} \left[ \frac{1}{6} x^3 + \frac{a}{2\pi} x^2 \sin \frac{\pi x}{a} + \frac{a^2}{\pi^2} x \cos \frac{\pi x}{a} - \frac{a^3}{\pi^3} \sin \frac{\pi x}{a} \right] \begin{vmatrix} a \\ -a \end{vmatrix}$$
$$= \frac{1}{a} \left( \frac{1}{3} a^2 + 0 + \frac{-2a^3}{\pi^2} - 0 \right)$$
$$= a^2 \left( \frac{1}{3} - \frac{2}{\pi^2} \right)$$

Note that  $\langle x^2 \rangle \neq 0$  but  $\langle x \rangle = 0$ .

## 56. (C)

$$\mathbf{R}' = \mathbf{R}/\mathbf{n}; \mathbf{M}' = \mathbf{M};$$

Moment of inertia for a solid sphere is  $2/5 \text{ mr}^2$ .

Now the angular momentum is conserved. Hence,

$$\begin{split} I\omega &= I'\omega' \to (2/5 \text{ MR}^2)(2\pi/T) = (2/5 \text{ MR}'^2)(2\pi/T') \to \\ R^2/T &= R'^2/T' \to T' = (R'/R)^2T = (1/n)^2T = T/n^2 \end{split}$$

## 57. (C)

For an RLC circuit, Kirchhoff's law gives

$$-RI - LI' - \frac{Q}{C} = 0$$

or differentiating and defining

$$\omega_0^2 = \frac{1}{LC}$$
 and  $\gamma = \frac{R}{L}$   
 $I'' + \gamma I' + \omega_0^2 I = 0$ 



for the critical dumping case,

$$\omega_0^2 = \gamma^2/4$$

and the solution is

$$I = (A + Bt)e^{-\gamma t/2}$$

If  $I = I_0$  and I' = 0 at t = 0, then the desired solution is

$$I = I_0 \left( 1 + \frac{\gamma}{2} t \right) e^{-\gamma t/2}$$

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# **GRE PHYSICS**

# 58. (D)

An object propelled horizontally at distance r from the center of the Earth into a circular orbit feels a force

$$F = \frac{GmM}{r^2} = \frac{mv^2}{r}$$

Hence,

 $\sqrt{\frac{GM}{r}} = r\omega$ 

The linear frequency is then

It is interesting to note that this is the same as the period of an object dropped from distance  $r = r_E$  and falling through a hole in the Earth (see above figure) to execute simple harmonic motion.

# 59. (C)

By conservation of energy

 $mgh = \frac{1}{2}mv^2$  or  $v = \sqrt{2gh}$ 

is the pendulum bob velocity just before it hits the spring.

The conservative force is

$$F = -kx - bx^3$$

so that

$$U = -\int F \, dx = \frac{1}{2}kx^2 + \frac{1}{4}bx^4$$





$$T_r = rac{1}{arphi} = 2\pi \sqrt{rac{r^3}{GM_E}}$$

 $v = \frac{\omega}{2\pi} = \frac{1}{2\pi} \sqrt{\frac{GM}{r^3}}$ 

$$v^2 = \frac{GM}{r}$$
 and  $v = \sqrt{r}$ 

Again by conservation of energy

$$\frac{1}{2}mv^2 = mgh = \frac{1}{2}kx^2 + \frac{1}{4}bx^4$$

Rearranging

$$\left(x^2 + \frac{k}{b}\right)^2 = \frac{4mgh}{b} + \frac{k^2}{b^2}$$
$$x = \left(\sqrt{\frac{4mgh}{b} + \frac{k^2}{b^2}} - \frac{k}{b}\right)^{1/2}$$

or

60. (A)

This is Johann Bernoulli's famous brachistochrone problem. By conservation of energy

$$\Delta T = -\Delta V$$
$$\frac{1}{2}mv^2 = mgy \Rightarrow v = \sqrt{2gy}$$

The action integral

$$A = \int \frac{ds}{v}$$
$$= \int \sqrt{1 + {y'}^2} dx / \sqrt{2gy}$$

must be minimized. The second form of Euler's equation

$$\frac{\partial f}{\partial x} - \frac{d}{dx} \left( f - y' \frac{\partial f}{\partial y'} \right) = 0$$

is useful since

$$f = \sqrt{1 + {y'}^2} / \sqrt{y} \neq f(x)$$
$$\frac{d}{dx} \left( f - {y'} \frac{\partial f}{\partial y'} \right) = 0$$

Hence,

or

 $f - \operatorname{in} Qy' \frac{\partial f}{\partial y'} = d$ After some work, the cycloid equation is obtained.

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# 61. (A)

For one-dimensional vertical motion the basic classical mechanics equation is

$$mv' = mg - bv$$
  
 $v' = g - \gamma v, \gamma \equiv b/m$ 

or

Integrate

$$\int_0^v \frac{dv}{g - \gamma v} = \int_0^t dt$$



by using the *u*-substitution  $u = g - \gamma v$  to get

$$\frac{1}{\gamma} \ln \frac{g - \gamma v}{g} = -t \text{ or } v = \frac{g}{\gamma} (1 - e^{-\gamma t})$$

Integrate once again to get

$$y = \frac{gt}{\gamma} - \frac{g}{\gamma^2} (1 - e^{-\gamma t})$$

Note that dy/dt = v.

## 62. (A)

Gauss' law for gravitation is

$$\nabla \cdot \mathbf{g} = -4\pi G\rho$$

in differential form or

$$\oint \mathbf{g} \cdot d\mathbf{a} = -4\pi G m_{in}$$

in integral form. For a single infinite sheet of mass density  $\sigma = m/A$  use a Gaussian pillbox as in the figure. Then

$$\oint g \cdot d\mathbf{a} = -4\pi G m_{in}$$
$$-gA - gA = -4\pi G \sigma A$$
$$2gA = 4\pi G \sigma A$$
$$g = 2\pi G \sigma$$





For two sheets, the field will be  $4\pi G\sigma$  in magnitude in regions I and III and zero in region II.

# 63. (D)

The first condition of equilibrium, that of translational equilibrium, says

$$\Sigma \mathbf{F} = \mathbf{0}.$$

Hence,

$$\Sigma F_x = R_x - T_x = R_x - \frac{T}{\sqrt{2}} = 0$$
  
$$\Sigma F_y = R_y - T_y - W$$
  
$$= R_y + \frac{T}{\sqrt{2}} - mg = 0$$



The second equilibrium condition involves rotation, specifically  $\Sigma \tau = 0$ . So

$$R(0) - w\frac{1}{2} + Tl\sin(180 - \theta) = 0 \Rightarrow T = \frac{W}{\sqrt{2}}$$
$$R_x = \frac{W}{2} = 500 \text{ N}$$

Thus,

$$R_y = 1000 - 500 = 500 \text{ N}$$

In polar coordinate rotation,

$$\mathbf{R} = (707 \text{ N}, 45^{\circ})$$

# 64. (D)

The moment of inertia

$$I = \int r^2 dm$$
$$= \frac{1}{2}mr^2$$

for a right circular cylinder. Then by conservation of energy

$$mgh = \frac{1}{2}mv^{2} + \frac{1}{2}I\omega^{2}, v = r\omega$$
$$= \frac{1}{2}mv^{2} + \frac{1}{4}mr^{2}\frac{v^{2}}{r^{2}}$$
$$mgh = \frac{3}{4}mv^{2}$$
$$v_{R} = 2\sqrt{\frac{gh}{3}}$$



For normal translational motion

$$v_T = \sqrt{2gh}$$

Hence,

$$\frac{v_R}{v_T} = \frac{2}{\sqrt{3}} \times \frac{1}{\sqrt{2}}$$
$$= \sqrt{\frac{2}{3}}$$

#### 65. (A)

Kepler's Second Law is that the area swept out per unit time by a radius vector from the sun to a planet is constant. By the usual triangle area rule,

$$dA = \frac{1}{2}r \, rd\theta$$

Hence,

$$\frac{dA}{dt} = \frac{1}{2}r^2\omega$$
$$= \frac{L}{2m}$$



Since the angular momentum is  $L = I\omega = mr^2\omega$ . Thus,

$$L = \text{constant}$$
$$mv_{ap}2a = mv_{pe}a$$
$$v_{pe}/v_{ap} = 2$$

66. (A)

The given mass density is

$$\lambda = M/(\pi R/2)$$

The differential force dF between differential mass element

$$dM = \lambda ds = \lambda R d\theta$$

and *m* has components

$$dF_x = \frac{Gm \, dM}{R^2} \cos \theta$$

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and

 $dF_y = Gm \ dM \sin \theta / R^2$ 

Thus,

$$=Gm\frac{\lambda}{R}=rac{2GmM}{\pi R^2}$$

 $F_x = \frac{Gm}{R_2} \lambda R \int_0^{\pi/2} \cos\theta \ d\theta$ 

**DETAILED EXPLANATIONS OF ANSWERS — PRACTICE EXAM 2** 



and

$$F_{\rm y} = \frac{Gm}{R^2} \lambda \int_0^{\pi/2} \sin\theta \ d\theta$$

$$=rac{2GmM}{\pi R^2}$$

Finally, 
$$F = \sqrt{F_x^2 + F_y^2} = \frac{2\sqrt{2}GmM^2}{\pi R^2}$$

and 
$$\mathbf{F} = \left(\frac{2\sqrt{2}GmM^2}{\pi R^2}, 45^\circ\right)$$
 in polar coordinates.

67. (E)

By Newton's universal law of gravitation and the definition of weight

$$\frac{GMm}{r^2} = mg$$

Thus,

$$g = \frac{GM}{r^2}$$



is the gravitational field.

Use a Taylor expansion to get

$$g = g_0 \left[ 1 - 2\frac{h}{R} + 3\left(\frac{h}{R}\right)^2 \right]$$

 $= \frac{GM}{R^2} \frac{1}{(1+x)^2}, x = \frac{h}{R}$ 

to 2nd order.

68. (C)

The hotter fluid loses heat energy to the cooler one:

Heat lost = heat gained

$$c_1 m_1 (T_1 - T) = c_2 m_2 (T - T_2)$$

Assuming equal specific heats  $c_1 = c_2 = c$  and with the masses related as given  $m_1 = 2m_2$ , we get

$$c2m_{2}(2T_{2} - T) = cm_{2}(T - T_{2})$$

$$4T_{2} - 2T - T + T_{2} = 0$$

$$5T_{2} = 3T$$

$$T = \frac{5}{3}T_{2}$$

$$= \frac{5}{3}(30^{\circ})C$$

$$= 50^{\circ}C$$

#### 69. (E)

The tide-raising acceleration the moon produces is a differential acceleration between points A and B

$$\begin{split} \Delta g &= g_A - g_B \\ &= \frac{Gm}{a^2} - \frac{Gm}{(a+r)^2} \\ &= \frac{Gm}{a^2} \left( 1 - \frac{1}{\left(1 + \frac{r}{a}\right)^2} \right) \\ &= \frac{Gm}{a^2} \left( 1 - \left(1 - 2\frac{r}{a}\right) \right) \end{split}$$



using a series expansion, thus

$$\Delta g = 2Gmr/a^3$$

Tidal acceleration is hence inversely proportional to the distance cubed.

#### 70. (B)

Measuring the distance between the two stars in a binary system and knowing the period of revolution allows one to determine the total mass. Kepler's general law (3rd law) is

$$T^2 = 4\pi^2 a^3 / (m_1 + m_2)$$

For the solar system  $m_1 = m_0$  and  $m_2 = m_{\text{planet}}$ . Also T = sidereal period. For a binary star system, the period assigned to the system applies to each star. Hence

$$m_T = (m_1 + m_2) = 4\pi^2 a^3 / T^2$$

If  $m_1 = m_2 = m$ , then

$$m = 2\pi^2 a^3/T^2$$

gives the mass of one star.

#### 71. (D)

The 3-D harmonic oscillator can be used to develop a basic nuclear shell model. Use the Schrödinger equation

$$H\Psi = E\Psi$$
$$\frac{-\mathbf{h}^2}{2m}\nabla^2\Psi + U\Psi = E\Psi$$

with potential energy  $U = \frac{1}{2} kr^2$ . Because

$$r^2 = x^2 + y^2 + z^2,$$

we get energy eigenvalues

$$E = \left(n_x + n_y + n_z + \frac{3}{2}\right) \mathbf{h}\boldsymbol{\omega}$$

The nucleon has g = 4 since we have  $p, n, and \uparrow and \downarrow$  spin. For

$$E=\frac{3}{2}\hbar\omega,$$

we get 4 states; for

$$E=\frac{5}{2}\hbar\omega,$$

12 states; and

$$E=\frac{7}{2}\hbar\omega,$$

24 states. Thus 4, 4 + 12 = 16, 4 + 12 + 24 = 40 are magic numbers.

$${}^{4}_{2}$$
He,  ${}^{16}_{8}$ O, and  ${}^{40}_{20}$ Ca

are very stable.

We are given information on the classic Young double-slit interference experiment. Plane waves of monochromatic light are incident on the two slits. The condition for constructive interference is

 $d\sin\theta = n\lambda$ 

 $\sin \theta = \frac{\lambda}{d} = \frac{y}{\sqrt{l^2 + y^2}} \approx \frac{y}{l}$ 

Hence,

72. (A)

and

is the location of the 1st maximum.

## 73. (B)

In the Mayer and Jensen nuclear shell model, the spin-orbit interaction

$$H_{\rm so} = -a \, \boldsymbol{l} \cdot {\bf s}$$

splits levels with the same *l* but different

 $\mathbf{j} = \mathbf{s} + \mathbf{l}$ 

For example, the  $P_{1/2}$  and  $P_{3/2}$  states both have l = 1 and s = 1/2, but the different *j* values (1/2 and 3/2) produce different degeneracies (2 and 4). For the nuclide 17/8O, the proton configuration is

$$(1s_{1/2})^2 (1p_{3/2})^4 (1p_{1/2})^2$$

and the neutron configuration is

$$(1s_{1/2})^2(1p_{3/2})^4(1p_{1/2})^2(1d_{5/2})^1$$

The ground state nucleus spin is thus j = 5/2 from the unpaired neutron.



 $y = \lambda \frac{l}{d} = \frac{(6000 \times 10^{-10})(15)}{(0.15 \times 10^{-3})}$ 

= 0.006 m

#### 74. (E)

For a spherical shell, the potential energy is

$$dU = -\frac{G}{r} \left(\frac{4}{3}\pi r^2 \rho\right) (4\pi r^2 dr \rho)$$
$$= -16\pi^2 G \rho^2 r^4 dr/3$$

The stars total gravitational potential energy is then

$$U = -16\pi^2 G \rho^2 / 3 \int_0^R r^4 dr$$
  
= -(16\pi^2 G \rho^2 / 15) R^5

Since the volume V and mass M are

$$V = \frac{4}{3}\pi R^3$$
 and  $M = \frac{4}{3}\pi R^3\rho$ 

we may rewrite this as being

$$U = -\frac{3}{5}GM^2/R = -\frac{3}{5}\left(\frac{4\pi}{3}\right)^{1/3} GM^2V^{-1/3}$$

Finally, the gravitational pressure is then

$$P = \frac{\partial U}{\partial V} = -\frac{1}{5} \left(\frac{4\pi}{3}\right)^{1/3} GM^2 V^{-4/3}$$
$$= -\left(\frac{4\pi}{375}\right)^{1/3} GM^2 \propto V^{-4/3}$$

75. (C)

There are six leptons:

$$e, v_e; \mu, v_{\mu}; \text{ and } \tau, v_{\tau}$$

The electron, *mu*, and *tau* increase in mass from 0.511 MeV/c<sup>2</sup> to 105.6 MeV/c<sup>2</sup> to 1784 MeV/c<sup>2</sup>; each has charge -e where  $e = 1.6 \times 10^{-19}$  C is the fundamental electronic charge. The neutrinos  $v_e$ ,  $v_\mu$ , and  $v_\tau$  are thought to have no mass and also have zero charge. In nuclear reactions, electron lepton number  $L_e$ , mu-lepton number  $L_\mu$ , and tau-lepton number  $L_\tau$  are conserved quantities. The antileptons have opposite charge and lepton number; e.g., e has q = -e and  $L_e = 1$  but e<sup>+</sup> has q = +e and  $L_e = -1$ .

#### 76. (C)

There are six quarks up, down, strange, charm, bottom, and top or u, d, s, c, b, and t. The up, charm, and top quarks have charges 2/3 e and masses 350, 1800, and  $\approx 20,000 \text{ MeV/c}^2$ . The down, strange, and bottom quarks have charges -1/3 e and masses 350, 550, and 4500 MeV/c<sup>2</sup>. The strange quark has strangeness S = -1, the charm quark has charm quantum number C = 1, the bottom quark bottomness B = -1, and the top quark has top quantum number T = 1. Hadrons like the proton and pion are built of quarks and anti-quarks.

## 77. (A)

The theory of radioactive decay proceeds as follows. Let P = probability, then

 $P(1 \text{ decay}) = \lambda dt, \quad \lambda = \text{decay constant}$ 

The differential number of particles decaying is

$$dN = -NP$$
$$= -\lambda N dt$$
$$\int_{N_0}^N \frac{dN}{N} = -\int_0^t \lambda dt \Rightarrow N = N_0 e^{-\lambda t}$$

The half-life is calculated as an expectation value

$$t = \langle t \rangle = \int_0^\infty t e^{-\lambda t} dt \bigg/ \int_0^\infty e^{-\lambda t} dt$$
$$= \lambda \left[ -\frac{t}{\gamma} e^{-\lambda t} - \frac{1}{\lambda^2} e^{-\lambda t} \right] \bigg|_0^\infty = \frac{1}{\lambda}$$

where  $e^{-\lambda \tau}$  plays the role of a probability function.

78. (D)

$$2L(n-1) = m\lambda_{\text{vac}} \to 2(0.1)(n-1) = 100(580 \times 10^{-9}) \to n = 1.00029$$

# 79. (A)

The Mössbauer effect, discovered by R. Mössbauer involves the resonance emission or absorption of nuclear radiation without recoil. For the usual emission case  $A^* \rightarrow A + \gamma$  the initial and final states are shown in the preceding figure. By conservation of momentum the recoil and photon momentum are equal



$$P_R = P_{\gamma}$$

Hence, the recoil energy is

$$E_R = P_R^2 / 2m_A$$
  
= (129 × 10<sup>3</sup>)<sup>2</sup>/2(191)(931.5 × 10<sup>6</sup>)  
= 0.0468 eV

# 80. (E)

One must use a fat lens formula to solve this kind of problem:

$$-\frac{n}{s} + \frac{n'}{s'} = \frac{n'-n}{R}$$

We are given n' = 1.7 and n = 1.2. Also, R = 5 cm and s = -15 cm. Thus, s' = 85 cm.

$$\frac{1.2}{15} + \frac{1.7}{s'} = \frac{0.5}{5} \Rightarrow s' = 85 \text{ cm}$$

Further,

$$s = -\infty \Rightarrow -\frac{n'}{f'} = \frac{n'-n}{R}$$
 or  $f' = 17$  cm

and

$$s' = \infty \Rightarrow \frac{-n}{f} - (n'-n)/R \text{ or } f = -12 \text{ cm}$$

The ray diagram is:



# 81. (B)

Consider transitions between two molecular states *i* and *j*. We are usually given that

$$N'_{i} = N_{i}B_{ij}U(\omega)$$
$$N'_{j} = N_{j}(A_{ji} + B_{ji}U(\omega)).$$

and

Hence,

$$\frac{N_i}{N_j} = (A_{ji} + B_{ji}U(\omega))/B_{ij}U(\omega)$$
$$E_j - E_i = \hbar\omega$$

But from the Boltzmann factor

$$\frac{N_j}{N_i} = e^{-\hbar\omega B}$$

Thus,

$$U(\omega) = A_{ii} / (B_{ij} e^{\hbar \omega \beta} - B_{ji})$$

 $B_{ij}e^{\hbar\omega\beta}U(\omega) = A_{ji} + B_{ji}U(\omega)$ 

where  $A_{ji}$ ,  $B_{ij}$ , and  $B_{ji}$  are the Einstein transition coefficients.

# 82. (A)

The B fields of a single circular current loop may be calculated from the Biot-Savart law

$$d\mathbf{B} = \frac{\mu_0}{4\pi} \frac{I\mathbf{d}\mathbf{l} \times \mathbf{s}}{s^2}$$

to be

$$\mathbf{B} = \frac{\mu_0 I r^2}{2(r^2 + z^2)^{3/2}} \mathbf{z}$$

along the *z*-axis

For Helmholtz coils, we have two such loops, each with N turns, and the field at z = R/2 is

$$B = 2N \frac{\mu_0 I R^2}{2 \left(R^2 + \frac{R^2}{4}\right)^{3/2}}$$
$$= N \mu_0 I R^2 / \left(\frac{5}{4}\right)^{3/2} R^3$$
$$= 8 \mu_0 I / (5^{3/2} R)$$

# 83. (C)

Consider Faraday's law-Maxwell's second equation

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

A vector identity is

$$abla imes (
abla imes \mathbf{E}) = 
abla (
abla \cdot \mathbf{E}) - 
abla^2 \mathbf{E}$$

Now  $\nabla \cdot \mathbf{E} = 0$  in vacuum

$$\Rightarrow -\frac{\partial}{\partial t} \nabla \times \mathbf{B} = -\nabla^2 \mathbf{E}$$

Now use Maxwell's fourth equation

$$\nabla \times \mathbf{B} = \frac{1}{c^2} \frac{\partial \mathbf{E}}{\partial t}$$

to get

$$\nabla^2 E = \frac{1}{c^2} \frac{\partial^2 E}{\partial t^2}$$

A plane wave solution is

 $\mathbf{E} = \mathbf{y} E_{oy} \cos \left(\omega t - kx + \alpha\right) + \mathbf{z} \cos \left(\omega t - kx + \beta\right) E_{oz}$ 

If

$$\alpha = \beta$$
 and  $E_{oy} = E_{oz} = E_0$  then

$$E_x^2 + E_y^2 = E_0^2$$

and we get circular polarization.

## 84. (D)

The potential is that of a real charge q at z' = h and an image charge q at z'' = -h

$$\phi = kq \left( \frac{1}{\sqrt{s^2 + (z - z')^2}} - \frac{1}{\sqrt{s^2 + (z + z')^2}} \right)$$

Note the boundary condition  $\phi(z = 0) = 0$  is satisfied. The electric field is then

$$E_z = -\frac{\partial \phi}{\partial z} = kq \left[ \frac{z - z'}{(s^2 + (z - z')^2)^{3/2}} - \frac{z + z'}{(s^2 + (z + z')^2)^{3/2}} \right]$$
$$E(z = 0) = -\frac{1}{2\pi\varepsilon_0} qh/(s^2 + h^2)^{3/2} \text{ using } k = \frac{1}{4\pi\varepsilon_0}$$

and where  $s^2 = x^2 + y^2$ 



# 85. (B)

The Fraunhofer diffraction intensity pattern is

$$I = I_0 \sin^2 \left(\frac{\pi d_w}{\lambda l} y\right) \left/ \left(\frac{\pi d_w}{\lambda l} y\right)^2 \right.$$
$$\int_{-\infty}^{\infty} I \, dy = 2 \int_0^{\infty} I \, dy$$
$$\frac{\pi d_w}{\lambda l} y = x$$

Let

$$\int_{-\infty}^{\infty} I dy = 2I_0 \int_0^{\infty} (\sin^2(x)/x^2) dx \frac{\lambda l}{\pi d_w}$$
$$= (2I_0 \lambda l/\pi d_w) (\pi/2)$$
$$= I_0 \lambda l/d_w$$

To do this problem, one needs to know

$$\int_0^\infty \frac{\sin x}{x} dx = \int_0^\infty \frac{\sin^2 x}{x^2} dx = \frac{\pi}{2}$$

The physics student should recognize  $\sin(x)/\pi x$  as one representation of the Dirac delta function  $\delta(x)$ .

# 86. (A)

The electric potential is

$$V = k \int \rho d^3 r / r$$
$$= k \hat{r} / r^2 \cdot \int r \rho \, d^3 r$$
$$= k \hat{r} / r^2 \cdot p$$

by definition of the dipole moment. Continuing

$$V = k\hat{r} \cdot p/r^2$$
$$= k p \cos \theta/r^2$$
$$= p = p\hat{z}$$

for

The electric field is

$$\mathbf{E} = -\nabla V$$
$$= -\hat{r}\frac{\partial V}{\partial r} - \hat{\theta}\frac{1}{r} - \hat{\phi}\frac{1}{r\sin\theta}\frac{\partial V}{\partial\phi}$$

Using spherical coordinates for  $\nabla$ 

$$= kp((2 \cos \theta/r^3)\hat{r} + (\sin \theta/r^3)\theta)$$
$$= (k/r^3)(3\hat{r} \cdot p\hat{r} - p)$$

and this is the usual dipole field.

## 87. (B)

For a particle on the surface of a sphere in spherical coordinates

$$x = r \sin \theta \cos \phi$$
$$y = r \sin \theta \sin \phi$$
$$z = r \cos \theta$$
$$x^{2} + y^{2} + z^{2} = r^{2} = R^{2}$$

and

The potential energy is

$$U = mg z = mg R \cos \theta$$
$$v^{2} = R^{2} \theta'^{2} + R^{2} \phi'^{2} \sin^{2} \theta$$
$$r' = \frac{d}{dt}(R) = 0$$

since

and

Hence, the Lagrangian function is

$$L = T - U = \frac{1}{2} m(R^2 \theta'^2 + R^2 \phi'^2 \sin^2 \theta) - mg R \cos \theta$$

# 88. (C)

The electric field of an infinite sheet of surface charge may be found from Gauss' law.

$$\nabla \cdot \mathbf{E} = \rho / \varepsilon_0$$
$$\oint \mathbf{E} \cdot d\mathbf{a} = q_{in} / \varepsilon_0$$

Recognizing that  $\mathbf{E} \cdot d\mathbf{a}$  is only non-zero on the left and right side faces of the Gaussian pill box, we get

$$EA + EA = \sigma A / \varepsilon_0$$



or

$$\mathbf{E} = \begin{cases} \sigma/2\varepsilon_0 \mathbf{x}, & x > 0 \\ \\ -\sigma/2\varepsilon_0 \mathbf{x}, & x < 0 \end{cases}$$

The electric potential must be such that

$$\mathbf{E} = -\nabla V = -\frac{\partial V}{\partial x}\mathbf{x}$$

thus

$$V = -\int \mathbf{E} \cdot d\mathbf{r} = -\int E \, dx$$
$$= -\sigma x/2\varepsilon_0$$

for x > 0 as desired.

# 89. (D)

The electric potential of a grounded conducting sphere in a uniform electric field is easily found and given as

$$\Phi(r,\theta) = -E_o r \left(1 - \left(\frac{a}{r}\right)^3\right) \cos\theta$$

The electric field has two components

$$E_{\theta} = -\frac{1}{r} \frac{\partial \Phi}{\partial \theta} = -E_0 \left( 1 - \left(\frac{a}{r}\right)^3 \right) \sin \theta$$

(As  $r \to a$ , clearly  $E_{\theta} \to 0$ .) and

$$E_r = -\frac{\partial \Phi}{\partial r} = E_0 \left[ 1 + 2\frac{a^3}{r^3} \right] \cos \theta$$

the surface charge is thus

$$\sigma_{\theta} = \varepsilon_0 E(r = a) = 3 \varepsilon_0 E_0 \cos \theta$$

## 90. (B)

The limiting angle of resolution of a circular aperture is

$$\theta_m = 1.22 \frac{\lambda}{\rho}$$

where  $\lambda$  is the light wavelength and *D* is the lens diameter. We are given  $\lambda = 6000$  Å and D = 3 mm. Hence,

$$\theta_m = 1.22 \frac{6000 \times 10^{-10}}{3 \times 10^{-3}} = 2.44 \times 10^{-4} \text{ rad}$$
By the definition of radian angle

then

$$\theta_m = d/l$$
  
 $l = d/\theta_m$   
 $= 2.0/2.44 \times 10^{-7}$   
 $= 8.2 \times 10^3 \text{ m}$   
 $= 8.2 \text{ km}$ 

Because of smog and haziness, one can rarely see this distance.

### 91. (B)

The deviation angle of a prism at nearly normal incidence is found from Snell's law and geometry

$$n_{i} \sin \theta_{i1} = n_{r} \sin \theta_{r1}$$
$$n_{i} \theta_{i1} = n_{r} \theta_{r1}$$
$$n_{r} \theta_{i2} = n_{i} \theta_{r2}$$

and

or

in the small angle approximation.



The total deviation angle has two parts:

$$\delta = (\theta_{i1} - \theta_{r1}) + (\theta_{r2} - \theta_{i2})$$
$$= \frac{n_r}{n_i}\theta_{r1} - \theta_{r1} + \frac{n_r}{n_i}\theta_{i2} - \theta_{i2}$$
$$= \left(\frac{n_r}{n_i} - 1\right)(\theta_{r1} + \theta_{i2})$$

Observe that

Thus,

$$\delta = \left(\frac{n_r}{n_i} - 1\right)\alpha$$

 $\alpha = \theta_{r1} + \theta_{i2}.$ 

for  $\theta_{i1}$  small at near normal incidence.

### 92. (E)

Fluorescence is an emission of electromagnetic radiation which takes place rapidly in  $t \sim 10^{-8}$  s. This is the natural lifetime of the molecular or atomic excited state. At STP, a molecule undergoes 100 or more collisions in this time and hence the fluorescence is usually quenched. Ultraviolet photons are commonly used to generate lower energy quanta via the Stokes transition. When there is an appreciable delay  $t >> 10^{-8}$  s, the emission is called phosphorescence. A quantum of light is scattered in the Compton effect off of electrons, but no such scattering occurs in fluorescence.

### 93. (E)

The Orbital Rule also known as Laporte rule states that in any electron transition in atoms,  $\Delta l = \pm 1$ .

### 94. (B)

$$z+1 = \frac{\lambda_o}{\lambda_e} = \sqrt{\frac{1+v/c}{1-v/c}}$$

$$v = 0.5 \ c \rightarrow z + 1 = \sqrt{[(1 + 0.8)/(1 - 0.8)]} = \sqrt{(1.8/0.2)} = 3 \rightarrow z = 3 - 1 = 2$$

#### 95. (A)

According to Hubble's Law, z is directly proportional to D, and v is directly proportional to z, hence v is directly proportional to D.

#### 96. (C)

According to the London theory, a weak magnetic field is able to penetrate only a small distance  $\lambda$  into the superconductor surface before decaying exponentially to zero.

### 97. (C)

Bragg's equation is  $d\sin\theta = m\lambda \rightarrow d\sin 30^\circ = (1)(0.25 \text{ nm}) \rightarrow d(1/2) = 0.25 \text{ nm} \rightarrow d = 0.50 \text{ nm}.$ 

### **DETAILED EXPLANATIONS OF ANSWERS — PRACTICE EXAM 2**

### 98. (E)

This problem is very similar to motion in a constant gravitational field. By the definition of electric field:

$$F = qE = -eE = ma$$
  

$$\rightarrow a = -eE/m$$
  

$$= -(1.6 \times 10^{-19})(100)/(9.1 \times 10^{-31})$$
  

$$= -1.76 \times 10^{13} \text{ m/s}^2$$

Then from kinematics

$$v_y = v_{oy} + at$$

$$0 = v_0 \sin \theta + at$$

$$t = -v_0 \sin \theta/a$$

$$T = -2 v_0 \sin \theta/a$$

$$= -2(4 \times 10^5)(\sin 30^\circ)/(-1.76 \times 10^{13})$$

$$= 2.3 \times 10^{-8} \text{ s}$$

$$= 23 \text{ ns}$$

since the time of flight is twice the time to reach the apex.

### 99. (E)

A retarder is an optical element used to change the polarization of an incident wave. The phase difference produced  $\Delta \phi$  is called the retardance. The optical devise is a full wave plate when  $\Delta \phi = 2\pi$  rad. The optical device is a half-wave plate when the retardance is  $\pi$  rad. Similarly, the quarter-wave plate causes a phase shift of  $\pi/2$  rad. A compensator impresses a controllable retardance on a wave; this retardance can be varied continuously.

#### 100. (A)

By Faraday's law or Maxwell's second equation:

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$
$$\oint \mathbf{E} \cdot d\mathbf{r} = -\frac{d\phi_B}{dt}$$

Thus, the induced voltage is

$$V = \frac{d}{dt} \int \mathbf{B} \times d\mathbf{a}$$
$$= \frac{d}{dt} B \times w$$
$$= Bwv$$

By Kirchhoff's law

Thus,

$$V = RI.$$
  

$$Bwv = RI$$
  

$$v = RI/Bw$$
  

$$= (2 \ \Omega)(0.5 \text{ A})/(1T)(0.5 \text{ m})$$
  

$$= 2 \text{ m/s}$$

# GRE Physics Exam 3

### **Answer Sheet: Practice Exam 3**

1.	A B C D E	34. A B C D E	68. (	AB	$\odot$ $\bigcirc$	E
2.	A B C D E	35. A B C D E	69. (	AB	© D	E
3.	A B C D E	36. A B C D E	70. (	AB	© D	E
4.	A B C D E	37. A B C D E	71. (	AB	© D	E
5.	A B C D E	38. A B C D E	72. (	AB	© D	E
6.	A B C D E	39. A B C D E	73. (	AB	© D	E
7.	A B C D E	40. A B C D E	74. (	AB	© D	E
8.	A B C D E	41. A B C D E	75. (	AB	© D	E
9.	A B C D E	42. A B C D E	76. (	AB	© D	E
10.	A B C D E	43. A B C D E	77. (	AB	© D	E
11.	A B C D E	44. A B C D E	78. (	AB	© D	E
12.	A B C D E	45. A B C D E	79. (	AB	© D	E
13.	A B C D E	46. A B C D E	80. (	AB	© D	E
14.	A B C D E	47. A B C D E	81. (	AB	© D	E
15.	A B C D E	48. A B C D E	82. (	AB	© D	E
16.	A B C D E	49. A B C D E	83. (	AB	© D	E
17.	(A) (B) (C) (D) (E)	50. A B C D E	84. (	AB	© D	E
18.	A B C D E	51. A B C D E	85. (	AB	© D	E
19.	A B C D E	52. A B C D E	86. (	AB	© D	E
20.	A B C D E	53. A B C D E	87. (	AB	© D	E
21.	A B C D E	54. A B C D E	88. (	AB	© D	E
22.	A B C D E	55. A B C D E	89. (	AB	© D	E
23.	A B C D E	56. A B C D E	90. (	AB	© D	E
24.	A B C D E	57. A B C D E	91. (	AB	© D	E
25.	A B C D E	58. A B C D E	92. (	AB	© D	E
26.	A B C D E	59. A B C D E	93. (	AB	© D	E
27.	(A) (B) (C) (D) (E)	60. A B C D E	94. (	AB	© D	E
28.	A B C D E	61. A B C D E	95. (	AB	© D	E
29.	A B C D E	62. A B C D E	96. (	AB	© D	E
30.	(A) (B) (C) (D) (E)	63. A B C D E	97. (	AB	© D	E
31.	A B C D E	64. A B C D E	98. (	AB	© D	E
32.	A B C D E	65. A B C D E	99. (	AB	© D	E
33.	(A) (B) (C) (D) (E)	66. A B C D E	100. (	AB	© D	E
		67. A B C D E				

### **Practice Exam 3**

Time: 170 Minutes 100 Questions

Directions: Each of the questions or incomplete statements below is followed by five answer choices or completions. Choose the best answer to each question.

- 1. Let the point of application of a force  $\mathbf{F} = (5, 3, -2) N$  be at position  $\mathbf{r} = (-2, 1, -3)m$ . Calculate the torque  $\tau$  due to this force.
  - (A) 7x 19y 11zN m
  - (B) 11x + 11y + 1zN m
  - (C) -10x + 3y + 6zN m
  - (D) -11x 11y 1zN m
  - (E)  $-7\mathbf{x} + 19\mathbf{y} + 11\mathbf{z}N m$
- 2. Julie, a physics student, stands on top of a 50 m cliff. She releases one stone with a downward speed of 1.0 m/s. With what speed must she project a second stone 0.5 s later at a 35° angle if both stones are to hit the bottom at the same time?
  - (A) 3.28 m/s(D) 1.51 m/s(B) 2.00 m/s(E) 8.60 m/s
  - (C) 4.00 m/s
- 3. A cylindrical bucket of cross-sectional area A, initial mass  $m_0$ , and initial speed  $v_0$  moves through space, picks up space debris of density  $\rho$  (uniformly distributed over A), and slows down. Use Newtonian mechanics to find the velocity of the bucket as a function of time t.

(A) 
$$v = v_0$$

(B) 
$$v = v_0 e^{-\rho A v_0 t/m_0}$$

(C)  $v = v_0 / \sqrt{1 + 2\rho A v_0 t / m_0}$ 

(D) 
$$v = v_0 e^{-2\rho A v_0 t/m_0}$$

(E)  $v = v_0 / \sqrt{1 + \rho A v_0 t / m_0}$ 



- 4. Consider two reference frames K and K', where K' moves relative to K with speed  $v = c(1 \delta)$  along the xx' axes. Consider a particle moving with speed  $u'_x = c(1 \delta)$  in K'. Find this particle's speed in the lab frame K to order  $\delta^2$ . Note  $\delta << 1$ .
  - (A)  $2c(1 \delta)$ (B)  $c(1 - \delta^2/2)$ (C)  $c(1 - \delta)$
  - (D)  $c(1 \delta/2)$
  - (E)  $c(1 \delta^2)$



- 5. Consider two equal masses  $m_1 = m_2 = m$  that are attracted gravitationally. Suppose that the masses are initially a distance  $\mathbf{r}_0$  apart and that one mass is given a velocity  $\mathbf{v}_0$  perpendicular to  $\mathbf{r}_0$ . For what values of  $v_0$  will the masses be bound in elliptical motion?
  - (A)  $v_0 > 2\sqrt{Gm/r_0}$ (B)  $v_0 > \sqrt{Gm/r_0}$ (C)  $v_0 = \sqrt{Gm/r_0}$ (D)  $v_0 < 2\sqrt{Gm/r_0}$ (E)  $v_0 < \sqrt{Gm/r_0}$
- 6. According to special relativity, a clock at the North Pole must measure a longer time interval than a clock at the equator of the Earth. Suppose that the polar clock reads T = 100 years. By how many seconds does the clock at the equator differ? ( $r_E = 6.4 \times 10^6$  m)

(A)	$1.90 imes10^{-3}~ m s$	(D)	$7.58  imes 10^{-3}  \mathrm{s}$
(B)	$3.79  imes 10^{-3}  ext{ s}$	(E)	$9.48 \times 10^{-3}  \mathrm{s}$
(C)	$5.70  imes 10^{-3}  ext{ s}$		

7. Two balls are thrown vertically upward at the same time. Suppose that the balls have initial velocities  $v_1 = 20$  m/s and  $v_2 = 24$  m/s, respectively. Find the distance between the two balls when ball one is at its maximum height.

(A)	20.40 m	(D)	8.14 m
(B)	28.56 m	(E)	14.28 m
(C)	16.28 m		

8. Consider two masses  $m_1$  and  $m_2$  moving on a frictionless surface as shown. Find the distance x of maximum compression of the spring.



- 9. Imagine a type of cylindrical shell as pictured. Initially a solid cylinder of mass M and radius R is rotating with angular velocity  $\omega_0$ ; then the shell, of mass *m*, collapses onto the cylinder. What is the final angular velocity of the system?
  - (A)  $M\omega_0/(M+m)$ (B)  $m\omega_0/(M+m)$ (C)  $M\omega_0/(M+2m)$ (D)  $m\omega_0/M$

  - (E)  $M\omega_0/m$
- 10. A mass m is subject to the gravitational force and attached by a string to a second mass m'. Supposing that the pulley has a finite moment of inertia *I*, find the acceleration of mass m. Assume friction is negligible.
  - (A)  $mg/(I/R^2 + m + m')$
  - (B) g
  - (C) mg/m'
  - (D) mg/(m + m')
  - (E)  $(m m')g/(I/R^2)$
- 11. A skier leaves a ski jump ramp at an angle of 14° with an initial speed of 11 m/s. Later he lands down the slope a distance *l* from where he started the jump. If the slope is inclined at  $45^{\circ}$ , then find *l*.
  - (A) 20.5 m
  - (B) 41.1 m
  - (C) 82.0 m
  - (D) 61.5 m
  - (E) 10.2 m
- 12. Consider the problem of a spherical pendulum of length *l* and mass m subject to gravity as shown. Derive the Lagrangian for this problem in spherical coordinates. The pendulum is free to move both in the x and y directions and would hang (at rest) parallel to the z-axis.
  - (A)  $1/2 m l^2(\theta'^2 + \phi'^2 \sin^2\theta) mg l(1 + \cos \theta)$
  - (B)  $1/2 m l^2 \theta'^2 mg l(1 + \cos \theta)$
  - (C)  $1/2 m l^2(\theta'^2 + \phi'^2 \sin^2\theta) mg l(1 \cos\theta)$
  - (D)  $1/2 m l^2 \phi'^2 \sin^2 \theta mg l(1 + \cos \theta)$
  - (E)  $1/2 m l^2 (\theta'^2 + \phi'^2) mg l \cos \theta$





45°

- 13. Picture a particle of mass *m* that is constrained to move on the surface of a cylinder  $x^2 + y^2 = R^2$ and subject to a force  $\mathbf{F} = -k\mathbf{r}$ . Find the Hamiltonian in the appropriate cylindrical coordinate system.
  - (A)  $H = 1/2 m z'^2 + 1/2 k z^2$ (B)  $H = 1/2 m (R^2 \theta'^2 + z'^2) + 1/2 k (R^2 + z^2)$ (C)  $H = 1/2 m R^2 (\theta'^2 + \phi'^2 \sin^2 \theta) + 1/2 k R^2$ (D)  $H = 1/2 m R^2 \phi'^2 \sin^2 \theta + 1/2 k R^2$ (E)  $H = 1/2 m R^2 \theta'^2 + 1/2 k R^2$
- 14. Suppose that the disk of radius *R* shown is in equilibrium. Note that the incline has a coefficient of static friction  $\mu_s = \mu$ . Find the tension in the cord.
  - (A)  $T = mg \sin \theta$ (B)  $T = mg \cos \theta$ (C)  $T = mg \sin \theta / (1 + \cos \theta)$ (D)  $T = mg \cos \theta / (1 + \sin \theta)$ (E)  $T = mg(1 + \cos \theta) / \sin \theta$



- 15. Imagine that an object of mass m (m = 2 kg) has position vector  $\mathbf{r} = (3t + 5t^3)\mathbf{x}$ . Calculate the work done on the particle over the time interval from 0 to 1 s.
  - (A) 78 J
     (D) 157 J

     (B) 235 J
     (E) 315 J
  - (C) 393 J
- 16. Derive Kepler's 3rd law from the assumption that the Earth moves in a circular orbit about the sun of radius  $r = 1.50 \times 10^{11}$  m. Use this information to calculate the mass of the sun.

(A)	$2.5 \times 10^{28} \mathrm{kg}$	(D)	$1.0 \times 10^{30} \mathrm{kg}$
(B)	$5.0 imes 10^{28}\mathrm{kg}$	(E)	$2.0  imes 10^{30}$ kg
(C)	$3.5 imes10^{29}\mathrm{kg}$		

17. A block of mass *m* moving at speed *v* collides with a spring of restoring force  $F = -k_1x - k_2x^3$  on a frictionless surface. Find the maximum compression of the spring.

(A) 
$$\sqrt{k_1/k_2}$$
  
(B)  $k_1/k_2 \left(\sqrt{1+mv^2k_2/k_1^2}-1\right)$   
(C)  $\sqrt{k_1/k_2} \left(\sqrt{1+mv^2k_2/k_1^2}-1\right)^{1/2}$   
(D)  $k_1/k_2 \left(\sqrt{1+2mv^2k_2/k_1^2}-1\right)$   
(E)  $\sqrt{k_1/k_2} \left(\sqrt{1+(2mv^2k_2/k_1^2)}-1\right)^{1/2}$ 



- 18. A dumbbell type molecule is modelled as two spheres of radius *r* and mass *m* separated by distance 2*l* as pictured. Suppose that the angular frequency  $\omega = \omega_y \mathbf{y} + \omega_z \mathbf{z}$ . Find the angular momentum of the molecule.
  - (A)  $L_x = 0, L_y = 4/5 mr^2 \omega_y, L_z = 4/5 mr^2 \omega_z$
  - (B)  $L_x = 0, L_y = 2 m^2 \omega_y, L_z = 4/5 m r^2 \omega_z$
  - (C)  $L_x = 2/5 mr^2 \omega_z, L_y = 4/5 mr^2 \omega_y, L_z = 0$
  - (D)  $L_x = 0, L_y = (4/5 mr^2 + 2ml^2)\omega_y, L_z = 4/5 mr^2\omega_z$
  - (E)  $L_x = 0, L_y = (2/5 \ mr^2 + 2ml^2)\omega_y, L_z = 2/5 \ mr^2\omega_z$
- 19. The executive toy in its simplest form is made of two identical masses hanging from a pivoting rod as shown. If each mass is m and the lengths are L for each arm and l for the pivot, derive the condition under which the toy is stable.
  - (A)  $L \cos \theta > l$ (B)  $\theta < 45^{\circ}$ (C)  $L \cos \theta > l$ (D)  $\theta < 20^{\circ}$
  - (D)  $\theta < 30^{\circ}$
  - (E) L > l





20. Consider the flow of an incompressible fluid through a horizontal pipe as shown. Determine the pressure difference  $p_2 - p_1$  in terms of the cross-sectional areas and the flow velocity  $v_1$  at the left.

σ

- (A)  $1/2\rho v_1^2 (1 A_1^2/A_2^2)$
- (B)  $1/2\rho v_1^2$
- (C)  $1/2\rho v_1^2 (1 + A_1^2/A_2^2)$
- (D)  $1/2\rho v_1^2 A_1^2/A_2^2$
- (E)  $1/2\rho v_1^2 (1 A_2^2/A_1^2)$



- 21. Consider the problem of four infinite charge planes situated as shown. Find the electric field in the region |x| < a/2. (See figure.)
  - (A)  $\sigma/2\varepsilon_0 \mathbf{x}$ (B)  $-\sigma/2\varepsilon_0 \mathbf{x}$ (C)  $2\sigma/\varepsilon_0 \mathbf{x}$ (D)  $-2\sigma/\varepsilon_0 \mathbf{x}$ (E) 0



22. A wedge capacitor has potential  $\Phi$  ( $\phi = 0$ ) = 0 and  $\Phi$  ( $\phi = \alpha$ ) =  $V_0$ . Assume that the two plates are infinite. Find the electric field between the capacitor plates.

(A) 
$$\mathbf{E} = -\frac{V_0\phi}{\alpha\rho}\hat{\phi}$$
  
(B)  $\mathbf{E} = -\frac{V_0\phi}{\alpha}\hat{\phi}$   
(C)  $\mathbf{E} = -\frac{V_0}{\alpha}\hat{\phi}$   
(D)  $\mathbf{E} = -\frac{V_0}{\alpha\rho}\hat{\phi}$   
(E)  $\mathbf{E} = -\frac{V_0}{\rho}\hat{\phi}$ 



- 23. A neutral hydrogen atom may be thought of as a proton orbited by an electron. Supposing that the electron's charge density is  $\rho(\mathbf{r}) = e(\delta(\mathbf{r}) (\alpha^3/\pi)e^{-2\alpha r})$ , calculate the radial electric field  $E_r$ .
  - (A)  $E_r = (e/4\pi\varepsilon_0)r^2$
  - (B)  $E_r = (e/4\pi\varepsilon_0)r^2e^{-2\alpha r}$
  - (C)  $E_r = (e/4\pi\varepsilon_0)(2\alpha/r + 1/r^2)^2 e^{-2\alpha r}$
  - (D)  $E_r = (e/4\pi\varepsilon_0)(2\alpha^2 + 2\alpha/r)e^{-2\alpha r}$
  - (E)  $E_r = (e/4\pi\varepsilon_0)r^2(2\alpha^2 + 2\alpha/r + 1/r^2)e^{-2\alpha r}$
- 24. A charged pith ball of mass 2 g is suspended on a massless string in an electric field  $\mathbf{E} = (3\mathbf{x} + 4\mathbf{y}) \times 10^5 \text{ N/C}$ . If the ball is in equilibrium at  $\theta = 57^{\circ}$ , then find the tension in the string.
  - (A) 0.0500 N(B) 0.0250 N
  - (C) 0.0125 N
  - (D) 0.0063 N
  - (E) 0.0032 N



- 25. A rod 0.20 m long has a total charge  $q = -75 \ \mu C$ . Find the electric field along the axis of the rod 0.10 m from one end.
  - (A)  $-5.50 \times 10^7$  N/C x (B)  $-2.25 \times 10^7$  N/C x
  - $(C) \quad 0 \text{ N/C}$
  - (D)  $2.25 \times 10^7$  N/C x
  - (E)  $5.50 \times 10^7$  N/C x



- 26. A capacitor is constructed from two rectangular metal plates of area A separated by a distance d. Suppose that one-half of the space between the plates is filled by a dielectric  $\kappa_1$  and the other half by a dielectric  $\kappa_2$ . Find the capacitance in terms of the free space capacitance  $C_0$ , where  $C_0 = \varepsilon_0 A/d$ .
- 27. Find the electric field of a uniformly charged disk of radius a situated in the yz plane at point P along the x-axis. Let the surface charge density of the disk be  $\sigma$ . (See figure.)
  - (A)  $\sigma/2\varepsilon_0\left(1-x/\sqrt{x^2+a^2}\right)\mathbf{x}$
  - (B)  $\sigma/2\varepsilon_0 \mathbf{x}$

(C) 
$$\sigma/2\varepsilon_0 x/\sqrt{x^2+a^2x}$$

(D)  $\sigma a^2/4\varepsilon_0 \mathbf{x}$ 

(E) 
$$\sigma/2\varepsilon_0\left(\sqrt{x^2+a^2}-x\right)\mathbf{x}$$





28. A resistor is made from a hollow cylinder of length l, inner radius a, and outer radius b. The region a < r < b is filled with material of resistivity  $\rho$ . Find the resistance R of this component.



(D) 0.254 A

(E) 0.654 A

- 29. For the circuit shown, find the amount of current that passes through the 5  $\Omega$  resistor.
  - (A) 0.873 A
  - (B) 0.127 A
  - (C) 0.346 A



- 30. The Hall effect relates to
  - (A) the behavior of waves in regularly spaced lattices known as constructive interference.
  - (B) the generation of a voltage when a current carrying conductor is placed in a magnetic field.
  - (C) a more sophisticated statement of the consequences of Coulomb's inverse square law.
  - (D) magnetohydrodynamic waves (specifically).
  - (E) the determination of a substance's electric susceptibility.

- 31. Find the magnetic field of a circular ring of radius rsituated in the xy plane for an arbitrary point along the z axis. The ring carries a current I.
  - (A)  $\mu_0 I/2r$
  - (B)  $\mu_0 I r^2 / (r^2 + z^2)^{3/2}$
  - (C)  $\mu_0 I/4r$
  - (D)  $\mu_0 I r^2 / 2 (r^2 + z^2)^{3/2}$
  - (E)  $\mu_0 Ir^2/4(r^2+z^2)^{3/2}$



32. The differential statement of Ampere's law is incomplete without the displacement current. In fact, it is implied for the vector current density that  $\nabla \cdot \mathbf{j} = 0$ . Use Maxwell's equations to find the correct vector whose divergence is zero.

(A)	$\mathbf{j} + \partial \mathbf{E} / \partial t$	(D)	$\mu_0 arepsilon_0 \mathbf{j}$
(B)	$\mu_0 \mathbf{j}$	(E)	$\mathbf{j} + 1/\varepsilon_0 \partial \mathbf{E}/\partial t$
(C)	$\mathbf{j} + 1/\mu_0 \boldsymbol{\varepsilon}_0 \partial \mathbf{E}/\partial t$		

- 33. Ampere's law is different in a vacuum than in the presence of matter. In the presence of matter, consider that the total current density consists of a free or vacuum current density and a bound or magnetization current density. Hence, derive what vector must have a curl of  $\mu_0 \mathbf{j}_{\text{free}}$ .
  - (D)  $\mathbf{B} \mu_0 \mathbf{M}$ (E)  $\mu_0 \mathbf{B} \mathbf{M}$ (A) **H** (B)  $\mathbf{B} + \mu_0 \mathbf{M}$ (C)  $\mu_0 \mathbf{B} + \mathbf{M}$
- 34. In one version of the Millikan oil drop experiment, oil droplets of radius R are allowed to achieve terminal speed first with a downward electric field  $(v_{\perp})$  and then with an upward electric field  $(v_{\perp})$ . Let  $\eta$  be the viscosity of oil in air and E the electric field. Find the electronic charge e.



- (A)  $\frac{3\pi R\eta}{F}(v_--v_+)$ (D)  $\frac{6\pi R\eta}{E}(v_-+v_+)$ (B)  $\frac{6\pi R\eta}{E}(v_--v_+)$ (E)  $\frac{mg}{F}$ (C)  $\frac{3\pi R\eta}{F}(v_-+v_+)$
- 35. Gauss's law may be used to derive Coulomb's law. Let  $k_{\rm E}$  be the constant in Coulomb's law. Furthermore, Ampere's law may be used to derive the force per unit length between two currents. Let  $k_B$  be the constant in this magnetic Coulomb law. What is the ratio  $k_R/k_F$ ?
  - (A) c (D)  $\mu_0 \varepsilon_0$ (E)  $c^2$
  - (B)  $2\mu_0\varepsilon_0$
  - (C) 2c

- 36. Consider that a sliding conductive bar closes the circuit shown below and moves to the right with a speed v = 4 m/s. If l = 1.5 m,  $R = 12 \Omega$ , and B = 5 T, then find the magnitude of the induced power and the direction of the induced current.
  - (A) 75 W, counterclockwise
  - (B) 75 W, clockwise
  - (C) 2.5 W, counterclockwise
  - (D) 2.5 W, clockwise
  - (E) 0 W, there is no current flow



- 37. A lightly damped RLC circuit has  $R = 10 \Omega$ ,  $L = 10^{-3}$ H, and  $C = 1 \mu$ F. For this lightly damped circuit, find the ratio of the charge on the capacitor at  $t = 2 \times 10^{-4}$  s to the maximum charge.
  - (A) 0.732 (D) 0.400 (B) 0.600 (E) 0.366
  - (C) 0.549
- 38. As is well known, Maxwell's equations imply the existence of electromagnetic waves. Determine the appropriate wave equation for a magnetic field  $\mathbf{B} = B_z(x, y, z)\mathbf{z}$ .
  - (A)  $\partial^2 \mathbf{B} / \partial x^2 = \partial^2 B / \partial t^2$
  - (B)  $\partial^2 \mathbf{B}_z / \partial z^2 = \partial^2 B_z / \partial t^2$
  - (C)  $(\partial^2/\partial x^2 + \partial^2/\partial y^2 + \partial^2/\partial z^2)B_z = \mu_0 \varepsilon_0 (\partial^2 B_z/\partial t^2)$
  - (D)  $\partial^2 B_z / \partial y^2 = \partial^2 B_z / \partial t^2$
  - (E)  $(\partial^2/\partial x^2 + \partial^2/\partial y^2 + \partial^2/\partial z^2)B_z = \partial^2 E_v/\partial t^2$
- 39. An infinite cylinder with charge density  $\lambda$  and current flow I is at rest in reference frame K. Find the speed of reference frame K' where the electric field is zero, i.e., in that frame one observes a pure magnetic field.
  - (A)  $v = \lambda / \varepsilon_0 \mu_0 I$
  - (B) v = c/2
  - (C)  $v = \lambda \varepsilon_0 \mu_0 / I$
  - (D) v = c/4
  - (E) This is impossible.



40. The  $\mu$ -meson has half-life  $\tau_{1/2} = 1.5 \ \mu$ s. These particles are produced by the collision of cosmic rays with gas nuclei 60 km above the surface of the Earth. Find the speed parameter  $\beta$  with which the muons move if only 1/8 of them reach sea level without decaying.

(A)	$\beta = 0.975$	(D)	$\beta = 0.99975$
(B)	$\beta = 0.9975$	(E)	$\beta = 0.98$

- 41. Cerenkov radiation is observed from a beam of 700 MeV electrons travelling through air of index of refraction n = 1.00029. Find the half angle of the light cone of radiation.
  - (A) 1.38°
    (B) 2.76°
  - (C) 4.14°

- (D) 7.52° (E) 15.0°
- 42. A nuclear reaction occurs with 1.808 MeV deuterons incident on a target of deuterium. Protons are observed at  $\theta = 90^{\circ}$  with 3.467 MeV kinetic energy. Given that  $m_p = 938.791$  MeV and  $m_d = 1876.140$  MeV, find the
  - (A) 2814.931 MeV
    (B) 2809.462 MeV
    (C) 2816.373 MeV

mass of the triton.

- (D) 2814.210 MeV
- (E) 2814.840 MeV



43. Consider a symmetric elastic collision between a particle of mass *m* and kinetic energy *T* and a particle of the same mass at rest. Relativistically, what is the cosine of the angle between the two particles after the collision? Let  $E_0$  = rest mass.

			<u> </u>
		E <sub>0</sub>	E <sub>0</sub>
(A) (B)	1.00 T/(T + 5E)		(D) $T/(T + 3E_0)$ (E) $T/(T + 2E)$
(D) (C)	$T/(T + 3E_0)$ $T/(T + 4E_0)$		(E) $1/(1 + 2E_0)$

44. A particle of mass *m* moves in a delta function potential  $V(x) = -g\delta(x)$ . Find the energy eigenvalue of the particle.

(A)	$-mg^{2}/8h^{2}$	(D)	$-mg^2/2h^2$
(B)	$-mg^{2}/6h^{2}$	(E)	$-mg^2/h^2$
(C)	$-mg^{2}/4h^{2}$		

45. Consider a particle in a potential  $V(x) = -V_0$  for  $|x| \le l/2$  and V(x) = 0 otherwise. Determine the equation that must be solved to find the energy eigenvalues. Let

$$\theta = \sqrt{ml^2(V_0 - E)/2\hbar^2} \text{ and } \theta_0 = \sqrt{ml^2V_0/2\hbar^2}$$
(A)  $\cot \theta = \sqrt{\theta_0^2/\theta^2 - 1}$ 
(B)  $\sin \theta = \sqrt{\theta_0^2/\theta^2 - 1}$ 
(C)  $\tan \theta = \sqrt{\theta_0/\theta - 1}$ 
(D)  $\sin \theta = \sqrt{\theta_0^2/\theta^2 - 1}$ 
(E)  $\tan \theta = \sqrt{\theta_0^2/\theta^2 - 1}$ 

- 46. A potential well consists of a harmonic oscillator potential  $V(x) = 1/2 m\omega^2 x^2$  for x > 0 and an infinite barrier for x < 0. In the WKB approximation, what equation must be solved to find the energy eigenvalues? Let x = a specify the classical turning point.
  - (A)  $\int_0^a p \, dx/h = \pi/2, \, 3\pi/2, \, 5\pi/2, \ldots$  (D)  $\int_0^a p \, dx/h = \pi/4, \, 3\pi/4, \, 5\pi/4, \ldots$

(B) 
$$\int_0^a p \, dx/h = 3\pi/4, \, 7\pi/4, \, 11\pi/4, \dots$$
 (E)  $\int_0^a p \, dx/h = \pi/4, \, \pi/2, \, 3\pi/4, \dots$ 

- (C)  $\int_{0}^{a} p \, dx/h = \pi, 2\pi, 3\pi, \ldots$
- 47. In the Zeeman effect, it is found that a sample of Na placed in a magnetic field B has its spectral D line split into three lines. Find the amount of the shift  $\delta \omega$ , in cgs units, where  $\omega$  is the angular frequency of the spectral line.
  - (D)  $\delta \omega = \pm eB/4m_e c$ (E)  $\delta \omega = \pm eB/2m_e c$ (A)  $\delta \omega = \pm eB/8 m_{e}c$ (B)  $\delta \omega = \pm eB/3 m_e c$ (C)  $\delta \omega = \pm eB/m_e c$
- 48. Each hydrogenic spectral series has an upper and a lower limit. Which of the following spectral series has an upper limit  $\lambda = 18,760$  Å?
  - (A) Balmer series (D) Pfund series
  - (B) Lyman series (E) Paschen series
  - (C) Brackett series
- 49. Two metal rods of identical geometrical dimensions have thermal conductivities  $k_1$  and  $k_2$ , respectively. These two rods are connected laterally a composite rod. The thermal conductivity of the composite rod must be
  - (A)  $(k_1 + k_2)$ (D)  $2k_1k_2/(k_1 + k_2)$ (E)  $\sqrt{(k_1k_2)}$ (B)  $\frac{1}{2}(k_1 + k_2)$
  - (C)  $k_1 k_2 / (k_1 + k_2)$

- 50. Which of the following statements is NOT true about the Franck-Hertz experiment? Specifically, consider the case where the Franck-Hertz tube is filled with Hg vapor at low pressure.
  - (A) An electron loses most of its kinetic energy in an elastic collision with an atom.
  - (B) Electrons raise Hg atoms to the first excited state.
  - (C) The kinetic energy of the electrons may be changed simply by altering the voltage on the grid.
  - (D) The collected current peaks at multiples of 4.9 volts.
  - Any monoatomic gas at low pressure may be used. (E)
- 51. In the photoelectric effect, electromagnetic radiation is incident upon the surface of a metal. Which of the following is NOT a true statement about the photoelectric effect? Let v = threshold frequency and  $v_0$  = frequency.
  - (A) There is no photocurrent unless  $v > v_0$ .
  - (B)  $v_0$  is characteristic of the cathode material.
  - (C) Above v, the flux of electrons per second increases as the intensity of incident light.
  - (D) The stopping potential  $V_0$  is proportional to  $v^2$ .
  - (E) The stopping potential is independent of the intensity.

- 52. A beam of tritons is incident on an Au foil 1  $\mu$ m thick and scattered through an angle of 37° solely through the Coulomb interaction. Find the differential scattering cross section in *b*/sr.
  - (A) 5.15b
  - (B) 6.23*b*
  - (C) 7.34*b*
  - (D) 8.95*b*
  - (E) 10.30*b*



53. Consider a cubical container of volume V containing a photon gas in equilibrium. Calculate the differential number of allowable normal modes of frequency  $\omega$ .

(D)  $V\omega^2 d\omega/\pi^2 c^3$ (E)  $8 V\omega^2 d\omega/\pi^2 c^3$ 

- (A)  $V\omega^2 d\omega/2\pi^2 c^3$ (B)  $2V\omega^2 d\omega/\pi^2 c^3$
- (C)  $4V\omega^2 d\omega/\pi^2 c^3$
- 54. The black body energy density  $u(\omega)$  has a maximum as a function of  $\omega$ . Which is the equation for this maximum, also known as Wien's displacement law? Let *k* be the Boltzmann constant.
- 55. Photoelectrons are found to be ejected from a metal surface when the wavelength of incident light is below 230 nm. If the wavelength of incident photons is 150 nm, then what must be the stopping potential  $V_0$  to stop the photoelectrons?

A)	8.27 V	(D)	2.32 V
B)	5.39 V	(E)	1.56 V
C)	2.88 V		

56. X-rays of wavelength  $1.50 \times 10^{-10}$  m are scattered by a metal through an angle of 90°. What is the kinetic energy of the recoil electrons?

(A)	132 eV	(D)	736 eV
(B)	264 eV	(E)	822 eV
(C)	368 eV		

- 57. A particle is bound in a potential well given by  $V(x) = \infty$  for  $x \le 0$  and V(x) = cx for x > 0. Estimate the ground state energy of this system.
  - (A)  $(\frac{hc}{2\sqrt{2m}})^{2/3}$
  - (B)  $\frac{\hbar^2}{2m} + c$
  - (C)  $(h^2 c^2/m)^{1/3}$
  - (D)  $(\frac{\hbar c}{\sqrt{2m}})^{2/3} + (\hbar^2 c^2/2m)^{1/3}$

(E) 
$$(\frac{\hbar c}{2\sqrt{2m}})^{2/3} + (\frac{\hbar^2 c^2}{2m})^{1/3}$$



- 58. A ball with a mass of 2  $\times$  10<sup>-3</sup> kg and a kinetic energy of  $1 \times 10^{-3}$  J is incident upon a potential barrier 0.20 m in height and 0.02 m in width. What is the probability that the ball will quantum-mechanically tunnel through the potential barrier and appear on the other side?
  - (A)  $10^{-5.6 \times 10^{29}}$
  - (B)  $10^{-1.3 \times 10^{30}}$
  - (C)  $10^{-4}$
  - (D) 10<sup>-6</sup>
  - (E)  $10^{-3.2 \times 10^{28}}$



- 59. The rms-speed and the internal energy of an ideal gas are  $v_{\rm rms}$  and U, respectively. If the absolute temperature of the gas were decreased to  $\frac{1}{4}$  the original value, the new values for rms-speed and internal energy would be
  - (D)  $\frac{1}{2} v_{\rm rms} \text{ and } \frac{1}{2} U$ (E)  $\frac{1}{8} v_{\rm rms} \text{ and } \frac{1}{4} U$
- 60. What is the transmission probability due to the tunnel effect of a 1 eV electron incident on a barrier 0.5 nm wide and 5 eV high.
  - (D)  $4.5 \times 10^{-5}$ (A)  $1.4 \times 10^{-4}$ (E)  $3.2 \times 10^{-30}$ (B)  $3.6 \times 10^{-14}$ (C)  $8.6 \times 10^{-20}$
- 61. A uniform wire of resistance R is bent into the form of a circle and the two ends are connected together. P and Q are two points for which the radii are an angle of  $\theta$  radians apart as shown in the diagram.
  - (A)  $R\theta/(2\pi)$ (B)  $R [1 - \theta/(2\pi)]$ (C)  $R\theta(360^{\circ})/(2\pi)$ (D)  $R [1 - \theta(360^{\circ})/(2\pi)]$
  - (E)  $[R \theta/(2\pi)][1 \theta/(2\pi)]$



- 62. Two particles of mass *m* move in a three-dimensional cubical box of side *a*. If the particles also repel each other via a weak short range force  $V(\mathbf{r}_1 - \mathbf{r}_2) = V_0 \delta^3(\mathbf{r}_1 - \mathbf{r}_2)$ , then calculate the ground state energy using perturbation theory.
  - (A)  $3h^2\pi^2/ma^2 + (3/2a)^3V_0$ (B)  $3h^2\pi^2/ma^2$
  - (B)  $3h^2\pi^2/ma^2$
  - (C)  $(3/2a)^3 V_0$

(D)  $h^2 \pi^2 / ma^2 + (3/2a)^3 V_0$ (E)  $h^2 \pi^2 / ma^2$ 

### **GRE PHYSICS**

- 63. An atom has three valence electrons in a p shell. Determine the total number of states in this configuration. That is, how many distinct three electron states can be constructed from the orbits in a p shell?
  - (A) 8 (D) 4
  - (B) 20 (E) 3
  - (C) 12
- 64. Let the potential energy of the NaCl molecule be described by

$$V(r) = -e^{2}/r + ke^{-r/r_{0}}$$

where *r* is the internuclear separation. If the equilibrium separation is  $r^* = 2.50 \times 10^{-10}$  and the dissociation energy is  $V(r^*) = 3.60$  eV, then find the constants  $r_0$  and *k*.

(A)  $r_0 = 2.50 \times 10^{-10} \text{ m}, k = 3.60 \text{ eV}$ (B)  $r_0 = 1.25 \times 10^{-10} \text{ m}, k = 7.20 \text{ eV}$ (C)  $r_0 = 0.94 \times 10^{-10} \text{ m}, k = 30.94 \text{ eV}$ (D)  $r_0 = 1.88 \times 10^{-10} \text{ m}, k = 15.47 \text{ eV}$ (E)  $r_0 = 2.50 \times 10^{-10} \text{ m}, k = 5.76 \text{ eV}$ 



65. Scientists use the Lennard-Jones 6 = 12 potential  $V(r) = A/r^{12} - B/r^6$  to describe the interaction between the atoms in a diatomic molecule. For small departures from the equilibrium separation  $r_0$ , find the angular frequency of oscillation. Let *m* be the mass of each atom.

(A) 
$$\omega = \sqrt{312A/mr_0^{14} - 84B/mr_0^8}$$
 (D)  $\omega = \sqrt{312A/mr_0^{14} + 84B/mr_0^8}$   
(B)  $\omega = \sqrt{A/mr_0^{14} + B/mr_0^8}$  (E)  $\omega = \sqrt{156/mr_0^{14} - 42B/mr_0^8}$   
(C)  $\omega = \sqrt{A/mr_0^{14} - B/mr_0^8}$ 

66. An object of mass 6.0 kg oscillates harmonically with negligible damping with a frequency of 1.0 Hz. With a small magnetic damping, the amplitude decreases from 0.25 m to 0.125 m after 10 seconds. Find the angular frequency for the damped system.

(A)	6.28 rad/s	(D)	4.21 rad/s
(B)	3.14 rad/s	(E)	5.28 rad/s

- (C) 1.07 rad/s
- 67. The bobs of two simple pendula each have mass *m* and are attached to a string of length *l* as shown. If the two pendula are coupled by a massless spring of constant *k*, then find the higher frequency of oscillation of the system. String tension can be neglected, as oscillations about equilibrium are small.

(A) 
$$\omega = \sqrt{g/l + k/m}$$

- (B)  $\omega = \sqrt{2g/l + k/m}$
- (C)  $\omega = \sqrt{2g/l + 3k/m}$
- (D)  $\omega = \sqrt{g/l + 2k/m}$
- (E)  $\omega = \sqrt{g/l + 3k/m}$



- 68. The ionosphere may be viewed as a dielectric medium of refractive index  $n = n(\omega_p)$ , where  $\omega_p$  is the plasma frequency. Calculate the group velocity of a radio wave of frequency  $\omega = \sqrt{2}\omega_p$ .
  - (A)  $c/\sqrt{2}$  (B)  $c/\sqrt{3}$  (C) c/2
  - (D) c/4 (E)  $\sqrt{2}c$
- 69. In a laboratory experiment, two quantities x and y are measured. Then the formula

 $f = c\sqrt{x/y}$ 

is used to calculate the third quantity f. If  $\Delta x$  and  $\Delta y$  are the uncertainties in x and y, respectively, then what is the uncertainty in f?

(A)  $\Delta f = \sqrt{(\Delta x)^2 + (\Delta y)^2}$ (B)  $\Delta f = f\sqrt{(\Delta x/2x)^2 + (\Delta y/y)^2}$ (C)  $\Delta f = \frac{f}{2}\sqrt{(\Delta x/x)^2 + (\Delta y/y)^2}$ 

(D) 
$$\Delta f = f \sqrt{(\Delta x/x)^2 + (\Delta y/2y)^2}$$

(E) 
$$\Delta f = f \sqrt{(\Delta x/x)^2 + (\Delta y/y)^2}$$

- 70. A point charge initially at rest at the origin experiences an acceleration  $a = z + for a very short time and then proceeds to move with speed <math>u = a\Delta t$ . What is the magnitude of the Poynting flux of this accelerated charge at distance r and angle  $\theta$ ?
  - (A)  $q^2 a^2 \cos^2 \theta / 16\pi^2 \varepsilon_0 r^2 c^3$
  - (B)  $q^2 a^2 \sin^2 \theta / 8\pi^2 \varepsilon_0 r^2 c^3$
  - (C)  $q^2 a^2 \cos^2 \theta / 8\pi^2 \varepsilon_0 r^2 c^3$
  - (D)  $q^2 a^2 \sin^2 \theta / 16\pi^2 \varepsilon_0 r^2 c^3$
  - (E)  $q^2 a^2 \sin^2 \theta \cos^2 \theta / 4\pi^2 \varepsilon_0 r^2 c^3$



- 71. The nitrogen molecules in the atmosphere have an ultraviolet transition at  $\lambda_0 = 75$  nm. The density of  $N_2$  there is  $\rho_t = 1.68 \times 10^{29}$  particles/ $m^2$ . Find the fraction of blue sunlight  $\lambda = 450$  nm scattered out of the atmosphere.
  - (A) 0.43%(B) 0.86%(C) 1.29%(D) 1.72%(E) 2.15%
- 72. Consider the field of a point charge q moving with constant velocity. What must the speed  $\beta$  of the charge be for its field in the lab frame at  $\theta' = 90^{\circ}$  to be twice the normal non-relativistic value?
  - (A) c/4 (B) c/2 (C)  $\sqrt{3}c/2$
  - (D) c/3 (E)  $c/\sqrt{2}$

- 73. A particle of mass m follows the Maxwell-Boltzmann distribution at temperature T. Find the most probable speed for this particle.
  - (A)  $\sqrt{2kT/m}$  (B)  $\sqrt{kT/m}$  (C)  $\sqrt{8kT/\pi m}$ (D)  $\sqrt{3kT/m}$  (E)  $\sqrt{3kT/\pi m}$
- 74. Consider the validity of the classical theory of the ideal gas. Let  $\rho$  be the number of particles per unit volume, *m* the particle mass, and *T* the temperature. Derive a condition for the classical Maxwell-Boltzmann distribution to be valid.
  - (A)  $h\rho^{1/3}/\sqrt{2mkT} \ll 1$  (D)  $\rho^{-1/3} \ll 1$  Å
  - (B)  $h\rho^{1/3}/\sqrt{mkT} >> 1$  (E)  $h\rho^{1/3}/\sqrt{mkT} << 1$ (C)  $\rho^{-1/3} >> 1\text{\AA}$
- 75. A Michelson interferometer is used to measure the wavelength of a monochromatic light. When the movable mirror is displaced by a distance 0.12 mm the number of fringes moving into the center of the pattern is 400. The wavelength of the light must be
  - (A) 300 nm(B) 400 nm(C) 500 nm(D) 600 nm(E) 1200 nm
- 76. A person is riding a Ferris wheel that is spinning at constant angular speed  $\omega$ . At the top of the wheel, her apparent weight is 5% more than her true weight. At the bottom of the wheel, her apparent weight will be
  - (A) 5% more than the true weight
  - (B) 5% less than the true weight
  - (C) 95% more than the true weight
  - (D) 95% less than the true weight
  - (E) the same as her true weight
- 77. A square wave f(t) = a for 0 < t < T/2 and f(t) = -a for -T/2 < t < 0 is generated on a laboratory oscilloscope. Decompose this wave into a Fourier series.

(A) 
$$f(t) = 4a/\pi \sum_{j=0}^{\infty} \cos(j\omega t)/(2j+1)$$
  
(B)  $f(t) = 2a/\pi \sum_{j=0}^{\infty} \cos(2j+1)\omega t/(2j+1)$   
(C)  $f(t) = 4a/\pi \sum_{j=0}^{\infty} \sin(2j+1)\omega t/(2j+1)$   
(D)  $f(t) = 2a/\pi \sum_{j=0}^{\infty} \sin(2j+1)\omega t/(2j+1)$   
(E)  $f(t) = 4a/\pi \sum_{j=0}^{\infty} \sin(j\omega t)/(2j+1)$ 

- 78. Consider a system which contains N magnetic atoms per unit volume in a magnetic field **B**. Supposing that each magnetic atom has spin 1/2 and magnetic moment  $\mu_0$ , find the magnetization of the system at temperature T.
  - (A)  $N\mu_0 \operatorname{coth}(\mu_0 B/kT)$

(B)  $N\mu_0(e^{\mu_0 B/kt} + e^{-\mu_0 B/kT})$ 

- (C)  $N\mu_0 \tanh(\mu_0 B/kT)$
- 79. In the derivation of the Stefan-Boltzmann law from the Planck distribution, one must evaluate the integral

$$\int_0^\infty z^3 \, dz \, / \left( e^z - 1 \right)$$

where z = h v/kT. What is the value of this integral?

- (A)  $\pi^{4/15}$  (D)  $\pi^{3/15}$ (B)  $\pi^{3/90}$  (E)  $\pi^{4/90}$ (C)  $\pi^{2/40}$
- 80. A typical commercially available diatomic gas will actually consist of a diatomic and a monatomic portion. Let the degree of dissociation be  $\delta = m_1/m$  where  $m_1$  is the mass of the monatomic portion and *m* the total system mass. Use Dalton's law of partial pressures to obtain the equation of state of the gas. The monatomic mass has a mass of *A* g per mole.
  - (A)  $pV = m(1 + \delta)RT/A$ (B)  $pV = m(1 + \delta)RT/2A$ (C)  $pV = m_1RT/A$ (D) pV = mRT/2A(E)  $pV = m_1(1 + \delta)RT/A$
- 81. For a gas with a van der Waals equation of state, calculate the coefficient of cubical expansion  $\beta$ , where  $\beta = \frac{1}{V} \left( \frac{\partial V}{\partial T} \right)_P$ .
  - (A)  $\beta = RV^2(V-b)/(RTV^3 2a(V-b)^2)$ (B)  $\beta = 1/T$ (C)  $\beta = RV^2(V-a)/(RTV^3 - b(V-a)^2)$ (D)  $\beta = T$ (E)  $\beta = RV(V-b)/(RTV^2 - a(V-b)^2)$
- 82. A cubical box of edge length *L* contains *n* molecules per cc of radius *a*. Estimate the mean number of collisions that an ideal gas molecule will undergo in crossing the box. Let L = 100 m and  $a = 1 \times 10^{-10}$  m.
  - (A)  $10^{20}$  (D)  $10^{10}$
  - (B)  $10^{17}$  (E)  $10^{24}$
  - (C)  $10^8$

(D)  $N\mu_0(e^{\mu_0 B/kt} - e^{-\mu_0 B/kT})$ (E)  $N\mu_0 \sinh(\mu_0 B/kT)$ 

83. Consider a one-dimensional anharmonic oscillator of energy

$$E = p^2/2m + bx^4$$

Find the mean total energy of this oscillator at a temperature T.

- (A) 1/2 kT (B) kT (C) 3/4 kT(D) 3/2 kT (E) 7/4 kT
- 84. A thermally insulated ideal gas is compressed quasi-statically from an initial macrostate of volume  $V_0$  and pressure  $p_0$  to a final macrostate of volume  $V_f$  and pressure  $p_f$ . Calculate the work done on the gas in this process.
  - (A)  $p_f V_f p_0 V_0$ (B) 0 (C)  $(c_p - c_V)/R(p_f V_f - p_0 V_0)$ (D)  $c_V/R(p_f V_f - p_0 V_0)$ (E)  $c_p/R(p_f V_f - p_0 V_0)$
- 85. The pictured one-dimensional system is in equilibrium at temperature *T*. The normal mode frequencies are  $a\omega_0$ ,  $b\omega_0$ , and  $c\omega_0$ , where a = 1.0, b = 0.54, c = 1.3, and  $\omega_0 = \sqrt{2k/m}$ . Compute the total average energy of the system including quantum mechanical effects.



- (B)  $\hbar\omega_0(an_a + bn_b + cn_c + 3/2)$
- (C)  $\hbar\omega_0 \left[ae^{-\hbar\omega_0/kT} + be^{-\hbar\omega_0/kT} + ce^{-\hbar\omega_0/kT}\right]$

(D) 
$$\hbar\omega_0[a/(e^{\hbar\omega_0/kT}+1)+b/(e^{\hbar\omega_0/kT}+1)+c/(e^{\hbar\omega_0/kT}+1)]$$

(E) 
$$\hbar\omega_0[a/(e^{\hbar\omega_0/kT}-1)+b/(e^{\hbar\omega_0/kT}-1)+c/(e^{\hbar\omega_0/kT}-1)]$$

- 86. A  $K_{\alpha}$  x-ray emitted by one hydrogen atom strikes a second hydrogen atom and undergoes photoelectric absorption with an *L* shell electron. What energy does the ejected electron have?
  - (A) 13.6 eV
     (B) 10.2 eV
     (C) 4.6 eV

     (D) 3.4 eV
     (E) 6.8 eV

87. In deriving the Rayleigh-Jeans law, it is necessary to count the number of modes dn corresponding to a wave number k for a photon gas in a cubical box. Consider this same problem, but in two dimensions for a square of side length L. What is the number of modes dn? Let  $A = L^2$ .

- (A)  $\frac{A}{2\pi}k \, dk$  (D)  $\frac{A}{\pi}k \, dk$
- (B)  $\frac{2A}{\pi}k \, dk$  (E)  $\frac{A}{4\pi}k \, dk$
- (C)  $\frac{4A}{\pi}k dk$





- 88. Consider that the energy of an incident photon becomes very large. The photon is then scattered by an electron as shown here in the initial state. Find the upper limit of energy for the Compton scattered photon.
- $\mathcal{M}_{\gamma}$
- (A)  $m_{\rho}c^{2}$ (D) *E*/2 (E)  $E/2 + 1/2 m_c c^2$ (B) *E* (C)  $1/2 m_c c^2$
- 89. Suppose that the parameters in J.J. Thomson's e/m apparatus are path length of deflecting plates = 0.05 m, plate separation = 0.015 m, potential between deflecting plates = 50 volts, and deflection of the beam when the magnetic field is off =  $1.25 \times 10^{-3}$  m. Further suppose that no deflection is observed when  $B = 1.2 \times 10^{-4}$  T. Find *e/m*.
  - (A)  $4.62 \times 10^{11}$  coul/kg (B)  $2.31 \times 10^{11}$  coul/kg
  - (C)  $1.76 \times 10^{11}$  coul/kg

(A) 7.87 fm

(B) 15.74 fm

(C) 5.32 fm

(D)  $3.52 \times 10^{-10}$  (E)  $2.04 \times 10^{11}$  coul/kg

90. A deuteron is incident on a lead nucleus at Brookhaven National Laboratory. The terminal voltage of the accelerator is 15 megavolts. Find the distance of closest approach in a head-on collision.



- 91. The problem of *n* identical harmonic oscillators with negligible interactions in a microcanonical ensemble of energy E is solved by considering an *n*-dimensional sphere. What is the volume of such a sphere?
  - (A)  $4/3\pi R^n$ (D)  $\pi^n/n! R^n$ (E)  $\pi^{n-1}/(n-1)! R$ (B)  $\pi R^2$ (C)  $\pi^{n/2}/(n/2)! R^n$
- 92. Consider an ensemble of systems consisting of N harmonic oscillators of total energy  $E = 1/2\hbar\omega N + \hbar\omega M$  subject to the constraint

$$\sum_{i=1}^{N} n_i = M$$

Find the number of microstates  $\Omega(E)$  and from that the entropy in Stirling's approximation.

- (A)  $S = (M + N)\ln(M + N)$
- (B)  $S = M \ln M$
- (C)  $S = N \ln N$
- (D)  $S = (M + N) \ln (M + N) M \ln M N \ln N$
- (E)  $S = (M + 2N) \ln (M + 2N) M \ln M 2N \ln 2N$

### **GRE PHYSICS**

93. A solid at absolute temperature T contains N negative impurity ions per cm<sup>3</sup> with lattice spacing a. An equal number of positive ions are free to propagate throughout the solid. If a small electric field E is applied along the x direction, then find the electric polarization  $P_x$ .



- 94. An ion of mass m and electric charge e is moving in a dilute gas of molecules experiencing collisions. Suppose that the mean time between collisions is  $\tau$  and that a uniform electric field E is applied in the x direction. What is the mean distance  $\langle x \rangle$  that the ion travels between collisions?
  - (A)  $1/2eE\tau^2/m$ (B)  $eE\tau^2/m$ (E)  $eE\tau^2/\sqrt{3m}$ (D)  $eE\tau^2/\sqrt{2m}$
- 95. A hollow cube has conducting walls defined by six planes: x = 0, y = 0, z = 0, x = a, y = a, and z = a. The walls at z = 0 and z = a are held at constant potential  $V_0$ , whereas the other sides have  $\phi = 0$ . (See diagram.) Find the potential at the center  $\phi_c$ .
  - (A)  $V_0/6$
  - (B)  $V_0/4$
  - (C)  $V_0^{(0)}$ (D)  $V_0^{(0)}/2$

  - (E)  $V_0^{'}/3$



(C)  $\sqrt{2}eE\tau^2/m$ 

- 96. If the Boltzmann equation, which describes the scattering of two particles with initial momenta **p** and  $\mathbf{p}_2$  to final momenta  $\mathbf{p}'_1$  and  $\mathbf{p}'_2$ , is integrated with a weight of the mass, then what is the result? Let  $\rho$  be the local density, **u** the local mean velocity, **P** the pressure tensor, **Q** the general energy flux, and F the external force.
  - (A)  $\rho \frac{d\mathbf{u}}{dt} = -\nabla \cdot \boldsymbol{P} + \frac{\rho}{m}\mathbf{F}$
  - (B) One finds that momentum is not conserved locally.

(C) 
$$\frac{\partial(\rho E)}{\partial t} + \nabla \cdot Q = \rho \mathbf{u} \cdot \mathbf{F}/m$$

(D) 
$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0$$

(E) One finds that mass is not conserved locally.



97. Consider a sphere of radius *R* whose surface is held at a potential given by  $\Psi(R, \theta, \phi) = B \sin \theta \sin \phi$ . Find the charge density on the sphere.

(A) 
$$\frac{B\varepsilon_0}{R}$$
 (D)  $\frac{B\varepsilon_0}{R}\cos\theta\sin\phi$   
(B)  $\frac{3B\varepsilon_0}{R}\sin\theta\sin\phi$  (E)  $\frac{B\varepsilon_0}{R}\sin\theta\sin\phi$   
(C)  $\frac{2B\varepsilon_0}{R}\sin\theta\sin\phi$ 

98. A beam of singly ionized boron is accelerated through a potential difference of 4 kilovolts and then passed through a mass spectrometer with magnetic field B = 0.5 Tesla. What is the radius *R* through which the boron is bent? Note A = 10.0129 amu for boron.

(A)	$5.76 \times 10^{-2} \mathrm{m}$	(D)	$11.52 \times 10^{-2} \mathrm{m}$
(B)	$2.88  imes 10^{-2} \mathrm{m}$	(E)	$14.40 \times 10^{-2} \mathrm{m}$
(C)	$8.64 \times 10^{-2} \mathrm{m}$		

99. A nuclear magnetic resonance experiment is performed with protons. The frequency may be adjusted so as to resonate when the sweeping field crosses its zero value. What resonance frequency is expected for a magnetic field of 0.6642 T?

(A)	7.02 MHz	(D)	21.1 MHz
(B)	28.1 MHz	(E)	3.51 MHz

- (C) 14.0 MHz
- 100. A stretched string is vibrating with four loops. If the frequency is increased by a factor of 4, the number of loops formed would be
  - (A) 1 (B) 4 (C) 16
  - (D) 2 (E) 8

## **Answer Key**

1.	(A)	18. (D)	35. (B)	52. (E)	69. (C)	86. (E)
2.	(E)	19. (C)	36. (A)	53. (D)	70. (D)	87. (D)
3.	(C)	20. (A)	37. (E)	54. (B)	71. (B)	88. (C)
4.	(B)	21. (C)	38. (C)	55. (C)	72. (C)	89. (B)
5.	(D)	22. (D)	39. (A)	56. (A)	73. (A)	90. (A)
6.	(B)	23. (E)	40. (D)	57. (E)	74. (A)	91. (C)
7.	(D)	24. (C)	41. (A)	58. (A)	75. (D)	92. (D)
8.	(E)	25. (B)	42. (B)	59. (B)	76. (A)	93. (A)
9.	(C)	26. (A)	43. (C)	60. (D)	77. (C)	94. (B)
10.	(A)	27. (A)	44. (D)	61. (E)	78. (C)	95. (E)
11.	(B)	28. (C)	45. (E)	62. (A)	79. (A)	96. (D)
12.	(A)	29. (E)	46. (B)	63. (B)	80. (B)	97. (B)
13.	(B)	30. (B)	47. (E)	64. (C)	81. (A)	98. (A)
14.	(C)	31. (D)	48. (E)	65. (A)	82. (B)	99. (B)
15.	(E)	32. (E)	49. (B)	66. (A)	83. (C)	100. (C)
16.	(E)	33. (D)	50. (A)	67. (D)	84. (D)	
17.	(E)	34. (C)	51. (D)	68. (A)	85. (E)	

### **Practice Exam 3**

### **Detailed Explanations of Answers**

### 1. (A)

$$\mathbf{r} = -2\mathbf{x} + \mathbf{y} - 3\mathbf{z} \, m$$
$$\mathbf{F} = 5\mathbf{x} + 3\mathbf{y} - 2\mathbf{z} \, N$$

One must evaluate the cross product as a determinant.

$$\tau = \mathbf{r} \times \mathbf{F} = \begin{vmatrix} \mathbf{x} & \mathbf{y} & \mathbf{z} \\ -2 & 1 & -3 \\ 5 & 3 & -2 \end{vmatrix} \begin{vmatrix} \mathbf{x} & \mathbf{y} \\ -2 & 1 \\ -2 & 1 \\ 5 & 3 \end{vmatrix}$$
$$= \mathbf{x}(-2+9) + \mathbf{y}(-15-4) + \mathbf{z}(-6-5)$$
$$= 7\mathbf{x} - 19\mathbf{y} - 11\mathbf{z} N - m$$

2. (E)

$$\Delta t = 0.5 \text{ s}$$

$$\downarrow V_{2y} \qquad \downarrow V_1 = 1 \text{ m/s}$$

$$h = 50 \text{ m}$$

Using basic translational kinematics, we have

$$y_1 = y_0 + v_0 t + \frac{1}{2} a t^2 = v_1 t + \frac{1}{2} g t^2 = t + 4.9 t^2$$
  
 $h = t + 4.9 t^2$ 

thus

$$50 = t + 4.9t^2$$
$$t^2 + 0.204t = 10.204$$

Completing the square, we get

$$(t + 0.102)^2 = 10.204 + (0.102)^2$$



### **GRE PHYSICS**

So 
$$t = 3.094s$$

is the time for the first stone.

The key here is to find the time from the first stone and then use that information. Again, we apply the basic 1-D kinematics formula with  $x_0 = 0$ . But now there is a time delay  $\Delta t = 0.5$ s.

$$y_2 = v_2(t - 0.5) + 4.9(t - 0.5)^2$$
  

$$50 = v_2(2.594) + 4.9(2.594)^2$$
  

$$v_2 = 17.028/2.594$$
  

$$v_2 = 6.56 \text{ m/s}$$

The initial velocity of the second stone in the y-direction must therefore be 6.56 m/s.

$$v_{2y} = 6.56 \text{ m/s}$$
  
 $\frac{6.56}{v_2} = \cos \theta$   
 $v_2 = \frac{6.56}{\cos \theta} = \frac{6.56}{\cos(35^\circ)} = 8.60 \text{ m/s}$ 

3. (C)

Conservation of momentum means that

$$mv = m_0 v_0$$
.

Yet, there is a mass increase of the bucket system

$$\frac{dm}{dt} = \rho A v$$

 $m\frac{dm}{dt} = \frac{1}{2}\frac{d}{dt}(m^2)$ 

Note

thus

 $\frac{d}{dt}(m^2) = 2m\frac{dm}{dt} = 2m\rho Av = 2\rho Amv = 2\rho Am_0 v_0$ 

$$\int_0^m d(m^2) = \int_0^t 2\rho A m_0 v_0 dt$$

Integrating, we obtain

$$m^2 = m_0^2 + 2\rho A m_0 v_0 t$$

$$v = \frac{m_0 v_0}{m} = v_0 \left/ \sqrt{1 + \frac{2\rho A v_0}{m_0} t} \right|$$



### 4. **(B)**

The velocity of the particle in the rest frame K' is



The Lorentz addition of velocities formula is

$$v_{3} = \frac{v_{1} + v_{2}}{1 + \frac{v_{1}v_{2}}{c^{2}}} = \frac{c(1-\delta) + c(1-\delta)}{1 + c(1-\delta)^{2}} = \frac{2c(1-\delta)}{1 + (1-\delta)^{2}}$$
$$= 2c\frac{1-\delta}{2-2\delta+\delta^{2}} = \frac{c}{1 + \frac{\delta^{2}}{2(1-\delta)}}$$
$$\approx c\left(1 - \frac{\delta^{2}}{2(1-\delta)}\right) \approx c\left(1 - \frac{\delta^{2}}{2}\right)$$

to order  $\delta^2$ .

### 5. (D)

The motion is elliptical when the mechanical energy is negative:

$$E = K + U = \frac{1}{2}\mu v_0^2 - \frac{Gm_1m_2}{r_0} < 0$$

Now the reduced mass is  $\mu = m_1 m_2 / (m_1 + m_2) = m/2$ . Thus,

$$E = \frac{1}{4}mv_0^2 - \frac{Gm^2}{r_0} < 0$$
  
$$\Rightarrow Gm^2/r_0 > \frac{1}{4}mv_0^2, v_0 < 2\sqrt{Gm/r_0}$$

### 6. (B)

The clock at the North Pole is at rest in an approximate inertial system. The equatorial clock moves with speed  $v = \omega R_E$ .

Now  $T = T_0 \gamma$  is the time at the pole and

$$T = 100yr \frac{365dy}{yr} \frac{86,400s}{dy} = 3.15 \times 10^9 s$$
$$\Delta T = T - T\sqrt{1 - \beta^2}$$
$$\approx T - T\left(1 - \frac{1}{2}\beta^2\right)$$
$$= T\frac{1}{2}\frac{v^2}{c^2} = \frac{T}{2}\frac{\omega^2 R_E^2}{c^2}$$

Now the angular frequency of the Earth's rotation is

$$\omega = 2\pi v = 2\pi/t_E = 2\pi/86,400 = 7.27 \times 10^{-5} \text{ rad/s}$$

Hence,

$$\Delta T = \frac{1}{2} (3.15 \times 10^9) ((7.27 \times 10^{-5})^2 (6.4 \times 10^6)^2 / (3 \times 10^8)^2)$$
$$= 3.79 \times 10^{-3} s$$

7. (D)

The two balls have velocities

$$v_1 = 20$$
 m/s and  $v_2 = 24$  m/s at time  $t = 0$ .

For the first object:

$$v^{2} = v_{0}^{2} + 2a(y - y_{0})$$
$$0 = 20^{2} - 2(9.8)h$$

Since the velocity reduces to zero at the maximum height,

$$h = 20^2/2(9.8) = 20.41 \ m = y_1$$

Also,

$$0=20-9.8t$$

 $v = v_0 + at$ 

$$t = 2.04s$$

For the second object:

$$y = y_0 + v_0 t + \frac{1}{2} a t^2$$
$$= 24t - 4.9t^2$$
$$= 24(2.04) - 4.9(2.04)^{2}$$
$$= 28.55 \text{ m} = y^{2}$$
$$\Delta y = y_{2} - y_{1} = 8.14 \text{ m}$$

8. (E)



We must use both conservation of momentum and energy. By the law of conservation of energy:

$$\frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 = \frac{1}{2} (m_1 + m_2) v^2 + \frac{1}{2} k x^2$$

By the law of conservation of momentum:

$$m_1v_1 + m_2v_2 = (m_1 + m_2)v \Rightarrow v = \frac{m_1v_1 + m_2v_2}{m_1 + m_2}$$

$$= m_1 v_1^2 + m_2 v_2^2 = (m_1 + m_2) \left(\frac{m_1 v_1 + m_2 v_2}{m_1 + m_2}\right)^2 + kx^2$$
$$kx^2 = \frac{(m_1 v_1^2 + m_2 v_2^2)(m_1 + m_2) - m_1^2 v_1^2 - m_2^2 v_2^2 - 2m_1 m_2 v_1 v_2}{m_1 + m_2}$$
$$= m_1 m_2 (v_1 - v_2)^2 / (m_1 + m_2)$$
$$= \sqrt{\frac{\mu}{k}} (v_1 - v_2)$$

where  $\mu = m_1 m_2 / (m_1 + m_2)$ 

# 9. (C)

The cylindrical shell has moment of inertial  $I_2 = mR^2$  while the solid cylinder has  $I_1 = \frac{1}{2}MR^2$ .



We must use the conservation of angular momentum:

$$\Sigma L = I_1 \omega_0 = (I_1 + I_2) \omega_f$$

Since initially only the cylinder is rotating, while finally the cylinder and shell rotate with the same angular velocity,

$$\omega_f = I_1 \omega_0 / (I_1 + I_2) = \frac{1}{2} M r^2 \omega_0 / (\frac{1}{2} M r^2 + m r^2)$$
$$= M \omega_0 / (M + 2m)$$

Note that energy is not conserved in this inelastic interaction.

### 10. (A)

This is a basic dynamics problem. Applying Newton's second law to the hanging mass m and then the second mass m', we get:

$$\Sigma \tau = (T_1 - T_2)R = Ia,$$
$$mg - T_1 = ma$$
$$T_2 = m'a$$

Applying Newton's second law for rotation, we obtain:

$$v = R\omega \Rightarrow a = R\alpha$$
  

$$(T_1 - T_2)R = Ia/R$$
  

$$m(g - a) - m'a = Ia/R^2$$
  

$$mg - (m + m')a = Ia/R^2$$
  

$$a(I/R^2 + m + m') = mg$$
  

$$\Rightarrow a = mg/(I/R^2 + m + m')$$



11. **(B)** 

$$v_{0x} = v_0 \cos \theta$$
  
= 11 cos 14° = 10.67 m/s  
$$v_{0y} = v_0 \sin \theta$$
  
= 11 sin 14° = 2.66 m/s  
$$x = v_{0x}t$$
  
 $l \cos 45° = 10.67 t$   
 $t = l/15.09$ 

$$y = y_0 + v_{0y}t + \frac{1}{2}a_yt^2$$
$$-l\sin 45^\circ = 2.66t - 4.9t^2 = 0.1763 \ l - 0.02152 \ l^2$$

 $0.02152 l^2 - 0.8834 l = 0$ 

l = 0.8834/0.02152 = 41.05 m



# 12. (A) Consider the usual spherical coordinates $(r, \theta, \phi)$ . The velocity then has two components

$$v_{\theta} = l \frac{d\theta}{dt}$$
 and  $v_{\phi} = l \sin \theta \frac{d\phi}{dt}$ 

Hence, the kinetic energy is

$$K = \frac{1}{2} m l^2 (\theta'^2 + \phi'^2 \sin^2 \theta).$$

Also, the potential energy comes solely from the *z* coordinate:

$$z = r \cos \theta = l \cos \theta$$
$$U = m\sigma l (1 + \cos \theta)$$

Note that

$$\theta = 180^{\circ} \Rightarrow U = 0$$

Hence, the Lagrangian is

$$L = K - U = \frac{1}{2} m l^2 (\theta'^2 + \phi'^2 \sin^2 \theta) - mg l(1 + \cos \theta)$$

# 13. (B)

The potential is

$$U = -\int \mathbf{F} \cdot d\mathbf{r} = \int kr \, dr = \frac{1}{2} \, kr^2$$
$$= \frac{1}{2} \, k(R^2 + z^2)$$



In cylindrical coordinates the velocity is

$$\mathbf{v} = r'\hat{r} + r\theta'\hat{\theta} + z'\hat{z}$$

Thus,

$$K = \frac{1}{2} mv^{2}$$
  
=  $\frac{1}{2} m(r'^{2} + r^{2} - \theta'^{2} + z'^{2})$ 

since r = R = constant

$$= \frac{1}{2}m(R^2\theta'^2 + z'^2)$$

Hence,

$$H = K + U = \frac{1}{2} m(R^2 \theta'^2 + z'^2) + \frac{1}{2} k(R^2 + z^2)$$

# 14. (C)

This is a static equilibrium problem.



The first condition of equilibrium gives:

 $\Sigma F_x = T_x + \mu N - W_x = 0$ 

 $T\cos\theta + \mu(mg\cos\theta + T\sin\theta) - mg\sin\theta = 0$ 

and

$$\Sigma F_y = N - T_y - W_y = 0$$

Thus, 
$$N = mg \cos \theta + T \sin \theta$$

which we already used above. The second condition of equilibrium is:

$$\Sigma \tau = RT - \mu NR = 0 \Rightarrow \mu N = T$$
$$\Rightarrow \mu (mg \cos \theta + T \sin \theta) = T$$
$$T \cos \theta + T = mg \sin \theta$$

Finally,

 $T = mg\sin\theta/(1 + \cos\theta).$ 

# 15. (E)

The given position vector is:

$$r = (3t + 5t^3)\mathbf{x} \quad \text{or} \quad x = 3t + 5t^3$$
$$v = \frac{dx}{dt} = 3 + 15t^2$$
$$a = \frac{dv}{dt} = 30t \Rightarrow F = ma = 60t$$

since m = 2kg is given. The power is

$$P = \mathbf{F} \cdot \mathbf{v} = 180t + 900t^3$$

The work is then

$$W = \int_0^1 P \, dt = \int_0^1 (180t + 900t^3) dt$$
$$= 90t^2 + 225t^4 \begin{vmatrix} 1 \\ 0 \end{vmatrix} = 315J$$

# 16. (E)

If the motion is circular, the gravitational force causes centripetal acceleration so that

$$F = mv^{2}/r$$
  

$$\Sigma F = ma$$
  

$$G mm_{s}/r^{2} = m\omega^{2}r$$
  

$$= 4\pi^{2}mv^{2}r$$
  

$$= 4\pi^{2}mr/T^{2}$$
  

$$G mm_{s}/r^{2} = 4\pi^{2}mr/T^{2}$$
  

$$T^{2} = (4\pi^{2}/Gm_{s})r^{3}$$
  

$$m_{s} = (4\pi^{2}/G)r^{3}/T^{2}$$
  

$$= (4\pi^{2}/6.673 \times 10^{-11})(1.50 \times 10^{11})^{3}/(\pi \times 10^{7})^{2}$$
  

$$= 2.0 \times 10^{33} \text{ g} = 2.0 \times 10^{30} \text{ kg}$$

Hence,

# 17. (E)

The given spring force is nonlinear but conservative.

$$F = -k_1 x - k_2 x^3$$

Using the work-energy theorem:

$$x^{4} + 2\frac{k_{1}}{k_{2}}x^{2} = \frac{2m}{k^{2}}v^{2}$$
$$\left(x^{2} + \frac{k_{1}}{k_{2}}\right)^{2} = 2\frac{m}{k_{2}}v^{2} + \left(\frac{k_{1}}{k_{2}}\right)^{2}$$
$$x^{2} = -\frac{k_{1}}{k_{2}} + \sqrt{2\frac{m}{k_{2}}v^{2} + \frac{k_{1}^{2}}{k_{2}^{2}}}$$
$$x = \sqrt{k_{1}/k^{2}} \left(\sqrt{1 + \frac{2mv^{2}k_{2}}{k_{1}^{2}} - 1}\right)^{1/2}$$

# 18. (D)

The products of inertia or off-diagonal elements of the inertia tensor vanish for a symmetric body, e.g.,  $I_{xy} = -\Sigma mxy = 0$ . Thus,

$$L_x = I_{xx}\omega_x, L_y = I_{yy}\omega_y$$
, and  $L_z = I_{zz}\omega_z$ 

 $I_{zz}$  just comes from the moment of inertia of the two spheres:

$$I_{zz} = \frac{2}{5} mr^2 + \frac{2}{5} mr^2 = \frac{4}{5} mr^2$$

 $I_{yy}$  comes from the parallel axis theorem:



$$I_{vv} = 2(I_{cm} + md^2)$$

where d = l has been used

$$= 2(\frac{2}{5} mr^{2} + ml^{2})$$
  
=  $\frac{4}{5} mr^{2} + 2ml^{2}$   
 $L_{x} = 0, L_{y} = (\frac{4}{5} mr^{2} + 2ml^{2})\omega_{y}$ , and  $L_{z} = \frac{4}{5} mr^{2}\omega_{z}$ 

thus

19. (C)

Consider that the executive toy is tilted at angle  $\alpha$ :



 $= 2mg(l - L\cos\theta)\cos\alpha$ 

Equilibrium occurs at

$$\frac{dU}{d\alpha} = -2mg(I - L\cos\theta)\sin\alpha = 0$$

As we expect,  $\Rightarrow \alpha = 0$  so that stability at equilibrium requires

$$\left. \frac{d^2 U}{d\alpha^2} \right| = -2mg(I - L\cos\theta)\cos\alpha > 0$$
  
$$\alpha = 0$$

Thus,  $L \cos \theta > l$  which means that the masses must hang below the pivot.

# 20. (A)

Bernoulli's equation is

$$\frac{1}{2}\rho v^2 + p + \rho gh = \text{constant}$$

Here we may take h = 0 so that

$$\frac{1}{2}\rho v_1^2 + p_1 = \frac{1}{2}\rho v_2^2 + p_2$$

Now the equation of continuity implies that

$$A_1 v_1 = A_2 v_2$$

 $p_2 = p_1 + \frac{1}{2}\rho v_1^2 \left(1 - \frac{A_1^2}{A_2^2}\right)$ 

Hence,

Note that  $A_1 > A_2 \Rightarrow p_2 < p_1$ 

#### 21. (C)

For an infinite charged plane

# $\int \mathbf{E} \cdot d\mathbf{a} = q/\varepsilon_0$

by Gauss's law or Maxwell's 1st equation:

->

σ

σ

$$EA + EA = \sigma \cdot A/\varepsilon_0$$
$$E = \sigma/2\varepsilon_0$$

In between the planes where  $-\frac{a}{2} < x < \frac{a}{2}$ , there are four identical contributions

$$\mathbf{E} = 4\sigma/2\varepsilon_0 \mathbf{x} = 2\sigma/\varepsilon_0 \mathbf{x}$$

 $-\sigma$ 



->

# **DETAILED EXPLANATIONS OF ANSWERS — PRACTICE EXAM 3**

# 22. (D)

Here cylindrical coordinates are appropriate. Laplace's equation is:

 $\nabla^2 \Phi = 0$ 

ZA

α

and

$$\nabla^2 = \frac{1}{\rho} \frac{\partial}{\partial \rho} \left( \rho \frac{\partial}{\partial \rho} \right) + \frac{1}{\rho^2} \frac{\partial^2}{\partial \phi^2} + \frac{\partial^2}{\partial z^2}$$

in orthogonal curvilinear cylindrical coordinates. By symmetry  $\Phi = \Phi(\phi)$  only. Thus, Laplace's equation is

$$\frac{1}{\rho^2}\frac{d^2\Phi}{d\Phi^2} = 0$$

Integrating we obtain

$$\frac{d\Phi}{d\Phi} = a, \int d\Phi = \int a d\phi$$
$$\Phi = a\phi + b$$

The boundary conditions are at

$$\phi = 0, \Phi = 0 \Rightarrow b = 0$$

and at

$$\phi = \alpha, \Phi = V_0 \Rightarrow a = V_0 / \alpha,$$
$$\Phi = -\frac{V_0}{\alpha} \phi$$

Finally,

$$\mathbf{E} = -\nabla \Phi = -\frac{1}{\rho} \frac{\partial \Phi}{\partial \phi} \hat{\phi} = -\frac{V_0}{\alpha \rho} \hat{\phi}$$

## 23. (E)

The given charge density is:

$$\rho(\mathbf{r}) = e\left(\delta(\mathbf{r}) - \frac{\alpha^3}{\pi}e^{-2\alpha r}\right)$$



≻ y Now we use Gauss's law.

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0} \text{ or } \oint E \cdot da = \int \nabla \cdot E dV$$

$$E_r \cdot 4\pi r^2 = \frac{e}{\varepsilon_0} \left( \int \delta(\mathbf{r}) dV - \frac{\alpha^3}{\pi} \int e^{-2\alpha r} dV \right)$$

$$E_r = \frac{e}{4\pi\varepsilon_0 r^2} \left( 1 - \frac{\alpha^3}{\pi} 4\pi \int_0^r e^{-2\alpha r} r^2 dr \right)$$

$$= \frac{e}{4\pi\varepsilon_0 r^2} \left( 1 - 4\alpha^3 \left[ 1\frac{1}{2\alpha} \left( r^2 + \frac{r}{\alpha} + \frac{1}{2\alpha^2} \right) e^{-2\alpha r} \middle| \begin{array}{c} R \\ O \end{array} \right] \right)$$

$$= \frac{e}{4\pi\varepsilon_0 r^2} (2\alpha^2 r^2 + 2\alpha r + 1) e^{-2\alpha r}$$

$$= \frac{e}{4\pi\varepsilon_0 r^2} \left( 2\alpha^2 + \frac{2\alpha}{r} + \frac{1}{r^2} \right) e^{-2\alpha r}$$

24. (C)

We are given that

$$m = 2g$$
  

$$\mathbf{E} = (3\mathbf{x} + 4\mathbf{y}) \times 10^5 \text{ N/C}$$
  

$$\theta = 33^{\circ}$$

and





We resolve **E** into parallel and perpendicular components:

$$E_{\parallel} = 5 \times 10^5 \sin 20.13 = 1.72 \times 10^5 \text{ N/C}$$
  
 $E_{\perp} = 5 \times 10^5 \cos 20.13 = 4.70 \times 10^5 \text{ N/C}$ 

mg

mg cos 33°

mg sin 33°

Since the pith ball is in equilibrium

$$qE_{\perp} = mg \sin \theta$$

$$q = (0.002)(9.8) \sin (33^{\circ})/5 \times 10^{5} \cos (20.13^{\circ})$$

$$= 2.27 \times 10^{-8} \text{ C}$$

$$T = mg \cos \theta - qE_{\parallel}$$

$$= (0.002)(9.8) \cos (33^{\circ}) - (2.27 \times 10^{-8})1.72 \times 10^{5}$$

$$= 0.0125 \text{ N}$$

25. (B)



The given charge is

 $q = -75 \ \mu C$ 

Therefore, the linear charge density is

$$\lambda = \frac{q}{l} = \frac{-75 \times 10^6 C}{0.20M} = -3.75 \times ^{-4} C/m$$

Now calculate the electric field

$$E = \int \frac{dq}{4\pi\epsilon_0 r^2} \mathbf{r} = \frac{1}{4\pi\epsilon_0} \int_{-0.30}^{-0.10} \frac{\lambda dx}{x^2} (-x)$$
$$= \frac{\lambda}{4\pi\epsilon_0} \left(\frac{1}{0.10} - \frac{1}{0.30}\right) x, = -2.25 \times 10^7 \text{in Q: N/Cx}$$

Substituting

$$\varepsilon_0 = 8.85 \times 10^{-12}$$

26. (A)

In vacuum,

$$C_0 = \varepsilon_0 A/d$$

In a dielectric,

$$C = \kappa \varepsilon_0 A/d$$

Here we really have two capacitors in series:

$$C_1 = \frac{\kappa_1 \varepsilon_0 A}{d/2}, \ C_2 = \frac{\kappa_2 \varepsilon_0 A}{d/2}$$

The rule for adding capacitance in series is:

$$C = C_1 C_2 / (C_1 + C_2)$$
  
=  $\kappa_1 \kappa_2 (2\varepsilon_0 A/d)^2 / (\kappa_1 + \kappa_2) (2\varepsilon_0 A/d)$   
=  $2\kappa_1 \kappa_2 C_0 / (\kappa_1 + \kappa_2)$ 

Hence,

# 27. (A)

Using the symmetry of the disk, the differential change element of an annulus is

$$\sigma = \frac{dq}{da}$$

$$V(x) = \int \frac{dq}{4\pi\varepsilon_0 r}$$

$$= \int \frac{2\pi\sigma s}{\sqrt{s^2 + x^2}} \frac{ds}{4\pi\varepsilon_0}$$

$$= \frac{1}{4\pi\varepsilon_0} 2\pi\sigma\sqrt{x^2 + s^2} \begin{vmatrix} s = a \\ s = 0 \end{vmatrix}$$

$$V(x) = \frac{\sigma}{2\varepsilon_0} \left(\sqrt{x^2 + a^2}\right), x > 0$$





Now the electric field is the negative gradient of the electric potential.

$$\mathbf{E} = -\nabla V = \frac{\sigma}{2\varepsilon_0} \frac{\partial}{\partial x} \left( \sqrt{x^2 + a^2} - x \right) \mathbf{x}$$
$$= \frac{\sigma}{2\varepsilon_0} \left( \frac{1}{2} (x^2 + a^2)^{-1/2} 2x - 1 \right) \mathbf{x}$$
$$= \frac{\sigma}{2\varepsilon_0} \left( \frac{x}{\sqrt{x^2 + a^2}} - 1 \right) \mathbf{x}$$

# 28. (C)

Now the current density is

$$j = I/A = I/\rho(b^2 - a^2)$$

and by Ohm's law

$$j = \sigma E = \frac{1}{\rho} = \frac{1}{\rho}E = \frac{1}{\rho}\frac{V}{l}$$

where s is the conductivity and r is the resistivity. Hence,

$$\frac{1}{\rho}\frac{V}{l} = \frac{1}{\pi(b^2 - a^2)}$$

Finally,

$$R = V/I = \rho l/\pi (b^2 - a^2)$$

# 29. (E)

The bottom two resistors are in parallel,



Thus,

$$R = \frac{R_1 R_2}{R_1 + R_2} = \frac{15(5)}{15 + 5} = 3.75\Omega$$

The last two resistors now add up since they are in series:

$$R = R_1 + R_2$$
  
= 10 + 3.75  
= 13.75 $\Omega$ 

The current in the reduced circuit is then

I = V/R = 12/13.75 = 0.873 A

Applying Kirchhoff's voltage law to the second circuit, we get:

$$12V = 5I_1 + 10I = 5I_1 + 8.73$$
  
 $I_1 = 3.27/5 = 0.654$  A

Thus,

### 30. (B)

If a magnetic field is applied to a current-carrying conductor perpendicularly, an electrical potential difference is generated. This physical phenomenon was discovered by E. H. Hall in 1879 and, consequently, is known as the Hall effect.

#### 31. (D)

The Biot-Savart law states that:

$$d\mathbf{B} = \frac{\mu_0}{4\pi} I \frac{d\mathbf{l} \times s}{s^2}$$
$$d\mathbf{B}_z = d\mathbf{B}\cos\theta$$
$$= \frac{\mu_0 I d\mathbf{l}}{4\pi s^2} \cos\theta = \frac{\mu_0 I d\mathbf{l}}{4\pi s^2} \frac{r}{s}$$
$$\mathbf{B}_z = \frac{\mu_0 I r}{4\pi s^2} \int_0^{2\pi r} d\mathbf{l}$$
$$= \frac{\mu_0 I 2\pi r^2}{4\pi s^3}$$
$$= \mu_0 I r^2 / 2(r^2 + z^2)^{3/2}$$



Where we have used the theorem of Pythagoras

 $s^2 = r^2 + z^2$ 

32. (E)

Ampere's law states that

$$\nabla \times \mathbf{B} = \boldsymbol{\mu}_0 \mathbf{j}$$

Since

$$\mathbf{A} \cdot (\mathbf{A} \times \mathbf{B}) = 0, \nabla \cdot (\nabla \times \mathbf{B}) = 0 \Rightarrow \nabla \cdot \mathbf{j} = 0$$

0

But really 
$$\nabla \cdot \mathbf{j} + \frac{\partial \rho}{\partial t} =$$

is the continuity equation. Now

$$abla imes \mathbf{E} = rac{
ho}{arepsilon_0}$$

is Gauss's Law. Hence,

$$\nabla \cdot \mathbf{j} + \frac{\partial}{\partial t} \nabla (\varepsilon_0 \cdot E) = 0$$
$$\nabla \cdot \left( \mathbf{j} + \varepsilon_0 \frac{\partial E}{\partial t} \right) = 0$$

### 33. (D)

In a vacuum  $\nabla \times \mathbf{B} = \mu_0 \mathbf{j}$ . But really  $\mathbf{j} = \mathbf{j}_{\text{free}}$ . In the presence of matter

Hence,  

$$\mathbf{j} = \mathbf{j}_{\text{free}} + \mathbf{j}_{\text{bound}}$$

$$\nabla \times \mathbf{B} = \mu_0 (\mathbf{j}_{\text{free}} + \mathbf{j}_{\text{bound}})$$

$$\nabla \times \mathbf{B} - \mu_0 \mathbf{j}_{\text{bound}} = \mu_0 \mathbf{j}_{\text{free}}$$

Defining the magnetization vector by

$$\nabla \times (\mathbf{B} - \boldsymbol{\mu}_0 \mathbf{M}) = \boldsymbol{\mu}_0 \mathbf{j}_{\text{free}}$$
$$\nabla \times \mathbf{M} \equiv \mathbf{j}_{\text{bound}}$$

where

the problem is solved.

#### 34. (C)

The electron drifts in a different direction depending on the direction of the electric field. When the electron drifts upward:

$$mg = eE - 6\pi R\eta v_+$$

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When the electron drifts downward:

$$mg = -eE + 6\pi R\eta v_{-}$$

Subtracting the two equations we obtain:

or

35. (B)

Derivation of Coulomb's law from Gauss's law:

$$\nabla \cdot \mathbf{E} = \rho/\varepsilon_0$$
$$\oint \nabla \cdot \mathbf{E} dV = \oint \mathbf{E} \cdot d\mathbf{a}$$

by the divergence theorem.

$$\oint \mathbf{E} \cdot d\mathbf{a} = \oint \rho dV / \varepsilon_0$$

For a point charge

$$E \cdot 4\pi r^2 = q/\varepsilon_0$$
$$E = q/4\pi\varepsilon_0 r^2$$

for a change  $q = q_1$ 

$$\begin{split} E_1 &= q_1 / 4\pi\varepsilon_0 r^2 \\ F &= q_2 E_1 = q_1 q_2 / 4\pi\varepsilon_0 r^2 = k_E q_1 q_2 / r^2 \end{split}$$

for the force on another charge  $q_2$  due to  $q_1$ .

Using Ampere's law:

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{j}$$
$$\int \nabla \times \mathbf{B} \cdot d\mathbf{a} = \oint \mathbf{B} \cdot d\mathbf{l}$$

by Stokes' theorem.

$$\int (\nabla \cdot \mathbf{B} \cdot d\mathbf{a} = \boldsymbol{\mu}_0 \int \mathbf{j} \cdot d\mathbf{a}$$

For a line current

$$2\pi rB = \mu_0 I$$
$$B = \mu_0 I / 2\pi r$$
$$B_1 = \mu_0 I_1 / 2\pi r$$

For a current  $I = I_1$ .

The Lorentz force is

 $\mathbf{F} = q_2 \mathbf{v} \times \mathbf{B} = I_2 \mathbf{I} \times \mathbf{B}$  $F/l = \mu_0 I_1 I_2 / 2\pi r = k_B I_1 I_2 / r$  $k_B / k_E = \frac{\mu_0}{2\pi} 4\pi \varepsilon_0$  $= 2\mu_0 \varepsilon_0$ 

Hence,

### **36.** (A)

We must use Faraday's law:

$$\nabla \times \mathbf{E} = -\partial \mathbf{B} / \partial t$$
$$V = \int \mathbf{E} \cdot d\mathbf{l} = -\frac{\partial \Phi}{\partial t}$$

 $\Phi = \int \mathbf{B} \cdot d\mathbf{a} = Bl\mathbf{x}$ 

where

$$V = -Blv$$
 and  $V = RI \Rightarrow I = V/R = Blv/R$ 

$$R = 12\Omega \underbrace{\underbrace{\begin{array}{c} x \\ I \\ \otimes \\ B \\ \end{array}}}_{B = 5T} v = 4 \text{ m/s} \qquad \int I = 1.5 \text{ m}$$

The power is

$$P = VI$$
  
=  $(Blv)^2/R$   
=  $(5 \cdot 1.5 \cdot 4)^2/12 = 75$  W

Furthermore, *I* must be *counterclockwise* by Lenz's law to counteract the increase of magnetic flux due to the motion of the bar.

### 37. (E)

By Kirchhoff's voltage law,

$$\Sigma V = 0$$

So

$$-RI - LI' - \frac{Q}{C} = 0$$

$$Q'' + \frac{R}{L}Q' + \frac{1}{LC}Q = 0$$

$$\omega_0 = \sqrt{1/LC}, \gamma = R/L, \ \omega = \sqrt{\omega_0^2 - \gamma^2/4}$$

$$Q'' + \gamma Q' + \omega_0^2 Q = 0$$

Then for the light damping case, the solution is

$$Q = Q_m e^{-1/2\gamma t} \cos(\omega t + \delta)$$
  
=  $Q_m e^{-1/2\gamma t} \cos \omega t$  if  $\delta = 0$   
 $R = 10 \ \Omega, L = 10^{-3} H, C = 10^{-6} F$   
 $\omega_0 = 1/\sqrt{LC} = 1/\sqrt{10^{-9}} = 31,620 \text{ rad/s}$   
 $\omega = \sqrt{\omega_0^2 - \frac{1}{4} \frac{R^2}{L^2}}$   
=  $\sqrt{31,620^2 - \frac{1}{4} \frac{10^2}{10^{-6}}} = 31,220 \text{ rad/s}$   
 $\frac{Q}{Q_m} = e^{-\frac{1}{2} \times \frac{10}{10^{-3}} \times 2 \times 10^{-4}} \cos(31,220 \times 2 \times 10^{-4}) = 0.366$ 



# 38. (C)

Maxwell's equations in vacuum are:

$$\nabla \cdot \mathbf{E} = 0$$
$$\nabla \cdot \mathbf{B} = 0$$
$$\nabla \times \mathbf{E} = -\partial \mathbf{B} / \partial t$$
$$\nabla \times \mathbf{B} = \mu_0 \varepsilon_0 \partial \mathbf{E} / \partial t$$

Now use a vector identity:

$$\nabla \times (\nabla \times \mathbf{B}) = \nabla (\nabla \cdot \mathbf{B}) - \nabla^2 \mathbf{B}$$
$$\nabla \times (\mu_0 \varepsilon_0 \partial \mathbf{E} / \partial t) = -\nabla^2 \mathbf{B} \text{ since } \nabla \cdot \mathbf{B} = 0$$
$$\mu_0 \varepsilon_0 \frac{\partial}{\partial t} \frac{\partial \mathbf{B}}{\partial t} = \nabla^2 \mathbf{B}$$
$$\nabla^2 \mathbf{B} = \mu_0 \varepsilon_0 \frac{\partial^2 \mathbf{B}}{\partial t^2}$$

is the general wave equation. For

$$\mathbf{B} = B_z(x, y, z)\mathbf{z}$$
  
we get 
$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)B_z = \mu_0\varepsilon_0 \frac{\partial^2 B_z}{\partial t^2}$$

**39.** (A)

Start with Gauss's law in integral form:

$$\oint \mathbf{E} \cdot d\mathbf{a} = q/\varepsilon_0$$
$$E 2\pi \rho l = q/\varepsilon_0$$
$$\mathbf{E} = (\lambda/2\pi\rho\varepsilon_0)\hat{\rho}$$

Now use Ampere's law:



By the Lorentz transformation,

$$E'_{\rho} = \gamma (E_{\rho} - vB_{\phi})$$
$$= \gamma \left(\frac{\lambda}{2\pi\rho\varepsilon_0} - v\frac{\mu_0 I}{2\pi\rho}\right) = \frac{\gamma}{2\pi\rho} \left(\frac{\lambda}{\varepsilon_0} - v\mu_0 I\right)$$

Hence,

$$E'_{\rho} = 0 \text{ if } v\mu_0 I = \frac{\lambda}{\varepsilon_0}, \Rightarrow v = \frac{\lambda}{\varepsilon_0 \mu_0 I}$$

# 40. (D)

The altitude is

$$x = 60 \text{ km}$$

x = ct

Also

so that

 $t = \frac{60 \times 10^3}{3 \times 10^8} = 2 \times 10^{-4} s$ 

Every half-life, one loses one-half of the particles

$$\left(\frac{1}{2}\right)^3 = \frac{1}{8} \Rightarrow t_0 = 3 \times 1.5 \times 10^{-6} = 4.5 \times 10^{-6}s$$

three half-lives. Using time dilation:

$$t = t_0 \gamma \Rightarrow \gamma = \frac{2 \times 10^{-4}}{4.5 \times 10^{-6}} = 44.44$$
$$\gamma = \frac{1}{\sqrt{1 - \beta^2}}$$
$$\beta = \sqrt{1 - \frac{1}{\gamma^2}} = 0.99975$$

$$1 - \beta = 0.00025$$

# 41. (A)

The kinetic energy of the electron is

$$K = 700 \text{ MeV}$$

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Hence, the total relativistic energy is

$$E = K + m = 700.511 \text{ MeV}$$

The  $\gamma$  parameter is thus

$$\gamma = E/m = 700.511/5.11 = 1371$$
  
 $\beta = \sqrt{1 - \frac{1}{\gamma^2}} = 1 - \frac{1}{2\gamma^2} = 0.9999997$ 

Now we must use the index of refraction:

$$n = c/v = 1.00029$$

The Cherenkov radiation formula is needed

$$\beta' = 1/n = 0.9997100$$
  
 $\cos \phi = \beta'/\beta = 0.9997100/0.9999997$   
 $= 0.9997104$   
 $\phi = 1.38^{\circ}$ 

42. (B)

The nuclear reaction is



and

The relativistic 4-momentum must be conserved

$$(\underline{p})_0 = (\underline{p})_f \Rightarrow (\mathbf{p}_1 i E/c)_0 = (\mathbf{p}_1 i E/c)_f$$

The fourth component of the 4-momentum is the total energy:

$$E_2 + m_2 = E_1 + E_3$$

The momenta components are assumed to be in the xy plane.

$$p_{2x} = p_{3x}$$

$$p_{1y} = -p_{3y}$$

$$m_3^2 = E_3^2 - \mathbf{p}_3^2 = (E_2 + m_2 E_1)^2 - p_{2x}^1 - p_{1y}^2$$

$$= E_2^2 - p_{2x}^2 + E_1^2 - p_{1y}^2 - 2E_1 E_2 + m_2^2 + 2m_2 (E_2 - E_1)$$

$$= 2m_2^2 + m_1^2 + 2m_2 (E_2 - E_1) - 2E_1 E_2$$

We are given the mass of the deuteron and the mass of the proton:

$$m_{2} = 1876.140 \text{ MeV}$$

$$m_{1} = 938.791 \text{ MeV}$$

$$E_{2} = T_{2} + m_{2} = 1877.948 \text{ MeV}$$

$$E_{1} = T_{1} + m_{1} = 942.258 \text{ MeV}$$

$$m_{3}^{2} = 2(1876.140)^{2} + (938.791)^{2} + 2(1876.140)(1877.948 - 942.258)$$

$$- 2(1877.948)(942.258)$$

Finally,  $m_3 = 2809.462 \text{ MeV}$ 

43. (C)

Hence,

and

Classically  $2\theta = 90^{\circ}$ .

In a relativistic equation, rest energy (mass,  $E_0$ ) must be considered as part of the total energy of the system. The total initial energy is thus:

$$E = T + E_0$$

# **DETAILED EXPLANATIONS OF ANSWERS — PRACTICE EXAM 3**

By symmetry, the total final energy of each particle is:



The relativistic momentum must then be:

$$p = \sqrt{E^2 - E_0^2}$$

$$p' = \sqrt{E'^2 - E_0^2}$$

$$= \sqrt{\frac{1}{4}T^2 + E_0^2 + E_0T - E_0^2}$$

$$= \sqrt{\frac{1}{4}T^2 + E_0T}$$



Using the 3-momentum triangle,

$$2p'\,\cos\,\theta=p$$

Thus,

$$\cos^2 \theta = \frac{p^2}{4p'^2}$$
$$= (T^2 + 2E_0T)/(T + 4E_0)$$

Using a trigonometric identity,

$$\frac{1}{2}(1+\cos 2\theta) = (T+2E_0)/(T+4E_0)$$
$$\cos(2\theta) = (2T+4E_0-T-4E_0)/(T+4E_0)$$
$$= T/(T+4E_0)$$

# 44. (D)

Where the Schrödinger equation is  $H\Psi = E\Psi$ 

$$H = T + U$$
$$= \frac{p^2}{2m} - g\delta(x)$$
$$= \frac{-h^2}{2m} \frac{d^2\Psi}{dx^2} - g\delta(x)$$

We now integrate the equation

$$\int_{-\varepsilon}^{\varepsilon} \left( -\frac{\hbar^2}{2m} \frac{\partial^2 \Psi}{\partial x^2} - g\delta(x)\Psi \right) dx = \int_{-\varepsilon}^{\varepsilon} E\Psi \, dx$$

to get

$$\frac{-\mathbf{h}^2}{2m} \frac{d\Psi}{dx} \begin{vmatrix} \boldsymbol{\varepsilon} \\ -\boldsymbol{\varepsilon} \end{vmatrix} - g\Psi(0) = 0$$

The solution to the Schrödinger equation is

$$\Psi(x) = \begin{cases} Ae^{kx}, & x < 0\\ Ae^{-kx}, & x > 0 \end{cases}$$

thus

$$\Psi'(x) = \begin{cases} k\Psi(x), & x < 0\\ -k\Psi(x), & x > 0 \end{cases}$$

Upon substitution, we obtain

$$\frac{\hbar^2 k}{m} = g$$
, and thus  $E = \frac{\hbar^2 k^2}{2m} = -\frac{1}{2} \frac{mg^2}{\hbar^2}$ 

## 45. (E)

Upon solving the Schrödinger equation, one obtains the following eigenfunctions:

$$\Psi(x) = \begin{cases} Ae^{k_1x} & x < -l/2 \\ B\cos k_2x & |x| < l/2 \\ Ae^{-k_1x} & x > l/2 \end{cases}$$

The continuity of  $\psi(\pm l/2)$  gives the condition that

$$Ae^{-k_1 l/2} = B \cos(k_2 l/2)$$

The continuity of  $\psi'(\pm l/2)$  gives the further condition that

$$k_1 A e^{-k_1 l/2} = k_2 B \sin(k_2 l/2)$$

 $k_2 \tan(k_2 l/2) = k_1$ 

 $\tan(k_2 l/2) = k_1 k_2$ 

Dividing produces the equation

or

Substituting the appropriate wave number values, one obtains

$$\tan\left(\sqrt{2m(V_0 - E)}l/2\mathbf{h}\right) = \sqrt{2mE}/\mathbf{h}/\sqrt{2m(V_0 - E)}/\mathbf{h}$$
$$\tan\sqrt{ml^2(V_0 - E)/2\mathbf{h}^2} = \sqrt{E/(V_0 - E)}$$
$$\tan\theta = \sqrt{\theta_0^2/\theta^2 - 1}$$

or

with

 $heta = \sqrt{ml^2(V_0 - E)/2\hbar^2}$   $heta_0 = \sqrt{ml^2V_0/2\hbar^2}$ 

and

# 46. (B)

The given potential is:

$$V(x) = \begin{cases} \frac{1}{2}m\omega^2 x^2 & x > 0\\ \\ \infty & x < 0 \end{cases}$$

The WKB or Wentzel-Kramers-Brillouin approximation may be used. This approximation is valid for a slowly changing potential energy.

$$\int_{0}^{a} p \frac{dx}{h} = n \frac{\pi}{2} + \frac{\pi}{4}, n = 1, 3, 5 \dots$$
$$\int_{0}^{a} p dx = \left(\frac{3\pi}{4}, \frac{7\pi}{4}, \frac{11\pi}{4}, \dots\right) h$$



where

$$p = \sqrt{2m(E-V)}$$

$$=\sqrt{2m\left(E-\frac{1}{2}m\omega^2x^2\right)}$$

Note that for the case of the full harmonic oscillator, the WKB solution is exact.

# 47. (E)

The Zeeman effect may be explained semi-classically. Consider an electron orbiting in a circular orbit. The centripetal force is

$$F_0 = m\omega_0^2 r$$
 with  $B = 0$   
 $F = m\omega_0^2 r \pm \frac{evB}{c} = m\omega^2 r$  with finite B

and

or

Combining the two equations, one gets

$$m(\omega^{2} - \omega_{0}^{2})r = \pm e\omega rB/c$$
$$(\omega + \omega_{0})(\omega - \omega_{0}) = \pm eB\omega/mc$$
$$2\omega\delta\omega = \pm eB\omega/mc$$

Using the approximation that

 $\omega = \omega_0$  $\delta \omega = \pm eB/2m_{\rm e}c$ 

thus

#### 48. (E)

According to the Bohr Theory

$$1/\lambda = R(1/n^2 - 1/m^2)$$
  
= 1.0977373 × 10<sup>-3</sup>(1/n<sup>2</sup> - 1/m<sup>2</sup>)/Å

Where *R* is the Rydberg constant

$$R = 1.097 \times 10^7 \, m^{-1}$$
$$= 1.097 \times 10^{-3} \text{\AA}^{-1}$$

For the Lyman series, n = 1. For the series upper limit, m = n + 1 = 2. Hence,

$$\lambda = 1216 \text{ Å}$$

For the Balmer series, n = 2. Thus,

 $\lambda = 6565 \text{ Å}$ 

For the Paschen series, n = 3. Therefore,

$$\lambda = 18760 \text{ Å}$$

For the Brackett series, n = 4. Hence,

 $\lambda = 40520 \text{ Å}$ 

For the Pfund series, n = 5. Thus,

 $\lambda = 74600 \text{ Å}$ 

Clearly the Paschen series, discovered in 1908, is the answer.

#### 49. (B)

Let each rod have area of cross section A and length L and thermal conductivities  $k_1$  and  $k_2$  respectively. Let the end temperatures be  $T_1 \& T_2$  for each rod hence also for the composited rod

For the composite rod:  $Q/t = k(2A)(T_1 - T_2)/)$ 

For the first rod:  $(Q/t)_1 = k_1 A(T_1 - T_2)/(L)$ 

For the composite rod:  $(Q/t)_2 = k_2 A(T_1 - T_2)/(L)$ 

Adding the second and third equation

$$(Q/t)_1 + (Q/t)_2 = (k_1 + k_2) A(T_1 - T_2)/(L)$$
  
 $Q/t = \frac{1}{2} (k_1 + k_2) (2A)(T_1 - T_2)/(L)$ 

Comparing this equation to the first equation above we get

$$k = \frac{1}{2}(k_1 + k_2)$$

#### 50. (A)

In the Franck-Hertz experiment, an electron loses most of its kinetic energy in an *inelastic* collision with an atom. The electrons raise Hg atoms to the 1st excited state 4.9 eV above the ground state. Any monatomic gas can be used in the Franck-Hertz experiment. Of course, the excited state energy will be different for other gases.

51. (D)



In the photoelectric effect, by conservation of energy

$$hv = \frac{1}{2}mv^2 + \phi$$

where  $\phi$  is the work function of the metal. Furthermore,

$$eV_0 = \frac{1}{2}mv^2$$

Thus, the stopping potential  $V_0$  is directly proportional to the incident light frequency v:

$$eV_0 = h\upsilon = \phi$$
 or  $V_0 = \frac{h}{e}\upsilon - \frac{\phi}{e}$ 

In fact, this is one way of determining Planck's constant.

# 52. (E)

The Rutherford scattering differential cross section is

$$\frac{d\sigma}{d\Omega} = \left(\frac{Z_1 Z_2 e^2}{4K}\right)^2 \sin^{-4} \left(\frac{\theta}{2}\right)$$
  

$$Z_1 = 1, Z_2 = 79, K = 8.8 \text{ MeV}$$
  

$$\frac{d\sigma}{d\Omega} = \left(\frac{(1)(79)(1.44)}{(4)(8.8)}\right)^2 \sin^{-4} \left(\frac{37^\circ}{2}\right)$$
  

$$= 1030 fm^2 \times \frac{10mb}{fm^2} \times \frac{1b}{1000mb}$$
  

$$= 10.30 b$$

## 53. (D)

For a cavity of volume

$$V = l^{3}$$
$$k_{x}l = n_{x}\pi$$
$$n_{x} = l/\pi k_{x}$$

or

thus

$$dn_x = l/\pi \ dk_x$$

Also, the same is true for the y and z directions. But

$$dn = dn_x dn_y dn_z$$
$$= (l/\pi)^3 d^3k$$

However, only the 1st octant of number space is physically meaningful

$$dn = (l/2\pi)^3 d^3k$$

Finally, photons have two possible polarizations

$$dn = 2\frac{V}{(2\pi)^3}d^3k \text{ and } k = \frac{\omega}{c}$$
$$= \frac{2}{8\pi^3}V4\pi\left(\frac{\omega}{c}\right)^2d\left(\frac{\omega}{c}\right)$$
$$= V\omega^2\pi^2\frac{d\omega}{c^3} = \frac{-\omega^2d\omega}{\pi^2c^3}$$

### 54. (B)

By inspection or recollection,

$$\lambda_{\max}T = \frac{hc}{4.965k} = 0.002898$$

# 55. (C)

The threshold wavelength allows us to determine the work function  $\phi$  of the metal:

$$\phi = hv_0 = hc/\lambda_0$$
  
= 1.24 × 10<sup>-6</sup>/230 × 10<sup>-9</sup> = 5.39 eV

The incident light has energy

$$hv = hc/\lambda = 1.24 \times 10^{-6}/1500 \times 10^{-10} \text{ m}$$
  
= 8.27 eV

Hence, the kinetic energy of the photoelectrons is

$$K = hv = \phi$$
$$= 8.27 - 5.39 = 2.88 \text{ eV}$$

Finally,  $eV_0 = K \Rightarrow V_0 = 2.88$  Volts

# 56. (A)

In the Compton effect, photons scatter from electrons.



Final

Initial

The given wavelength is:

$$\lambda = 1.50 \times 10^{-8} \,\mathrm{m}$$

then

$$\Delta \lambda = 2\lambda_c \sin^{2\phi}/2$$
 is the Compton shift

where the Compton wavelength

$$\lambda_c = h/m_e c = (6.626 \times 10^{-27})/(9.109 \times 10^{-28})(3 \times 10^{10})$$
$$= 2.43 \times 10^{-10} \text{ cm} = 0.0243 \times 10^{-8} \text{ m}$$
$$\Delta \lambda = 2(0.0243 \times 10^{-8}) \sin^2 90^{\circ}/2 = 0.0243 \times 10^{-8} \text{ m}$$

thus and

 $\lambda' = \lambda + \Delta \lambda = 1.5243 \times 10^{-8} \,\mathrm{m}$ 

Finally, the electron kinetic energy is

$$K = hc/\lambda - hc/\lambda' = 12,400(1/1.50 - 1/1.5243)$$
  
= 131.8 eV

57. (E)

The given potential energy is

$$V(x) = \begin{cases} \infty & x \le 0\\ cx & x > 0 \end{cases}$$
$$E = p^2/2m + cx$$

Using the uncertainty principle:

$$\Delta p \ \Delta x = \mathbf{h}$$

or

$$px = \mathbf{h}$$

$$p = \mathbf{h}/x$$

$$E = (\mathbf{h}/x)^2/2m + cx$$

$$= \mathbf{h}^2/2mx^2 + cx$$

Now we look for the minimum energy value:

$$dE/dx = h^2/mx^3 + c = 0$$
  

$$\Rightarrow x_0 = (h^2/cm)^{1/3}$$
  

$$E_0 = (h^2/2m)(cm/h^2)^{2/3} + c(h^2/cm)^{1/3}$$
  

$$= (h_c/2\sqrt{2m})^{2/3} + (h^2c^2/2m)^{1/3}$$

### 58. (A)

The transmission probability is

$$T = e^{-2kl}$$
$$k = \sqrt{(2m/\hbar^2)(V_0 - E)}$$

Note that

where

$$V_0 = mgh = (2)(980)(20) = 39,200 \text{ erg}$$
  
Hence,  
$$k = \sqrt{[2(2)/(1.054 \times 10^{-27})^2](39,200 - 10,000)}$$
$$k = 3.24 \times 10^{29}$$

Finally,

$$T = e^{-2(3.24 \times 10^{29})(2)}$$
  
=  $e^{-1.30 \times 10^{30}}$   
=  $10^{-1.30 \times 10^{30} \log e}$   
=  $10^{-5.63 \times 10^{29}}$ 

which is very, very small.

59. (B)

 $U = 3/2 NkT \rightarrow U$  is proportional to T

If 
$$T \to \frac{1}{4} T, U \to \frac{1}{4} U$$

Also,  $3/2 \ kT = \frac{1}{2} \ mv_{\rm rms}^2 \rightarrow v_{\rm rms}$  is proportional to  $\sqrt{T}$ . Therefore, if  $T \rightarrow \frac{1}{4} T$ , then  $v_{\rm rms} \rightarrow \frac{1}{2} v_{\rm rms}$ 

# 60. (D)

The transmission probability

 $T = e^{-2kl}$ 

where

$$K = \frac{\sqrt{2m(V_0 - E)}}{\hbar}$$

$$l = 5 \times 10^{-10} \, m, \, m_e = 9.10 \times 10^{-31} \, \text{kh}$$

$$V_0 = 5 \, \text{eV}, \, E = 1 \, \text{eV}, \, \hbar = 1.054 \times 10^{-34} \, \text{J} \cdot \text{s}$$

$$K = \frac{\sqrt{(2)(9.1 \times 10^{-31})(4)(1.6 \times 10^{-19} \, \text{J/eV})}}{1.054 \times 10^{-34}} = 1 \times 10^{10} m^{-1}$$

$$T = e^{-2(1 \times 10^{10})(5 \times 10^{-10})} = 4.5 \times 10^{-5}$$

### 61. (E)

The two points divide the circular wire into the upper and lower sections that become resistors in parallel. The lower resistor is  $R_1 = R\theta/(2\pi)$  and the upper resistor is  $R_2 = R [1 - \theta/(2\pi)]$ . Using the formula for resistors in parallel,

$$\begin{split} R &= (R_1 \times R_2)/(R_1 + R_2) = \{ R\theta/(2\pi) \times R \left[ 1 - \theta/(2\pi) \right] \} / \{ R\theta/(2\pi) \\ &+ R \left[ 1 - \theta/(2\pi) \right] \} \\ &= \left[ R \left( \theta/(2\pi) \right) \left[ 1 - \theta/(2\pi) \right] \end{split}$$

#### 62. (A)

The Schrödinger equation for the two particle system is

$$\left(-\frac{\mathbf{h}^2}{2m}(\nabla_1^2 + \nabla_2^2) + V_0 \delta^3(r_1 - r_2)\right) \Psi(r_1, r_2) = E \Psi(r_1, r_2)$$

For

$$V_0 = 0,$$
  

$$W_0 = (hk_1)^2 / 2m + (hk_2)^2 / 2m$$
  

$$= (h^2 / 2m)(3 + 3)\pi^2 / a^2$$
  

$$= 3h^2 \pi^2 / ma^2$$

where we have used the fact that

$$\mathbf{k}_1 = \pi/a(n_x\mathbf{x} + n_y\mathbf{y} + n_z\mathbf{z})$$
 and  $n_x = \mathbf{h}_y = n_z = 1$ 

for the ground state. (Similarly for  $\mathbf{k}_{2}$ .)

$$W_1 = \langle 0|V|0 \rangle$$
  
=  $(2/a)^6 \int V_0 \delta^3(r_1 - r_2) \sin^2(\pi/ax_1) \dots \sin^2(\pi/az_2) d^3r_1, d^3r_2$   
=  $(2/a)^6 V_0 \int \sin^4(\pi/ax_1) \sin^4(\pi/ay_1) \sin^4(\pi/az_1) \delta^3r_1$ 

where we have used the fact that

$$\begin{split} \psi_0 &= (2_a)^3 \sin(\pi_a x_1) \sin(\pi_a x_2) \sin(\pi_a y_1) \sin(\pi_a y_2) \\ &\quad \sin(\pi_a z_1) \sin(\pi_a z_2) \\ W_1 &= (2_a)^6 V_0 (\int \sin^4(\pi_a x_1) dx_1)^3 \\ &= (2_a)^6 V_0 (3_8 \pi a_\pi)^3 = (3_{2a})^3 V_0 \end{split}$$

 $E_0 = W_0 + W_1 = 3h^2 \pi^2 / ma^2 + (3/_{2a})^3 V_0$ 

Finally,

Thus,

# 63. (B)

The s shell has l = 0 and the p shell has l = 1. Hence,  $m_l = -1, 0, 1$ . The possible states taking each electron separately are then

-1	ſ	$\downarrow$	$\downarrow$	$\uparrow$	$\uparrow$	<b>↑</b>	$\downarrow$	$\downarrow$	
0	$\uparrow$	$\downarrow$	$\uparrow$	$\downarrow$	$\uparrow$	$\downarrow$	$\uparrow$	$\downarrow$	8 states
1	↑	$\downarrow$	$\uparrow$	$\uparrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\uparrow$	

where the first electron is on the first line, the second electron on the second line, and the third electron on the third line.

Now we count states with two electrons in one state and the other electron separate.

-1					$\uparrow$	$\downarrow$	↑↓	↑↓	Ŷ	$\downarrow$	↑↓	$\uparrow\downarrow$	
0	$\uparrow$	$\downarrow$	$\uparrow \downarrow$	$\uparrow \downarrow$					$\uparrow\downarrow$	$\uparrow \downarrow$	$\uparrow$	$\downarrow$	12 states
1	$\uparrow\downarrow$	↑↓	$\uparrow$	$\downarrow$	↑↓	$\uparrow\downarrow$	↑	$\downarrow$	_				

Hence, there are a total of 20 states, as one expects from the binomial coefficient

$$\begin{pmatrix} 6\\ 3 \end{pmatrix} = \frac{6!}{3!3!} = 20$$

64. (C)

The given potential energy is

$$V(r) = -e^{2/r} + ke^{-r/r_{0}}, V(r^{*}) = -3.60 \text{ eV}$$

$$\frac{dU}{dr} = e^{2}/r^{2} - \frac{k}{r_{0}}e^{-r/r_{0}} = 0$$
Na
C

when

$$e^2 = 1.44 M_e V - fm = 14.4 \times 10^{-10} eV \cdot m$$

 $r = r^* = 2.5 \times 10^{-10} \,\mathrm{m}$ 

Plugging into the two equations, we get

$$\begin{cases} 14.4/(2.5)^2 - k/r_0 e^{-2.5/r_0} = 0\\ -14.4/2.5 + k e^{-2.5/r_0} = -3.60 \end{cases}$$

Adding we get

$$2.30r_0 - 5.76 = -3.60$$
 and so  $r_0 = 0.94 \times 10^{-8}$  m

Substituting in the second equation,

$$-5.76 + ke^{-2.5/.94} = -3.6$$
  
 $k = 30.94 \text{ eV}$ 

65. (A)

Thus,

The Lennard-Jones potential is

\_\_\_\_\_ m

$$V(r) = \frac{A}{r^{12}} - \frac{B}{r^6}$$

We wish to find the minimum.

$$\frac{d}{dr}V(r) = \frac{-12A}{r^{13}} + \frac{6B}{r^7} = 0 \Rightarrow r_0 = \sqrt[6]{2A/B}$$

Using a Taylor expansion, we have

$$V(r) \approx V(r_0) + \frac{d}{dr}U(r) \left| \begin{array}{c} r = r_0 \end{array} + \frac{1}{2}\frac{d^2}{dr^2}U(r) \right| r = r_0 \quad (r - r_0)^2$$
$$V(r) = \frac{A}{r_0^{12}} - \frac{B}{r_0^{12}} + \frac{1}{2}\left(\frac{156A}{r_0^{14}} - \frac{42B}{r_0^8}\right)(r - r_0)^2$$

The constant may be eliminated by redefining the energy zero.

$$V(r) = \frac{1}{2}k(r - r_0)^2$$

The angular frequency and reduced mass are

$$\omega = \sqrt{k/\mu}$$
 and  $\mu = m/2$ 

so

$$\omega = \sqrt{2/m} \sqrt{\frac{156A}{r_0^{14}} - \frac{42B}{r_0^8}}$$

$$\omega = \sqrt{\frac{312A}{mr_0^{14}} - \frac{84B}{mr_0^8}}$$

66. (A)

We are given that

$$m = 6 \text{ kg}$$
 and  $v_0 = 1 \text{ Hz}$ 

According to Newton's second law,

$$F = -kx - bx' = mx''$$

The differential equation is then

$$x'' = \beta x' + \omega_0^2 x = 0,$$

where

$$\omega_0 = \sqrt{k/m}$$
 and  $\beta = b/m$ 

The solution is

$$x = ce^{-\beta t/2} \cos \omega t$$

We are also given that

 $x_0 = x(t = 0) = c = 0.25 m$  $x_{10} = \frac{1}{2}c = ce^{-5\beta}$  $e^{5\beta}=2,\,5\beta=\ln 2$  $b = \beta m = \frac{1}{5} m \ln 2$ = 0.83 kg/sec

Also, 
$$\omega = \sqrt{\omega_0^2 - \beta^2/4} = \sqrt{4\pi^2 - 0.139^2/4} = 6.283 \text{ rad/s}$$

thus

67. (D)



From Newton's second law, we obtain two equations

$$mx''_{1} = -mg\frac{x_{1}}{l} + k(x_{2} - x_{1})$$
$$mx''_{2} = -mg\frac{x_{2}}{l} + k(x_{1} - x_{2})$$

Assuming sinusoidal solutions, we get

$$x_1'' = -\omega^2 x_1$$
 and  $x_2'' = -\omega^2 x_2$ 

The coupled equations then become

$$\begin{cases} m\omega^2 x_1 - mg\frac{x_1}{l} + k(x_2 - x_1) = 0\\ m\omega^2 x_2 - mg\frac{x_2}{l} + k(x_2 - x_1) = 0 \end{cases}$$
$$(\omega^2 - g/l)(x_2 - x_1) - 2k/m(x_2 - x_1) = 0 \end{cases}$$

or

Combining the equations there are two solutions, the symmetric mode

$$\omega_+ = \sqrt{g/l}$$

and the antisymmetric mode

$$\omega_{-}=\sqrt{g/l+2k/m}$$

68. (A)

The refractive index is given by

$$n=\sqrt{1-\omega_p^2/\omega^2}$$

The wave number is

$$k = \frac{\omega}{v_p} = \frac{\omega n}{c} \text{ since } n = c/v_p$$
$$= \frac{\omega}{c} \sqrt{1 - \omega_p^2 / \omega^2}$$
The phase velocity is easy

$$v_p - \frac{\omega}{K} = c \left/ \sqrt{1 - \omega_p^2 / \omega^2} = \sqrt{2}c \right.$$

The group velocity requires differentiation

$$v_g = \frac{d\omega}{dk} = 1 \left/ \left( \frac{dk}{d\omega} \right) \right.$$
$$= 1 \left/ \frac{d}{dw} \left( \frac{\omega}{c} \sqrt{1 - \omega_p^2 \omega^2} \right) \right.$$
$$= 1 \left/ \left[ \frac{1}{c} \sqrt{1 - \omega_p^2 \omega^2} + \frac{\omega}{c} (1 - \omega_p^2 / \omega^2)^{-1/2} \frac{\omega_p^2}{\omega^3} \right] \right.$$
$$= c \left/ \left( \sqrt{1/2} + \frac{1}{2} \sqrt{2} \right) = c / \sqrt{2}$$

69. (C)

The observable is

$$f = c\sqrt{x/y}$$

and the standard error propagation formula is

$$\Delta f = \sqrt{\left(\frac{\partial f}{\partial x}\Delta x\right)^2 + \left(\frac{\partial f}{\partial y}\Delta y\right)^2}$$
$$= \sqrt{\left(\frac{c}{\sqrt{y}}\frac{1}{2\sqrt{x}}\Delta x\right)^2 + \left(c\sqrt{x}\frac{1}{2}\frac{1}{\sqrt{y^3}}\Delta y\right)^2}$$
$$= \sqrt{\left(c\sqrt{x/y}\right)^2 \left[\left(\frac{\Delta x}{2x}\right)^2 + \left(\frac{\Delta y}{2y}\right)^2\right]}$$
$$= \frac{1}{2}f\sqrt{\left(\frac{\Delta x}{x}\right)^2 + \left(\frac{\Delta y}{y}\right)^2}$$

# 70. (D)

By similar triangles

$$\frac{E_{\perp}}{E_{\parallel}} = \frac{v_{\perp}t}{c\Delta t}$$
$$= \frac{a_{\perp}\Delta t/t}{c\Delta t}$$

 $E_{\perp} = E_{\parallel} \frac{a_{\perp} r}{c^2}$ 

Thus,

z $u_{\perp}t$  $u_{\parallel}t$  $u_{\parallel}t$ 

since r = ct or t = r/c. But

$$E_{\parallel} = \frac{q}{4\pi\varepsilon_0 r^2}$$

is just the radial Coulomb field. Hence,

$$E_{\perp} = \frac{qa_{\perp}}{4\pi\varepsilon_0 rc^2} = \frac{qa\sin\theta}{4\pi\varepsilon_0 rc^2}$$

Also,

 $B = r \times E_{\perp}/c.$ 

 $|S| = |E \times B|/\mu_0$ 

Hence,

$$= q^2 a^2 \sin^2 \theta / 16\pi^2 \varepsilon_0 r^2 c^3$$

is the magnitude of the Poynting flux.

# 71. **(B)**

The scattering cross section is

$$\sigma_R = \sigma_T \left(\frac{\lambda_0}{\lambda}\right)^4$$
$$\sigma_T = \frac{8\pi}{3} \left(\frac{e^2}{4\pi\epsilon_0 m_e c^2}\right)^3$$

where

is the Thomson cross section

$$\sigma_T = 6.65 \times 10^{-29} \, m^2$$

The reduction in intensity is

$$F = 1 - e^{-\rho_l \sigma_R}$$
  
= 1 - exp(-(1.68 × 10<sup>29</sup>)(6.65 × 10<sup>-29</sup>)(750/4500)<sup>4</sup>)

= 0.0086= 0.86%

72. (C)

In the rest frame of the charge  $E = q/4\pi\varepsilon_0 r^2$ . In the lab frame where the charge is moving

$$E' = \frac{q}{4\pi\varepsilon_0 r'^2} \frac{1-\beta^2}{(1-\beta^2\sin^2\theta')^{3/2}}$$
$$= \frac{q}{4\pi\varepsilon_0 r'^2} \frac{1-\beta^2}{(1-\beta^2)^{3/2}} \text{ at } \theta' = 90^\circ$$

We are given that E' = 2E, thus

$$\frac{1-\beta^2}{(1-\beta^2)^{3/2}} = 2, \ \frac{1}{1-\beta^2} = 4$$
$$\beta = \sqrt{1-\frac{1}{4}} = \frac{\sqrt{3}}{2}, \ v = \frac{\sqrt{3}}{2}c$$

# 73. (A)

The Maxwell-Boltzmann probability density for speed is

$$p(v) = 4\pi c v^2 e^{-mv^2/2kT}$$

$$\frac{dp}{dv} = 4\pi c (2v e^{-mv^2/2kT} - v^2 \frac{2mv}{2kT} e^{-mv^2/2kT}) = 0$$

$$2v = \frac{m}{kT} v^3$$

$$v = \sqrt{2kT/m}$$

Thus,

is the most probable speed.

74. (A)

The typical magnitude of p is

$$p = mv = \sqrt{2mkT}$$

using the most probable speed for v. This corresponds to

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mkT}}$$

The classical description is valid for  $s >> \lambda$  where s is the typical separation between particles. Now

$$\rho = \frac{N}{V} \Rightarrow s^3 = \frac{1}{\rho}$$

taking N = 1 and  $V = s^3$ .

Hence,

$$\left(\frac{1}{\rho}\right)^{1/3} >> \frac{h}{\sqrt{2mkT}}$$
$$\frac{h\rho^{1/3}}{\sqrt{2mkT}} << 1$$

The classical description is thus valid for low density and/or high temperature.

# 75. (D)

$$2L = m\lambda \rightarrow 2(0.12 \times 10^{-3}) = 400\lambda \rightarrow \lambda = 6 \times 10^{-7} \text{ m} = 600 \text{ nm}$$

#### 76. (A)

Normal force F = Apparent weight

At the top:  $\text{mg} - F_{\text{top}} = m\omega^2 r \rightarrow \text{mg} - 0.95 \text{ mg} = m\omega^2 r \rightarrow m\omega^2 r = 0.05 \text{ mg}$ 

At the bottom:  $F_{bottom} - mg = m\omega^2 r \rightarrow F_{bottom} - mg = 0.05 \text{ mg} \rightarrow F_{bottom} = 1.05 \text{ mg}$ , which is 5% more than mg

# 77. (C)

The given wave form is

$$f(t) = \begin{cases} a & 0 < t < t/2 \\ -a & -t/2 < 0 < t \end{cases}$$

The square wave is an odd function  $\Rightarrow A_m = 0$ 

$$B_m = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin(mx) \, dx, \, x = \omega t$$
$$= \frac{\omega}{\pi} \int_{-T/2}^{T/2} f(t) \sin(m \, \omega t) dt$$
$$= \frac{\omega}{\pi} 2 \int_{0}^{T/2} a \, \sin(m \, \omega t) dt$$
$$= \frac{2a}{m \pi} (1 - \cos m\pi) = \frac{4a}{\pi}, \, 0, \, \frac{4a}{3\pi}, \, 0, \dots$$

 $f(t) = \frac{4a}{\pi} \left( \sin(\omega t) + \frac{1}{3} \sin(3\omega t) + \dots \right)$ 

....

78. (C)

Hence,

The energy of interaction is

$$E = -\mu \cdot \mathbf{B} \Rightarrow E_{+} = -\mu_{0}B, E_{-} = \mu_{0}B$$

0 D

Now we must find the probabilities

$$p_{+} = Ce^{-\beta E_{+}} = Ce^{\beta \mu_{0}B}, \beta = 1/kT$$

$$p_{-} = Ce^{-\beta E_{-}} = Ce^{-\beta \mu_{0}B}$$

$$p_{+} + p_{-} = 1 \Rightarrow C = 1/(e^{\beta \mu_{0}B} + e^{-\beta \mu_{0}B})$$

$$<\mu > = p_{+}\mu_{0} + p_{-}(-\mu_{0})$$

$$= (e^{\beta \mu_{0}B} - e^{-\beta \mu_{0}B})\mu_{0}/(e^{\beta \mu_{0}B} + e^{-\beta \mu_{0}B})$$

$$= \mu_{0} \tanh(\mu_{0}B/kT)$$

The magnetization is

$$m = N < \mu > = N\mu_0 \tanh(\mu_0 B/kT)$$

79. (A)

We are asked to evaluate

$$\int_0^\infty \frac{z^3}{e^z - 1} dz$$

Rewrite the integral as

$$= \int z^{3} \frac{e^{-z}}{1 - e^{-z}} dz$$
$$= \int z^{3} \sum_{n=1}^{\infty} e^{-nz} dz$$
$$= \sum_{n=1}^{\infty} \int_{0}^{\infty} z^{4-1} e^{-nz} dz$$
$$= \sum_{n=1}^{\infty} \frac{\Gamma(4)}{n^{4}}$$
$$= \sum_{n=1}^{\infty} \frac{3!}{n^{4}} = \frac{\pi^{4}}{90} \times 6 = \frac{\pi^{4}}{15}$$

80. (B)

By Dalton's Law

$$p = p_1 + p_2$$
$$= (n_1 + n_2)\frac{RT}{V}$$

Using the ideal gas law

$$= \left(\frac{m_1}{A} + \frac{m_2}{2A}\right)\frac{RT}{V}$$

Hence,

$$pV = \left(m_1 + \frac{1}{2}(m - m_1)\right) RT/A$$
$$= \frac{1}{2}m\left(1 + \frac{m_1}{m}\right) RT/A$$
$$= \frac{1}{2A}m(1 + \delta)RT$$

Thus each component acts as an ideal gas, but the mixture does not!

# 81. (A)

The coefficient of cubical expansion

$$\beta = \frac{1}{V} \left( \frac{\partial V}{\partial T} \right)_P = -\frac{1}{V} \frac{(\partial P / \partial T)_V}{(\partial P / \partial V)_T}$$

Use van der Waals equation of state

$$\left(p+\frac{a}{V^2}\right)(V-b)=RT$$

 $p = \frac{RT}{V-b} - \frac{a}{V^2}$ 

to get

Then 
$$\left(\frac{\partial P}{\partial T}\right)_V = \frac{R}{V-b}, \ \left(\frac{\partial P}{\partial V}\right)_T = \frac{-RT}{(V-b)^2} + \frac{2a}{V^3}$$

Hence, 
$$\beta = -\frac{1}{V} \frac{R/(V-b)}{-RT/(V-b)^2 + 2a/V^3} = \frac{RV^2(V-b)}{RTV^3 - 2a(V-b)^2}$$

82. (B)

The mean free path is

$$\lambda = \frac{1}{\sqrt{2}n\pi(2a)^2}$$

Where the factor  $\sqrt{2}$  comes from the motion of the molecules. Now

$$L = \sqrt{N}\lambda$$

for a random walk. So

$$N = L^{2}/\lambda^{2}$$
  
=  $L^{2}2n^{2}\pi^{2}16a^{4}$   
=  $32\pi^{2}n^{2}a^{4}L^{2}$   
=  $32\pi^{2}(2.69 \times 10^{19})^{2}(10^{-8}\text{cm})^{4}(10^{4})^{2}$   
=  $2.28 \times 10^{17} \approx 10^{17}$ 

83. (C)

We are given that

Now

$$E = \frac{p^2}{2m} + bx^4$$
$$< KE > = \left\langle \frac{p^2}{2m} \right\rangle = \frac{1}{2}kT$$

by the equipartition theorem. Also,

$$< PE > = -\frac{d}{d\beta} \ln \int_{-\infty}^{\infty} e^{-\beta bx^4} dx,$$

since

$$y^4 = \beta x^4, y = \beta^{1/4} x$$
, and  $dy = \beta^{1/4} dx$ 

$$\langle PE \rangle = -\frac{d}{d\beta} \ln \beta^{1/4} \int_{-\infty}^{\infty} e^{-by^4} dy$$
$$= -\frac{d}{d\beta} \left( -\frac{1}{4} \ln \beta + \ln \int (\ldots) dy \right) = \frac{1}{4}\beta = \frac{kY}{4}$$
$$\langle E \rangle = \frac{1}{2}kT + \frac{1}{4}kT = \frac{3}{4}kT$$

Hence,

# 84. (D)

The gas is thermally insulated

 $\Rightarrow dQ = 0$ 

and hence the expansion is adiabatic.

$$W = -\int_{V_0}^{V_f} p \, dV = -\int_{V_0}^{V_f} k^{-\gamma} dV \text{ since } pV^{\gamma} = k$$
$$= \frac{k}{-\gamma+1} V^{-\gamma+1} \begin{vmatrix} V_0 \\ V_f \end{vmatrix} = \frac{k}{\gamma-1} \left( \frac{V_f}{V_f^{\gamma}} - \frac{V_0}{V_0^{\gamma}} \right)$$
$$= \frac{1}{\gamma-1} (p_f V_f - p_0 V_0)$$
$$\gamma - 1 = \frac{C_p}{C_V} - 1 = \frac{C_p - C_V}{C_V} = \frac{R}{C_V}$$

 $W = \frac{c_V}{R} (p_f V_f - p_0 V_0)$ 

85. (E)

Now

$$\begin{array}{c}
 m & m & m \\
 \hline
 k & k & k & k
\end{array}$$

$$E_a = n_a \hbar a \omega_0$$
$$E_b = n_b \hbar b \omega_0$$
$$E_c = n_c \hbar c \omega_0$$

combine to give total energy

$$E = E_a + E_b + E_c$$

Hence, the partition function is

$$Z = \Sigma e^{-E\beta} = \Sigma e^{-E_a\beta} \Sigma e^{-E_b\beta} \Sigma e^{-E_c\beta}$$
$$= \frac{1}{1 - e^{-\hbar\omega_0 a\beta}} \frac{1}{1 - e^{-\hbar\omega_0 b\beta}} \frac{1}{1 - e^{-\hbar\omega_0 c\beta}}$$

This gives rise to a sum of Planck distributions with average energy

$$<\!E>\!=\hbar\omega_0\left(\frac{a}{e^{\hbar\omega_0a\beta}-1}+\frac{b}{e^{\hbar\omega_0b\beta}-1}+\frac{c}{e^{\hbar\omega_0c\beta}-1}\right)$$

86. (E)

For the hydrogen atom

$$E_n = -13.6 \text{ eV} \frac{1}{n^2}$$

The  $K_{\alpha}$  x-ray energy has energy

$$E_{\alpha} = -13.6 \text{ eV}\left(\frac{1}{2^2} - \frac{1}{1^2}\right) = 10.2 \text{ eV}$$

The initial energy of an *l*-shell electron is

$$E = -13.6 \text{ eV} \frac{1}{2^2} = -3.4 \text{ eV}$$

After absorbing the photon and escaping, the kinetic energy of the electron is

$$K = 10.2 - 3.4 = 6.8 \text{ eV}$$

#### 87. (D)

Thus,

This is a mode counting problem.



But since only the first quadrant in number space is physically meaningful

$$dn = \left(\frac{l}{2\pi}\right)^2 2\pi k \, dk$$

355

Finally the photon has two possible polarizations so that

$$dn = \frac{A}{\pi}k \, dk$$

88. (C)

In the Compton effect

$$\Delta\lambda = \lambda' - \lambda = 2\lambda_c \sin^2 \frac{\phi}{2}$$

For  $\phi = \pi$ 

$$\lambda' = \lambda + 2h/m_{\rho}c$$

High incident photon energy  $E = hc/\lambda$  means small wavelength  $\lambda$ . Nevertheless,

 $\lambda' \geq rac{2h}{m_e c}$  $E' = rac{hc}{\lambda'} \leq rac{1}{2}m_e c^2$ 

and

89. (B)

In the Thomson experiment, the electric force balances the magnetic force

$$eE = evB$$
  
 $v = E/B$ 

Thus,

In the absence of *B*, the deflection is

$$s = \frac{1}{2}at^{2}$$
$$= \frac{1}{2}\frac{eE}{m}\left(\frac{l}{V}\right)^{2}$$
$$= \frac{1}{2}\frac{eE}{m}\frac{l^{2}B^{2}}{E^{2}} = \frac{1}{2}\frac{e}{m}\frac{l^{2}B^{2}}{V/d}$$

Hence,

$$\frac{e}{m} = 2(1.25 \times 10^{-3})(50/1.5 \times 10^{-2})/(5 \times 10^{-2})^2(1.2 \times 10^{-4})^2$$

$$= 2.31 \times 10^{11}$$
 coul/kg

# 90. (A)

$$^{2}_{1}d + ^{208}_{82}Pb$$

is the reaction. We are given that K = 15 MeV.



In a head on collision, the KE is transformed into PE at the distance of closest approach.

91. (C)

$$V_n = \int dx_1 \int dx_2 \dots \int dx_n = C_n R^n$$

where the integrals are all subject to the constraint that

$$x_1^2 + x_2^2 + \ldots + x_n^2 < R^2$$

where R is the radius of the n-dimensional sphere. Now consider a different integral

$$\int dx_1 \dots \int dx_n e^{-(x_{n^2} + \dots + x^n)} = \left( \int dx \ e^{-x^2} \right)^2 = (\sqrt{\pi})^n$$
$$\int dV_n e^{-R^2} = \pi^{n/2}$$
$$\int nC_n R^{n-1} e^{-R^2} \ dR = \pi^{n/2}$$

Make a substitution  $R^2 = t$  so that

$$dR = \frac{dt}{2\sqrt{t}}$$
$$\frac{n}{2}C_n \int_0^\infty t^{n/2-1} e^{-t} dt = \pi^{n/2}$$
$$\frac{n}{2}C_n \left(\frac{n}{2} - 1\right)! = \pi^{n/2}$$
$$C_n = \pi^{n/2} / \left(\frac{n}{2}\right)! R^n$$

Thus,

# 92. (D)

 $\Omega(E)$  can be thought of as the number of ways of putting *M* indistinguishable balls (the quanta) among N - 1 partitions along a line. For example,



Hence,

$$\Omega = \begin{pmatrix} M+N-1\\ \\ M \end{pmatrix} \approx \frac{(M+N)!}{M!N!}$$

The entropy is

$$S = k \ln \Omega$$
  
=  $k \ln (M + N)! - \ln M! - \ln N!$   
 $\approx (M + N) \ln (M + N) - M \ln M - N \ln N$ 

using Sterling's approximation.

#### 93. (A)

The polarization is  $\mathbf{P} = Ne\mathbf{r}$  where  $\mathbf{r}$  is the vector from negative to positive ion. The positive ions tend to be found at  $\pm a/2$  with respect to the negative ions (by symmetry). The positive ion can thus have energy

$$E_{\pm} = eEa/2$$

Thus, the polarization is

$$P_x = \frac{Ne_2^a e^{eEa/2kT} - Ne_2^a e^{-eEa/2kT}}{e^{eEa/2kT} + e^{-eEa/2kT}}$$
$$= (Ne \ a/2) \tanh(eEa/2kT)$$

#### 94. (B)

According to Newton's second law,

$$F = eE = mx''$$



with solution

$$x = \frac{1}{2} \frac{eE}{m} t^2$$
$$P(t) = \frac{e^{-t/\tau}}{\tau}$$

is the probability that a particle after surviving without collisions for a time t suffers one collision by time t + dt. The mean distance is

$$\langle x \rangle = \int_0^\infty \frac{1}{2} \frac{eE}{m} t^2 \frac{1}{\tau} e^{-t/\tau} dt$$
$$= \frac{1}{2} \frac{eE}{m\tau} \frac{\Gamma(3)}{(1/\tau)^3} = \frac{eE}{m} \tau^2$$

95. (E)

We must solve Laplace's equation

 $\nabla^2 \phi = 0$ 

Use separation of variables

$$\phi = XYZ$$

$$\frac{1}{X}\frac{d^2X}{dX^2} + \frac{1}{Y}\frac{d^2Y}{dY^2} + \frac{1}{Z}\frac{d^2Z}{dZ^2} = 0$$

$$\frac{X''}{X} = \text{constant} = -\alpha^2, \ -\alpha^2 - \beta^2 + \gamma^2 = 0$$

$$X = a_m \sin(\alpha_m x), \ \alpha_m = \frac{m\pi}{a}$$

$$Y = b_n \sin(\beta_n y), \ \beta_n = \frac{n\pi}{a}$$

$$Z = c_{nm}^{\pm}e^{\pm\gamma_n m^2}, \ \gamma_{nm} = \frac{\pi}{a}\sqrt{m^2 + n^2}$$

To get the potential of the center, we can use symmetry. If all six sides had  $\phi = V_0$ , then  $\phi_c = V_0$ . The potential is the superposition of contributions from three pairs of opposing sides. With  $\phi = V_0$  for just one pair,  $\phi_c = \frac{1}{3}V_0$ .

# 96. (D)

The Boltzmann equation is

$$\frac{\partial f}{\partial t} + \frac{\mathbf{p}}{m} \cdot \frac{\partial f}{\partial \mathbf{r}} + \mathbf{F} \cdot \frac{\partial f}{\partial \mathbf{p}}$$
$$= \int (f_1' f_2 - f f_2) \sigma v \, d\Omega \, d^3 \, p_2'$$

The scattering process is



Integrating

$$\int d^{3}p \frac{\partial f}{\partial t}m = \int d^{3}p \frac{\partial (mf)}{\partial t} = \frac{\partial}{\partial t} \int d^{3}p \, mf = \frac{\partial \rho}{\partial t}$$
$$\int d^{3}p \frac{\mathbf{p}}{m} \cdot \frac{\partial f}{\partial \rho}m = \frac{\partial}{\partial \rho} \int d^{3}p f \mathbf{p} = \frac{\partial}{\partial \mathbf{r}}(\rho \mathbf{u})$$
$$\int d^{3}p \mathbf{F} \cdot \frac{\partial f}{\partial \mathbf{p}}m = \int d^{3}p \frac{\partial}{\partial p_{i}}(F_{i}fm) = F_{i}fm \bigg|_{-\infty}^{\infty} = 0$$

Likewise, the collision integral contributes zero since the net mass does not change in the collision between two particles. Hence, we get the conservation of mass equation.

$$\frac{\partial \rho}{\partial t} + \frac{\partial (\rho \mathbf{u})}{\partial \mathbf{r}} = 0$$

97. (B)

The general potential is

$$\Psi(r,\theta,\phi) = \sum_{l,m} \left( A_{im} r^{\ell} + \frac{B_{im}}{r^{l+1}} \right) Y_{im}(\theta,\phi)$$

The boundary condition is  $\Psi(R, \theta, \phi) = B \sin \theta \sin \phi$ . Now

 $Y_{1,1} = \sin \theta \ e^{i\phi},$ 

and

$$Y_{1,-1} = \sin \theta \; e^{-i\phi}$$

Hence,

$$\Psi(r, \theta, \phi) = \begin{cases} \frac{Br}{R} \sin \theta \sin \phi & r < R \\ \frac{BR^2}{r^2} \sin \theta \sin \phi & r > R \end{cases}$$

The charge density is

$$\sigma = \varepsilon_0 \left( \frac{\partial \Psi}{\partial r} \bigg|_{\text{in}} - \frac{\partial \Psi}{\partial r} \bigg|_{\text{out}} \right)$$
$$= \varepsilon_0 B \left( \frac{1}{R} \sin \theta \sin \phi + \frac{2R^2}{R^3} \sin \theta \sin \phi \right)$$
$$= \frac{3B\varepsilon_0}{R} \sin \theta \sin \phi$$

# 98. (A)

The potential energy is transformed into kinetic energy

$$\frac{1}{2}mv^2 = qV_0$$

The centripetal force is the magnetic force

$$\frac{mv^2}{R} = qvB \Rightarrow v = q\frac{BR}{m}$$

Thus,

$$\frac{q}{m} = v^2 / 2V_0 = (qBr/m)^2 / 2V_0$$

$$q/m = 2V_0 / B^2 R^2$$

$$R = \sqrt{2V_0 m / B^2 q}$$

$$= \sqrt{2(4000)(10.0129)(1.66 \cdot 10^{-27}) / (0.5^2 \cdot 1.602 \cdot 10^{-19})}$$

$$= 0.0576 \text{ m} = 5.76 \times 10^{-2} m$$

99. (B)

The energy splitting is

$$\Delta E = 2g\mu_{N}BI$$

where g is the Lande g factor and  $\mu_N = e^{h/2m_p}$  is the nuclear magneton. Also  $I = \frac{1}{2}$  is the intrinsic proton spin. Hence,

$$\Delta E = h v_{I} = 2g \mu_{N} B I$$

where  $v_L$  is the resonant Larmor frequency

$$v_L = 2g\mu_N BI/h$$
  
=  $\frac{(5.56)(0.5050 \times 10^{-26})(0.6642)(1/2)}{(6.626 \times 10^{-34})}$   
=  $2.81 \times 10^7$  Hz  
=  $28.1$  M Hz

#### 100. (E)

Speed of wave on the string is  $V = \lambda F$ . V stays constant since it depends on T and  $\mu$  only. If  $F \to 4F$  then  $\lambda \to \frac{1}{4}\lambda$ . Thus there will be 4 times as many loops on the string since each loop equals  $\frac{1}{2}\lambda$ .

# GRE Physics Exam 4

# **Answer Sheet: Practice Exam 4**

1. A B C D E	34. A B C D E	68. A B C D E
2. A B C D E	35. A B C D E	69. A B C D E
3. A B C D E	36. A B C D E	70. A B C D E
4. $(A \otimes C \otimes D \otimes E)$	37. A B C D E	71. A B C D E
5. A B C D E	38. A B C D E	72. A B C D E
6. A B C D E	39. (A) (B) (C) (D) (E)	73. A B C D E
7. A B C D E	40. A B C D E	74. A B C D E
8. A B C D E	41. (A) (B) (C) (D) (E)	75. A B C D E
9. A B C D E	42. A B C D E	76. A B C D E
10. A B C D E	43. (A) (B) (C) (D) (E)	77. A B C D E
11. A B C D E	44. (A) (B) (C) (D) (E)	78. A B C D E
12. A B C D E	45. A B C D E	79. A B C D E
13. A B C D E	46. A B C D E	80. A B C D E
14. A B C D E	47. A B C D E	81. A B C D E
15. A B C D E	48. A B C D E	82. A B C D E
16. A B C D E	49. A B C D E	83. A B C D E
17. A B C D E	50. A B C D E	84. A B C D E
18. A B C D E	51. A B C D E	85. A B C D E
19. A B C D E	52. A B C D E	86. A B C D E
20. A B C D E	53. A B C D E	87. A B C D E
21. A B C D E	54. A B C D E	88. A B C D E
22. A B C D E	55. A B C D E	89. A B C D E
23. A B C D E	56. A B C D E	90. A B C D E
24. A B C D E	57. A B C D E	91. A B C D E
25. A B C D E	58. A B C D E	92. A B C D E
26. A B C D E	59. A B C D E	93. A B C D E
27. A B C D E	60. A B C D E	94. A B C D E
28. A B C D E	61. A B C D E	95. A B C D E
29. A B C D E	62. A B C D E	96. A B C D E
30. A B C D E	63. A B C D E	97. A B C D E
31. A B C D E	64. A B C D E	98. A B C D E
32. A B C D E	65. A B C D E	99. A B C D E
33. A B C D E	66. A B C D E	100. A B C D E
	67. A B C D E	

# **Practice Exam 4**

# Time: 170 Minutes 100 Questions

Directions: Each of the questions or incomplete statements below is followed by five answer choices or completions. Choose the best answer to each question.

- 1. Consider the motion of a relativistic particle of mass *m*, momentum *p*, and energy *E*. Find the group velocity.
  - (A)  $v_g = pc^2/E$  (B)  $v_g = E/p$  (C)  $v_g = p/mc$ (D)  $v_g = E/p$  (E)  $v_g = 0$
- 2. Each of the hydrogen atom quantum mechanical wave functions has a characteristic symmetry. In the 3D below picture where we have plotted  $\Psi\Psi^*$  (y = 0), what is the quantum state?



# **GRE PHYSICS**

- 3. Which of the following is NOT a true statement about quantum physics?
  - (A) The wave function is always a real quantity.
  - (B) The wave function represents the complete physical state.
  - (C)  $\psi, \psi'$ , and  $\psi''$  are finite, single-valued, and continuous.
  - (D) For every observable, there is a quantum mechanical operator.

(E) In one dimension that  $\int_{-\infty}^{+\infty} \psi^* \psi \, dx = 1$  is required.

4. The first excited state of the one-dimensional harmonic oscillator has eigenfunction

$$\phi(x) = Nxe^{-\alpha^2 x^2/2}$$

Find N.

- (A)  $(\alpha^{2}/\pi)^{1/4}$  (B)  $\alpha/\pi$  (C)  $2\alpha^{3}/\sqrt{\pi}$ (D)  $(\alpha^{2}/\pi)^{1/2}$  (E)  $\sqrt{2\alpha^{3}/\sqrt{\pi}}$
- 5. A variable length tube closed at one end is used to determine the speed of sound by producing standing waves with a 1000-Hz tuning fork. As the length of the tube is increased, the change in the length of the tube between the fundamental and first overtone is 0.175 m. According to this data, the speed of sound must be
  - (A)87.5 m/s(B)175 m/s(C)350 m/s(D)700 m/s(E)1,400 m/s
- 6. The dispersion relationship for deep water waves is given by  $\omega^2 = gk + ak^3$  where g and a are constants. Find the phase velocity in terms of  $\lambda$ .
  - (A)  $\sqrt{g\lambda/2\pi + 2\pi a/\lambda}$  (D)  $\sqrt{2\pi a/\lambda}$ (B)  $\sqrt{g\lambda/2\pi}$  (E)  $\sqrt{g\lambda/\pi + \pi a/\lambda}$ (C)  $\sqrt{g\lambda/2\pi + 2a/\lambda}$
- 7. An important part of experimental physics involves the use of high-vacuum technology. Which of the following is an INCORRECT statement?
  - (A) Mechanical pumps pump down to about  $10^{-3}$  torr.
  - (B) Ion pumps are generally useful as roughing pumps.
  - (C) Molecular diffusion pumps take the system to  $10^{-7}$  torr.
  - (D) Ion pumps can evacuate a chamber to  $10^{-9}$  torr.
  - (E) Several different types of pumps are usually needed to get a high vacuum.

8. The potential energy of a particle moving in one dimension is given by  $U(x) = 1/2kx^2 + 1/4bx^4$ . Determine the force.

V A

P•

° Q

Х

- (A)  $-kx bx^3$ (B)  $kx + bx^3$ (C)  $1/6kx^3 + 1/20bx^5$ (D)  $-1/6kx^3 - 1/20bx^5$ (E)  $-kx - bx^2$
- 9. A particular curve connected from points *P* to *Q* and revolved about the *x*-axis generates the surface of minimum surface energy. What is the name of the surface of revolution?
  - (A) Conic section
  - (B) Catenoid
  - (C) Portion of a sphere
  - (D) Cycloid
  - (E) Geodesic
- 10. A pendulum of length *l* is attached to the roof of an elevator near the surface of the Earth. The elevator moves upward with acceleration a = 1/2 g. Determine the linear frequency of the pendulum's vibration.



- 11. A room of dimensions h = 2.5 m by w = 5.0 m by l = 5.0 m contains  $10^{27}$  air molecules. If all of the molecules in the room statistically conglomerated in a small corner of dimensions h = 2.5 cm by w = 5.0 cm by l = 5.0 cm, then a person in the room would be unable to breathe. Calculate the logarithm  $\log_{10}$  of the probability that this will happen.
  - (A)  $\log p = 0$ (B)  $\log p = -27$ (C)  $\log p = -6 \times 10^{27}$ (D)  $\log p = -6$ (E)  $\log p = -162$
- 12. An object is projected upward near the surface of the Earth, but also subject to a resistive force -bv. Determine the time taken to reach the maximum height. Let  $\gamma \equiv b/m$  (for simplicity).
  - (A)  $v_0/g$ (B)  $2g_0/g$ (C)  $(1/\gamma) \ln(1 + v_0\gamma/g)$ (D)  $\ln(1 + v_0\gamma/g)$ (E)  $(1/\gamma) \ln(1 - v_0\gamma/g)$

- 13. What is the gravitational field of an infinite line mass of linear mass density  $\lambda$ ?
  - (A)  $-(\lambda G/r)\mathbf{r}$ (B)  $-(2\lambda G/r^2)\mathbf{r}$ (C)  $(2\lambda G/r)\mathbf{r}$ (D)  $-(\lambda G/r^2)\mathbf{r}$ (E)  $-(2\lambda G/r)\mathbf{r}$
- 14. A circular annulus a < r < b of uniform mass density  $\sigma$  is situated with center at the origin in the *yz* plane. Determine the gravitational potential at distance *x*.
  - (A)  $-2\pi\sigma G\sqrt{b^2 + x^2}$ (B)  $-2\pi\sigma G\left[\sqrt{b^2 + x^2} - \sqrt{a^2 + x^2}\right]$ (C)  $+\pi\sigma[b^2 - a^2]/x$ (D)  $-\pi\sigma[b^2 - a^2]/x$
  - (E)  $+2\pi\sigma G\left[\sqrt{b^2+x^2}-\sqrt{a^2+x^2}\right]$





- 15. A stunt plane flies a loop-the-loop circle at uniform speed *v*. The pilot experiences an apparent weight at the bottom that is twice his apparent weight at the top of the circular path. What is the radius of the path?
  - (A)  $v^2/g$  (D)  $4v^2/g$ (B)  $2v^2/g$  (E)  $5v^2/g$
  - (C)  $3v^2/g$



- (A) g/2
- (B) 2g/3
- (C) 3g/5
- (D) 3g/4
- (E) 4g/5



- 17. Which of the following is NOT a true statement about nucleons?
  - (A) Protons and neutrons are fermions.
  - (B) Even Z even N nuclei have zero total angular momentum.
  - (C) The total angular momentum is integral for nuclei with even A.
  - (D) Protons and neutrons have integer spin.
  - (E) The total angular momentum is half-integral for nuclei with odd A.

- 18. The Roche limit for the earth-moon system is the distance at which the tidal action of the earth would start to rip the moon apart. Let a = earth-moon distance (assumed variable), r = radius of moon, M = earth's mass, and m = moon's mass. Find the Roche limit a.
  - (A)  $(4m/M)^{1/3}r$
  - (B)  $(4M/m)^{1/3}r$
  - (C)  $(4M/m)^{1/3}R$
  - (D)  $(8m/m)^{1/3}R$
  - (E)  $(8M/m)^{1/3}r$



- 19. Many nuclei either in the ground state or in an excited state assume an ellipsoidal shape. Let Z be the number of protons in the nucleus, N the number of neutrons, e the accuracy, a the semi-major axis, and b the semi-minor axis of the ellipse. What is the quadrupole moment Q?
  - (A)  $1/5Ze^2a^2$
  - (B)  $1/5Ze^2b^2$
  - (C)  $3/5Ne^2a^2$
  - (D)  $2/5Ze^2a^2$
  - (E)  $3/5Ze^2b^2$



20. The meson theory of nuclear forces suggests a nucleonnucleon potential of the form

$$U(r) = V_{R}e^{-k_{R}r}/r - V_{A}e^{-k_{A}r}/r.$$

Determine the form of the repulsive part of the force, the so-called hard core.

- (A)  $(V_R e^{-k_R r}/r)[k_R + 1/r] (V_A e^{-k_A r}/r)[k_A + 1/r]$ (B)  $V_A k_A e^{-k_A r}/r$ (C)  $V_R k_R e^{-k_R r}/r$ (D)  $(V_R e^{-k_R r}/r)[k_R + 1/r]$
- (E)  $(V_R e^{-k_A r}/r)[k_A + 1/r]$
- 21. A block of mass *m* is at rest on the horizontal bed of a truck as the truck is accelerating at *a*. The coefficient of static friction between the truck and the crate is  $\mu_s$ . The frictional force acting on the crate must be
  - (A) Zero
  - (B)  $\mu_s mg$
  - (C) *ma*
  - (D)  $\mu_s ma$
  - (E)  $\mu_s mg \mu_s ma$



- 22. Which of the following is a true statement about the elementary particles?
  - (A) The photon is a stable particle with non-zero mass and spin one.
  - (B) The electron is a stable particle with mass  $0.511 \text{ MeV/}c^2$  and spin 3/2.
  - (C) The proton is a stable particle with mass  $1836 MeV/c^2$  and spin 1/2.
  - (D) The kaon has mass  $134.96 MeV/c^2$ , lifetime longer than that of the pion, spin zero, and strangeness one.

(E) J = cc

- (E) None of the above.
- 23. Hadrons consist of baryons and mesons and their structure is investigated using quantum chromo-dynamics or QCD. Which of the following is NOT a correct quark assignment?
  - (A) p = uud (D)  $K^- = us$
  - (B) n = udd
  - (C)  $\pi^+ = us$
- 24. Consider the elastic scattering of two identical atoms. Let the scattering angles in the lab frame be  $(\theta_{lab}, \phi_{lab})$  and those in the center of mass (*cm*) frame be  $(\theta, \phi)$ . If the differential cross section isotropic is the *CM* with value  $\sigma_0/4\pi$ , then what is the lab cross section?
  - (A)  $\sigma_0/4\pi$ (B)  $\sigma_0/2\pi$  • lab frame (C)  $\sigma_0/8\pi$ (D)  $\sigma_0 \cos \theta_{\rm lab}/2\pi$  • CM frame
  - (E)  $\sigma_0 \cos \theta_{\rm lab} / \pi$
- 25. Much of radioactive dating is based on the nuclear reaction

$${}^{14}_{6}\text{C} \rightarrow {}^{14}_{7}\text{N} + \text{e}^- + \overline{v}_{\text{e}}$$

which has a half-life of 5760 years. What is the mean life for this reaction?

- (A) 11,520 years(D) 5760 years(B) 7985 years(E) 3993 years
- (C) 8310 years
- 26. The laboratory operation of a laser is related to the atomic transition problem. Let  $E_2 E_1 = h\omega$  for two atomic states,  $u(\omega)$  be the radiation density,  $N_1$  be the number of atoms in state 1, and  $N_2$  that in state 2.  $B_{12}$  is the Einstein coefficient for absorption,  $B_{21}$  the transition probability for emission, and  $A_{21}$  the spontaneous emission coefficient. Find  $N_2/N_1$  for thermal quilibrium.

(A)  $B_{12}u(\omega)/A_{21}$ (B)  $A_{21}u(\omega)/B_{21}$ (C)  $B_{12}u(\omega)/B_{21}$ (D)  $B_{12}/[A_2 + B_{21}]$ (E)  $B_{12}u(\omega)/[A_{21} + B_{21}u(\omega)]$ 

- 27. Which of the following is NOT a true statement about the Raman effect? Let  $h\omega$  be the energy per incident photon.
  - (A) A quantum of monochromatic light is scattered inelastically.
  - (B) Energy  $h\omega'$  is exchanged with a molecule.
  - (C) The frequency of the scattered radiation is  $\omega'' = \omega \pm \omega'$ .
  - (D) Raman scattering occurs as a result of the induced dipole moment.
  - (E) A quantum of monochromatic light is scattered elastically.
- 28. Find the magnetic field due to the finite current carrying wire pictured at point P. (See figure.)



- 29. Figure out the total electric potential energy of a single spherical object of uniform charge density  $\rho$ , total charge Q, and radius R. Let  $k = 1/4 \pi \varepsilon_0$  as usual.
  - (A) 0
  - (B)  $kQ^2/R$
  - (C)  $1/2kQ^2/R$
  - (D)  $3/5kQ^2/R$
  - (E)  $2/3kQ^2/R$



30. A plane wave solution of Maxwell's equations in free space is

$$\mathbf{E} = \mathbf{y} E_{ov} \cos(\omega t - kx + \alpha) + \mathbf{z} E_{oz} \cos(\omega t - kx + \beta).$$

Let  $\delta = \beta - \alpha$  be the phase difference. Under what conditions do we get elliptic polarization?

- (D)  $\delta = \pm \pi/2$  and  $E_{oy} = E_{oz}$ (E)  $\delta = \pm \pi$ (A)  $\delta = \pm \pi/2$
- (B)  $\delta = 0$
- (C)  $\delta = \pm \pi/4$  and  $E_{oy} = E_{oz}$

# **GRE PHYSICS**

- 31. A toroidal substance of inner radius a, outer radius b, and magnetic permeability  $\mu$  is warpped with N turns in which current I flows. Find the magnetic field B at the center of the toroid's crosssection.
- (D)  $N\mu I/\pi a$ (A)  $N\mu_0 I/\pi(a+b)$ (B)  $N\mu_0 I/\pi$ (E)  $NI/\pi(a+b)$ (C)  $N\mu I/\pi(a+b)$ 32. Monochromatic light waves of wavelength  $\lambda$  are incident on a single slit of width d and observed on a screen a distance l away and a height y above the slit optical axis. If the density y is halfway to the first minimum, then what is the relative intensity  $I(y)/I_0$ ? For destructive interference,  $d \sin \theta = n\lambda$ . (A) 1/2 light (B) 1/*e* slit 1 screen (C)  $2/\pi$ (D)  $4/\pi^2$ (E)  $1/e^2$ 33. A point charge of magnitude q is situated a distance habove an infinite conducting xy plane, as shown. What is the charge density  $\sigma$  on top of the plane as a function of h the cylindrical radius s?

(A) 
$$-qh/2\pi/(s^2+h^2)^{3/2}$$

- (B) -q/(s+h)
- (C) +q/(s+h)
- (D)  $+qh/4\pi(s^2+h^2)^{3/2}$
- (E) Zero since the plane is conducting
- 34. An infinite conducting xy plane is maintained at zero voltage everywhere except within a circular region s < r where the voltage is  $V_0$ . Given that the voltage along the z-axis is

$$\phi(z) = \frac{V_0 z}{2\pi} \int s \, ds \, d\phi / (s^2 + z^2)^{3-2},$$

determine the electric field along this axis.

- (A)  $V_0/(r+z)$
- (B)  $V_0 r^2 / (r^2 + z^2)^{3/2}$
- (C)  $V_0/r$
- (D)  $V_0/z$
- (E)  $V_0 r^2 / z^3$





- 35. Two metal rods of identical geometrical dimensions have thermal conductivities  $k_1$  and  $k_2$  respectively. These two rods are connected end-to-end to make a composite rod. The thermal conductivity of the composite rod must be
  - (A)  $(k_1 + k_2)$ (D)  $2k_1k_2/(k_1 + k_2)$ (E)  $\sqrt{(k_1k_2)}$ (B)  $\frac{1}{2}(k_1 + k_2)$ (C)  $k_1 k_2 / (k_1 + k_2)$
- 36. The magnetic vector potential can be used to derive the magnetic dipole **m** field. What is the resultant field at distance **r**?
  - (A)  $\mu_0[3(\mathbf{m} \cdot \mathbf{r})\mathbf{r} \mathbf{m}]/4\pi r^3$
  - (B)  $\mu_0[3\mathbf{m}\cdot\mathbf{rr}-\mathbf{m}]/4\pi r^2$
  - (C)  $\mu_0[2\mathbf{m}\cdot\mathbf{rr}-\mathbf{m}]/4\pi r^3$
  - (D)  $\mu_0[3\mathbf{m}\cdot\mathbf{rr}-\mathbf{m}]/4\pi r^2$
  - (E)  $\mu_0 \mathbf{m}/4\pi r^2$





z

- 37. A particle is constrained to move on the surface of a sphere of radius R near the surface of the earth. What is the Lagrangian in cylindrical coordinates  $(\rho, \theta, z)$ ?
  - (A)  $1/2m(\rho'^2 + \rho^2\theta'^2 + z'^2) mgz$
  - (B)  $1/2m(\rho'^2 + \rho^2\theta'^2) mgz$
  - (C)  $1/2m(\rho'^2 + z'^2) mgz$
  - (D)  $1/2m(\rho^2\theta'^2 + z'^2) mgz$
  - (E)  $1/2m(\rho'^2 + \rho^2\theta'^2\sin^2\theta + z'^2) mgz$
- 38. A grounded conducting sphere is placed in a uniform electric field  $\mathbf{E} = E_0 \mathbf{z}$ . Determine the electric potential  $\Phi(r, \theta)$  given that

$$\Phi(r,\theta) = [Ar + B/r^2] \cos \theta$$

(A) 
$$-E_0 r \cos \theta$$
  
(B)  $-E_0 r [1 - (a/r)^3] \cos \theta$ 

(C) 
$$-E_0 r(a/r)^3 \cos \theta$$

- (D)  $-E_0 r [1 + (a/r)^3] \cos \theta$
- (E)  $+E_0 r(a/r)^3 ]\cos\theta$



- (A)  $\nabla \cdot \mathbf{S} + \mathbf{j} \cdot \mathbf{E} = 0$ (B)  $\nabla \cdot \mathbf{S} + \frac{\partial u}{\partial t} + \mathbf{j} \cdot \mathbf{E} = 0$
- (C)  $\nabla \cdot \mathbf{S} + \partial u / \partial t = 0$
- (D)  $\nabla \cdot \mathbf{S} + \frac{\partial u}{\partial t} + \mathbf{j} \cdot \mathbf{E} + IR = 0$ (E)  $\nabla \cdot \mathbf{S} = 0$



g

y

# (C) 350 m/s (D) 700 m/s (E) 1,400 m/s

(B) 175 m/s

- 44. A variable length tube closed at one end is used to determine the speed of sound by producing standing waves with a 1000-Hz tuning fork. As the length of the tube is increased, the change in the length of the tube between the fundamental and first overtone is 0.175 m. According to this data, the speed of sound must be (A) 87.5 m/s
- (E)  $5s^1$ (B)  $3s^1$ (C)  $3p^1$
- 43. The atomic number of aluminum is 13. Its outermost electron is in the state (A)  $2s^1$ (D)  $3d^{1}$
- (C)  $f' = n_i / [(n_i n_r)/R_1 + (n_r n_0)/R_2]$ (D)  $f' = -n_0/[(n_i - n_r)/R_1 + (n_r - n_0)/R_2]$ (E)  $f' = -4/(R_1 + R_2]$

(A)  $f' = -n_0/[(n_r - n_0)/R_1 + (n_i - n_r)/R_2]$ 

(B)  $f' = n_i / [(n_r - n_0)/R_1 + (n_i - n_r)/R_2]$ 

sits in medium  $n_0$ , the convex lens is made of material  $n_r$ , and the image is found in medium  $n_i$ . If the curvature radii are  $R_1$  and  $R_2$  are shown, then find the secondary focal length f'.

42. Consider the general thin lens problem where the object

(A) 30° (D) 47° (E) 70° (B) 60° (C) 23°

light due to passage through the prism.

41. Light is incident on a prism of apex angle  $\alpha = 60^{\circ}$  at angle of incidence  $\theta_{i1} = 30^\circ$ . The prism is made of a sub-

stance of refractive index  $n_r = 1.5$  and is surrounded by a

near vacuum. Determine the net angle of deviation of the

(D)  $0.3 \times 10^{-2} \text{ m}$ (E)  $0.1 \times 10^{-2} \text{ m}$ (A)  $2.9 \times 10^{-2} \text{ m}$ (B)  $1.4 \times 10^{-2}$  m (C)  $0.73 \times 10^{-2} \text{ m}$ 

wavelength 600 nm, what is the minimum diameter circular lens needed?

**GRE PHYSICS** 

40. A woman wishes to resolve objects one meter apart at a distance of 10,000 m. Assuming light of







# **PRACTICE EXAM 4**

- 45. An observer and a source of sound are moving in the same direction with  $\frac{1}{2}$  the speed of sound. The source is emitting sound of a frequency of 1000 Hz. The speed of sound in air is 343 m/s. The frequency heard by the observer is
  - (B) 250 Hz (C) 500 Hz  $(A) \quad 0 \text{ Hz}$ (D) 1000 Hz (E) 2000 Hz

# 46. The wavelength of sound received by the observer is

- (A) 0 m (B) 0.08575 m (C) 0.1715 m
- (D) 0.343 m (E) 0.686 m
- 47. For the shown circuit, what best describes the output voltage compared to the input voltage at very high frequencies?

  - (A)  $V_0 = V_i$  and leads by 90°. (B)  $V_0 = 1/2 V_i$  and lags by 90°.
  - (C)  $V_0^{\prime} << V_i$  and is in phase.
  - (D)  $V_0 = 1/2 V_i$  and is in phase.
  - (E)  $V_0^0 \ll V_i$  and lags by 90°.



- 48. Which of the following is NOT a correct statement about the phenomenon of optical activity?
  - (A) The vibration plane of light undergoes rotation when passed through a substance like turpentine.
  - (B) The electric field of the incident plane wave rotates about the optic axis.
  - (C) Quartz is only dextro-rotatory.
  - (D) A substance that causes clockwise rotation (looking in the source direction) is dextro-type.
  - (E) A substance that causes counterclockwise rotation is levo-type.
- 49. Consider a plane transmission diffraction grating. Let d be the distance between ruled lines, m the order number, and  $\theta$  the observation angle. Find the angular dispersion  $d\theta/d\lambda$  for incident light of wavelength  $\lambda$ .
  - (A)  $\sin \theta / \lambda$ (B)  $\cos \theta / \lambda$ (C)  $\cot \theta / \lambda$ (E) sec  $\theta/\lambda$ (D)  $\tan \theta / \lambda$
- 50. Newton's rings are observed with a plano-convex lens resting on a plane glass surface. If R is the lens radius or curvature, m is the order number, and  $\lambda$  is the incident light wavelength, then find the radii of dark interference rings r, such that  $r \ll R$ .



# **GRE PHYSICS**

- 51. Determine the speed of the photoelectrons ejected from a metal surface. The threshold wavelength is 263.8 nm and the wavelength of incident light is 160 nm.
  - (A)  $5.2 \times 10^5$  m/s
  - (B)  $2.6 \times 10^5$  m/s
  - (C)  $2.08 \times 10^6$  m/s
- 52. Electromagnetic radiation of wavelength  $6.20 \times 10^{-10}$  m is incident on a substance and backscattered at an angle of 180°. Determine the Compton energy shift of the radiation.
  - (A) 31.0 eV
  - (B) 15.5 eV
  - (C) 2.0 keV
  - (D) 4.0 keV
  - (E) 1.0 keV



(D) 49.7 m

(E) 24.8 m

(D)  $1.66 \times 10^6$  m/s

(E)  $1.04 \times 10^6$  m/s

- 53. A particle of mass *m* moves in one dimension subject only to a resistive force  $F_R = -bv$ . Let  $\gamma = b/m = 2.0 \ s^{-1}$  and the initial speed be 100 m/s. Determine the distance moved at  $t = 2.5 \ s$ .
  - (A) 50.0 m
  - (B) 25.0 m
  - (C) 12.5 m

(A) 1.5 km/s

54. Consider the motion of a rocket in free space. If the rocket starts with initial velocity 0.5 km/s and its mass decreases by a factor of two due to exhaust emitted with speed 1.0 km/s, then find the final velocity of the rocket.



- (D) 1.7 km/s (E) 1.2 km/s
- 55. Obtain the expression for the acceleration of a mass in cylindrical coordinates. (Hint, find **r** then use to get v in cylindrical coordinates.)

(B) 0.5 km/s

- (A)  $(\rho'' + \rho\theta'^2)\hat{\rho} + (\rho\theta'' 2\rho'\theta'')\hat{\theta} + z''\hat{z}$ (B)  $\rho''\hat{\rho} + \rho\theta''\hat{\theta} + z''\hat{z}$ (C)  $(\rho'' - \rho\theta'^2)\hat{\rho} + (\rho\theta'' - 2\rho'\theta')\hat{\theta} + z''\hat{z}$ (D)  $(\rho'' - \rho \theta'^2)\hat{\rho} + (\rho \theta'' + 2\rho' \theta')\hat{\theta} + z''\hat{z}$ (E)  $\rho''\hat{\rho} - \rho\theta''\hat{\theta} + z''\hat{z}$
- 56. The position of a particle is given by P = (1.0, 1.0) min standard Cartesian coordinates. What is the position of this particle with respect to a frame rotated by 30°?
  - (A) (1.00, 1.00)
  - (B) (1.37, 0.37)
  - (C) (0.37, 1.37)
  - (D) (1.37, 1.37)
  - (E) (0.37, 0.37)



57. Stirling's approximation for *N*! may be found for large *N* by using the gamma function. What is the approximation to second order?

(D)  $(N/e)^N \sqrt{2\pi N} [1 - 1/12N]$ 

(E)  $(N/e)^N \sqrt{2\pi N}$ 

- (A)  $(N/e)^N$
- (B)  $N^{N}$
- (C)  $(N/e)^N \sqrt{2\pi}$
- 58. The microcanonical ensemble theory value for the cumulative number of states (ideal gas) is

$$\Gamma(E) = \frac{\pi^{3N/2}}{N! \left(\frac{3N}{2}\right)! (2\pi\mathbf{h})^{3N}} V^N P^{3N} \text{ where } P = \sqrt{2mE}$$

Use this information to determine the entropy, i.e., the Sackur-Tetrode equation. Let  $g = m/2\pi h^2$ and T = 2E/3N.

- (A)  $S = Nk \ln [(gT)^{3/2}V/N] + 5/2 Nk$
- (B)  $S = Nk \ln [(gT)^{3/2}V/N] + 3/2 Nk$
- (C)  $S = 3/2 Nk \ln T + Nk \ln V + 5/2 Nk$
- (D)  $S = 3/2 Nk \ln T + Nk \ln V + 3/2 Nk$
- (E)  $S = 3/2 Nk \ln T Nk \ln V/N + 5/2 Nk$
- 59. Two long straight wires carry an equal amount of currents. The wires are insulated and are placed at right angles so that they intersect at the origin while the two currents point in the positive x and positive y direction, respectively, as shown in the diagram.

The direction of the net magnetic field at point P(3m, 3m) must be

- (A) toward the origin
- (B) away from the origin
- (C) in the positive z direction
- (D) in the negative z direction
- (E) undefined because it is zero



- 60. Determine the threshold kinetic energy to produce proton-antiproton pairs in positron-electron collisions. The positron KE is  $T_e$  and the target electrons are at rest.
  - (A) 1.876 GeV
  - (B) 0.938 GeV
  - (C) 1.72 TeV
  - (D) 3.44 TeV
  - (E) 0.86 TeV



- 61. Obtain the correct classical Lagrangian for a particle subject to an electric field **E** due to potential  $\phi$  and a magnetic field due to vector potential **A** such that  $\mathbf{B} = \nabla \times \mathbf{A}$ .
  - (A)  $L = 1/2 mv^2 + q\phi q\mathbf{A} \cdot \mathbf{v}$ (B)  $L = 1/2 mv^2 - \int (q\mathbf{E} + \mathbf{v} \times \mathbf{B}) \cdot d\mathbf{r}$ (C)  $L = 1/2 mv^2 - q\phi + q\mathbf{A} \cdot \mathbf{v}$ (D)  $L = 1/2 mv^2 + \int q\mathbf{E} \cdot d\mathbf{r}$ (E)  $L = 1/2 mv^2 + q\phi + q\mathbf{A} \cdot \mathbf{v}$
- 62. Find the distance of closest approach for the elastic nuclear reaction

$${}^{7}_{3}\text{Li} + {}^{208}_{82}\text{Pb}$$

Assume that only the Coulomb force is important. The Li nuclear is accelerated to a kinetic energy of 50 MeV.

Li

beam

(A) 1.12 fm
(B) 2.24 fm
(C) 3.54 fm
(C) 5.00 fm

(D) 7.08 fm

- (E) 8.20 fm
- 63. A charge q is distributed throughout a sphere of radius R with uniform charge density  $\rho$ . Given that the potential is

$$\phi = -\rho r^2/6\varepsilon_0 + A/r + B \text{ for } r < R,$$

determine the constants A and B and hence  $\phi$ .

 $\begin{array}{ll} (A) & \phi = -\rho r^2/6\varepsilon_{\rm o} + \rho R^3/3\varepsilon_{\rm o}r \\ (B) & \phi = -\rho r^2/6\varepsilon_{\rm o} \\ (C) & \phi = -(\rho/2\varepsilon_{\rm o})[2R - r^2/3] \\ (D) & \phi = O \\ (E) & \phi = (\rho/2\varepsilon_{\rm o})[R - r^2/3] \end{array}$ 



.

Pb

target

64. Two small spheres are half immersed in a substance of conductivity  $\sigma$  as shown below. Find the resistance of the two sphere system for  $r_1 = r_2 = r$ . d >> r.



- 65. The nature of the gravitational field and the electric field are alike in that both are inverse square. Use this analogy to develop a differential Gauss's law for gravitation. Let  $\rho$  be the mass/volume.
  - $\begin{array}{ll} (\mathbf{A}) & \nabla \cdot \mathbf{g} = -4\pi G \rho / \varepsilon_{\mathrm{o}} \\ (\mathbf{B}) & \nabla \cdot \mathbf{g} = -G\rho \\ (\mathbf{C}) & \nabla \cdot \mathbf{g} = -4\pi\rho \\ (\mathbf{D}) & \nabla \cdot \mathbf{g} = -\rho/G \\ (\mathbf{E}) & \nabla \cdot \mathbf{g} = -4\pi G\rho \end{array}$
- 66. The first variation

$$\delta f = \frac{\partial f}{\partial x} \delta x + \frac{\partial f}{\partial x} \delta x'$$

is used to determine the Lagrange equation. What is the second variation of the integrand of the action integral

$$A = \int_{t_1}^{t_2} f(t, x, x') dt?$$

(A) 
$$\frac{\partial^2 f}{\partial x^2} (\delta x)^2 + 2 \frac{\partial^2 f}{\partial x \partial x'} \delta x \delta x' + \frac{\partial^2 f}{\partial x'^2} (\delta x')^2$$

(B) 
$$\frac{\partial^2 f}{\partial x^2} (\delta x)^2 + \frac{\partial^2 f}{\partial x'^2} (\delta x')^2$$

(C) 
$$\frac{\partial^2 f}{\partial x^2} (\delta x)^2 + 2 \frac{\partial^2 f}{\partial x \partial x'} \delta x \delta x'$$

(D) 
$$2\frac{\partial^2 f}{\partial x \partial x'} \delta x \delta x' + \frac{\partial^2 f}{\partial x'^2} (\delta x')^2$$

(E) 
$$\frac{\partial^2 f}{\partial x^2} (\delta x)^2$$

- 67. What is the fundamental physics basis of Snell's law?
  - (A) The first postulate of special relativity
  - (B) The Pauli exclusion principle
  - (C) The uncertainty principle
  - (D) Newton's first law
  - (E) Fermat's principle of least time

68. Determine the two-dimensional Lorentz transformation matrix  $K \rightarrow K'$  using the rapidity variable  $y = i\phi$ .



- 69. Discover the first correction term for the classical kinetic energy as relativistic effects become important.
  - (A)  $1/2 mv^2 \gamma^2$ (B)  $3/4 mv^2 \beta^2$ (C)  $1/2 mv^2 \beta^2 \gamma^2$ (D)  $3/8 mv^2 \beta^2$ (E)  $1/2 mv^2 \beta^2 \gamma^2$
  - (C)  $1/2 mv^2 \beta^2$
- 70. Consider the standard two body nuclear reaction  ${}^{14}N(\alpha, p){}^{17}O$  and determine the minimum kinetic energy needed (in the center of mass frame) for the reaction to occur. Given:

 $m_p = 1.0078, m_{\alpha} = 4.0026, m_N = 14.0031, \text{ and } m_O = 16.9991 \text{ all in amu.}$ 

(A)	0.0 MeV	(D)	0.6 MeV
(B)	1.1 MeV	(E)	1.6 MeV
(C)	2.2 MeV		

71. An ideal system of N spins each of magnetic moment  $\mu_0$  is under consideration. Each spin can either point up or down only, where  $P(\uparrow) = p$  and  $P(\downarrow) = q = 1 - p$ . Find the variance of the mean magnetic moment.

(A) $Npq\mu_0^2$	(D)	$6Npq\mu_0^2$
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- (B)  $2Npq\mu_0^2$  (E)  $8Npq\mu_0^2$
- (C)  $4Npq\mu_0^2$
- 72. Cosmic ray events are detected with a Geiger counter. The events occur randomly in time, but with a well-defined mean rate r = 1 Hz = 1 event/s such that P[1 event occurs in (t, t + dt)] = rdt. What is the probability of recording five counts with the Geiger counter?
  - (A) 0.009 (B) 0.019 (C) 0.038
  - (D) 0.047 (E) 0.057
- 73. The second law of thermodynamics is intimately connected with the transfer of heat and the operation of machines. Which of the following is NOT a correct statement in light of this law?
  - (A) It is not possible to *only* transform heat into work extracted from a uniform temperature source.
  - (B) It is impossible to construct a perpetuum mobile of the second kind.
  - (C) It is impossible to *only* transfer heat from a body at high temperature to one at lower temperature.
  - (D) If heat flows by conduction from body A to body B, then it is impossible to *only* transfer heat from body B to body A.
  - (E) It is not possible to *only* transform work into heat where the body is at a uniform temperature.
- 74. In a centripetal laboratory experiment, g is computed from spinning a bob of mass m = 50 grams in a circle of radius  $r = 20 \pm 1$  cm. The rotational period is measured to be  $T = 0.638 \pm 0.016 \ s$ . The body stretches a spring a distance equivalent to the force of a weight Mgwhere M = 100 grams. If the working equation is  $g_{exp} = 4\pi^2 mr/MT^2$ , then find the uncertainty  $\Delta g$ .



(A)  $24 \text{ cm/s}^2$ 

(D)  $69 \text{ cm/s}^2$ (E) 59 cm/s<sup>2</sup>

- (B)  $39 \text{ cm/s}^2$
- (C)  $49 \text{ cm/s}^2$
- 75. For one mole of ideal gas and the Carnot cycle pictured, find  $Q_H Q_C$ .



- $\begin{array}{ll} \text{(A)} & RT_{H}\ln V_{A}/V_{B} RT_{C}\ln V_{C}/V_{D} \\ \text{(B)} & RT_{H}\ln V_{B}/V_{A} RT_{C}\ln V_{D}/V_{C} \\ \text{(C)} & RT_{H}\ln V_{A}/V_{B} RT_{C}\ln V_{D}/V_{C} \end{array}$
- (D)  $R[T_H T_C]$ (E)  $RT_H \ln V_B / V_A RT_C \ln V_C / V_D$

## **GRE PHYSICS**

- 76. Use the Boltzmann factor to study the thermodynamics of N independent particles of a spin system in a magnetic field where the energies are  $E_{+} = \pm \mu_0 B$ . Find the total average energy at temperature  $T = 1/\beta k$ .
  - (A)  $-N\mu_0 B \coth(\beta\mu_0 B)$ (B)  $+N\mu_0 B \coth(\beta\mu_0 B)$ (C)  $-N\mu_0 B \tanh(\beta\mu_0 B)$ (D)  $-N\mu_0 B \sinh(\beta\mu_0 B)$
  - (E)  $+N\mu_0 B \tanh(\beta\mu_0 B)$

77. Evaluate the microcanonical ensemble theory density of states for a harmonic oscillator with Hamiltonian  $p^2/2m$  +  $kx^2/2$ . The phase space plot is shown here. Let  $\omega = \sqrt{k/m}$ .

- (A)  $E/\hbar\omega$
- (B)  $1/\hbar\omega$
- (C)  $E^2/\hbar\omega$
- (D)  $\sqrt{E}/\hbar\omega$
- (E)  $E^{3/2}/\hbar\omega$

78. Identify the phase space orbit shown here.

- (A) Lissajous figure
- (B) Damped motion
- (C) Simple harmonic motion
- (D) Gravitational motion (g = constant)
- (E) Logistic difference equation



 $\sqrt{2E/k}$ 

В



79. Consider the spectroscopy of the hydrogen atom in Bohr theory. Determine the upper limit for the Brackett series. Given that  $R = 109,677.6 \text{ m}^{-1}$ .

(A)	121.6 nm	(B)	656.3 nm	(C)	4050 nm
(D)	1880 nm	(E)	7450 nm		

80. By looking at an empty glass along the ray path shown (angle =  $\theta$ ), one sees the lower left-hand corner. Now when the glass is filled with a clear liquid of refractive index n = 1.3, one sees the middle of the bottom of the glass again looking along angle  $\theta$ . Given the width is 5.0 cm, find the height y.

(A)	2.73 cm	(D)	4.08 cm
-----	---------	-----	---------

- (E) 6.33 cm (B) 5.46 cm
- (C) 1.35 cm





- 81. A chamber contains uniform electric and magnetic fields E and B, respectively, at right angles to each other. An ion traveling at velocity v enters the chamber at a right angle to both E and B and passes through without deflection. The velocity of the charged particle must be
  - (A) E/B (B) EB
  - (D)  $\sqrt{(E^2 B^2)}$  (E)
- 82. The surface temperature of a blackbody, such as the sun or any star, can be found from plotting the intensity  $u(\lambda)$  versus the wavelength  $\lambda$ . Generally,  $u(\lambda)$  is proportional to  $1/[\lambda^5(e^x 1)]$  where  $x = hc\beta/\lambda$ . Find the equation for *x* which one could use to find the temperature.
  - (A)  $e^{x}(5-x) = 5$ (B)  $e^{x}(4-x) = 4$ (C)  $e^{x}(3-x) = 3$
  - (D)  $e^{x}(2-x) = 2$
  - (E)  $e^{x}(1-x) = 1$
- 83. A ball bounces elastically in the vertical *y* direction. Calculate the energy levels using Bohr-Sommerfeld quantization.
  - (A) nghm
  - (B)  $[9n^2\pi^2g^2h^2m]^{2/3}$
  - (C)  $[n^2 \pi^2 g^2 h^2 m/8]^{1/3}$
  - (D)  $[9n^2\pi^2g^2h^2m/8]^{1/3}$
  - (E)  $[9n^2\pi^2g^2h^2m/8]^{2/3}$
- 84. A metal ball is dropped into a deep well with water on the very bottom. The time taken between dropping the ball from rest to hearing it splash into the water is 6.83 s. Calculate the depth of the well assuming  $v_s = 330$  m/s.
  - (A) 229 m(B) 219 m
  - (C) 201 m
  - (D) 191 m
  - (E) 131 m





(C)  $\sqrt{(E^2 + B^2)}$ 

85. Consider a quantum mechanical problem where the eigenfunction is the spherical harmonic

$$Y_{lm_l}$$
 with  $l = 1$  and  $m_1 = 0$ , i.e.,  $Y_{10}(\theta, \phi) = N \cos \theta$ 

Find the normalization constant N.

(A)	$\sqrt{3/6\pi}$	(D)	$\sqrt{3/2\pi}$
(B)	$\sqrt{3/\pi}$	(E)	$\sqrt{3/4\pi}$
(C)	$\sqrt{3/8\pi}$		

- 86. What is the degeneracy of the energy for a hydrogen-like atom with principal quantum number *n* and orbital quantum number *l*?
  - (A) n(B)  $n^2$
  - (C)  $n^3$ (D) l(l+1)
  - (D) l(l+1)(E) 2l+1
- 87. Study the ballistic pendulum problem in which a bullet of mass m = 0.005 kg hits and becomes embedded in a block of mass 1 kg. Given that the initial velocity of the bullet is 200 m/s, determine the height the ballistic pendulum rises.
  - (A)  $5.05 \times 10^{-2} \text{ m}$
  - (B)  $10.1 \times 10^{-2}$  m
  - (C)  $15.15 \times 10^{-2} \text{ m}$
  - (D)  $20.2 \times 10^{-2} \text{ m}$
  - (E)  $25.25 \times 10^{-2}$  m
- 88. Determine the speed of the Lorentz transformation in the *x*-direction for which the magnitude of velocity in frame K of a particle is

$$\mathbf{u} = \left(c/\sqrt{2}, \ c/\sqrt{2}\right)$$

and the magnitude of velocity in frame K' is seen as

$$\mathbf{u}' = \left(-c/\sqrt{2}, \ c/\sqrt{2}\right)$$







89. In the quantum theory approach to the hydrogen-like atom for l = 0 using the Schrödinger equation, find the energy eigenvalue for the ground state radial wavefunction

(A) 
$$-k^2 Z \mu e^{4/2 h^2}$$
  
(B)  $-k^2 Z e^{4/2 \mu^2}$   
(C)  $-k^2 Z^2 \mu e^{4/2 h^2}$   
(D)  $k^2 Z^2 \mu e^{4/2 h^2}$   
(E)  $k^2 Z e^{4/2 \mu \eta h}$ 

- 90. Two events occur in the space-time continuum. Event A has coordinates (1m, 2m, 3m, 0s) and event B occurs at (2m, 3m, 4m, 1/c) where the fourth coordinate gives the time in seconds. Calculate the proper distance between these two events.
  - (A) 1 m (B) 2 m (C)  $\sqrt{3}$  m (D)  $\sqrt{2}$  m (E)  $\sqrt{5}$  m
- 91. Use the Fermi gas model for electrons in a metal to determine the Fermi momentum  $k_F = p_F/h$  for electrons in a metal of density 971 kg/m<sup>3</sup> and molar mass 0.02299 kg/mol.
  - $\begin{array}{lll} (A) & 0.11 \times 10^{10} \ m^{-1} \\ (B) & 0.22 \times 10^{10} \ m^{-1} \\ (C) & 0.45 \times 10^{10} \ m^{-1} \\ (D) & 0.91 \times 10^{10} \ m^{-1} \\ (E) & 1.82 \times 10^{10} \ m^{-1} \end{array}$



92. The complete wave function for a particular state of a hydrogen-like atom is

$$\psi(r, \theta, \phi) = Nr^2 e^{-Zr/3a_0} \sin^2 \theta \ e^{2i\phi}$$

Determine the eigenvalue of the angular momentum operator  $L_{z}$ .

- (A) h (B) 2h
- (D) 4h (E) h

93. Study the coupled harmonic oscillator problem pictured

here. Find the anti-symmetric mode frequency.

K K K

т

(C) 3<del>h</del>

(A)  $\sqrt{k/m}$ 

(B) 
$$\sqrt{\kappa/m}$$

(C)  $\sqrt{(k+2\kappa)/m}$ 

(D) 
$$\sqrt{(\kappa+2k)/m}$$

(E) 
$$\sqrt{(k+\kappa)/m}$$

## **GRE PHYSICS**

- 94. Determine the electric current due to two electrons in a  $1s^2$  quantum state orbiting a central nucleus at distance 1.0 Å in a circular orbit.
  - (A) 0.0004 A (D) 0.0016 A (B) 0.0008 A (E) 0.0020 A
  - (C) 0.0012 A
- 95. Evaluate the circuit shown below to determine the anti-symmetric mode frequency. Let k = 1/LC,  $\kappa_1 = 1/L\gamma$ , and  $\kappa_2 = \beta/L$ .
  - (A)  $\sqrt{k}$
  - (B)  $\sqrt{(k+2\kappa_1)}$
  - (B)  $\sqrt{(k+2\kappa_1)}$ (C)  $\sqrt{(k+2\kappa_1)/(1+2\kappa_2)}$
  - (D)  $\sqrt{(1+2\kappa_2)}$
  - (E)  $\sqrt{(k+\kappa_1)/(1+\kappa_2)}$



96. F. London and H. London in 1935 explained a superconductor as a single wave function describing a coherent collective state. Use the London equation  $\mathbf{j} = -\mathbf{A}/\mu_0 \lambda_L^2$  to determine the equation for the B field as a function of distance into the superconductor. Hence, explain the Meissner effect.



97. Consider a reversible isothermal expansion of a photon gas. Determine the entropy S for this gas at temperature T and volume V.

(A)	$\sigma T^4 V$	(D)	$2/3 \sigma T^3 V$
(B)	$\sigma T^{3}B$	(E)	$4/3 \sigma T^3 V$
(C)	$1/3 \sigma T^3 V$		

98. The problem of the rigid diatomic rotor is of fundamental importance in physics and physical chemistry. What is the differential equation for the  $\theta$  part of the wave function  $\psi(\theta, \phi) = P(\theta)Q(\phi)$ ?

(A) 
$$\frac{1}{\sin\theta} \frac{d}{d\theta} \left( \sin\theta \frac{dP}{d\theta} \right) = 0$$
  
(B) 
$$\frac{1}{Q} \frac{d^2 Q}{d\phi^2} = -m_1^2$$
  
(C) 
$$\frac{1}{\sin\theta} \frac{d}{d\theta} \left( \sin\theta \frac{dP}{d\theta} \right) + \left[ l(l+1) - \frac{m_l^2}{\sin^2\theta} \right] P = 0$$
  
(D) 
$$\frac{d}{d\theta} \left( \sin\theta \frac{dP}{d\theta} \right) + \left[ l(l+1) - \frac{m_l^2}{\sin^2\theta} \right] P = 0$$
  
(E) 
$$\left[ l(l+1) - \frac{m_l^2}{\sin^2\theta} \right] P = 0$$

- 99. Which of the following is NOT a true statement about the general theory of relativity?
  - (A) True physical laws hold only in an inertial coordinate system.
  - (B) The gravitational equations can be applied to any coordinate system.
  - (C) The gravitational equations are structure laws describing the changes of the gravitational field.
  - (D) The universe is not Euclidean.

,

- (E) The ellipse of the planet Mercury rotates with respect to the sun.
- 100. Choose the correct dynamics equation for the relativistic rocket (of initial mass  $m_1$ ) problem. A mass element *m* is ejected with speed *v* to the left and the remaining mass of the rocket  $m_2$  moves with velocity *u* to the right.



(A)	$m_1 c^2 =$	$m\gamma_v c^2 +$	$m_2 \gamma_u c^2$	(D)	$m_1 c^2 = m_2 \gamma_u c^2$
	2	2			

- (B)  $m_1 c^2 = m \gamma_v c^2$
- (C)  $m_1 \gamma_v v = m_2 \gamma_u u$

(E)  $m\gamma_v v = m_2 \gamma_u u$ 

# **Answer Key**

1.	(A)	18. (E)	35. (D)	52. (B)	69. (D)	86. (B)
2.	(B)	19. (D)	36. (A)	53. (D)	70. (B)	87. (A)
3.	(A)	20. (D)	37. (A)	54. (E)	71. (C)	88. (B)
4.	(E)	21. (C)	38. (B)	55. (D)	72. (C)	89. (C)
5.	(C)	22. (E)	39. (B)	56. (B)	73. (E)	90. (D)
6.	(A)	23. (C)	40. (C)	57. (E)	74. (D)	91. (D)
7.	(B)	24. (E)	41. (D)	58. (A)	75. (E)	92. (B)
8.	(A)	25. (C)	42. (B)	59. (E)	76. (C)	93. (C)
9.	(B)	26. (E)	43. (C)	60. (D)	77. (B)	94. (B)
10.	(A)	27. (E)	44. (C)	61. (C)	78. (C)	95. (C)
11.	(C)	28. (C)	45. (D)	62. (D)	79. (C)	96. (B)
12.	(C)	29. (D)	46. (C)	63. (E)	80. (A)	97. (E)
13.	(E)	30. (A)	47. (D)	64. (D)	81. (A)	98. (C)
14.	(B)	31. (C)	48. (C)	65. (E)	82. (A)	99. (A)
15.	(C)	32. (D)	49. (D)	66. (A)	83. (D)	100. (E)
16.	(C)	33. (A)	50. (C)	67. (E)	84. (D)	
17.	(D)	34. (B)	51. (E)	68. (A)	85. (E)	

## **Practice Exam 4**

## **Detailed Explanations of Answers**

## 1. (A)

The total relativistic energy is found from

$$E^{2} = p^{2}c^{2} + m^{2}c^{4}$$
Hence,  

$$E = \sqrt{p^{2}c^{2} + m^{2}c^{2}}$$
Using QM,  

$$\mathbf{h}\boldsymbol{\omega} = \sqrt{(\mathbf{h} \ kc)^{2} + (m^{2}c^{2})}$$

$$\boldsymbol{\omega} = \sqrt{(kc^{2}) + (mc^{2}/\mathbf{h})^{2}}$$
By definition  

$$V_{g} = \frac{\partial \boldsymbol{\omega}}{\partial k} = \frac{1}{2}((kc)^{2} + (mc^{2}/\mathbf{h})^{2})^{-1/2}c^{2}2k$$

$$= \frac{\mathbf{h}kc^{2}}{\sqrt{p^{2}c^{2} + m^{2}c^{4}}}$$

$$= \frac{pc^{2}}{E} = \frac{mv\gamma c^{2}}{m\gamma c^{2}} = v$$

## 2. (B)

Several of the hydrogen-like atom quantum mechanical wave functions are

$$\begin{split} \Psi_{100} &= \frac{2}{\sqrt{4\pi}} \left(\frac{Z}{a_0}\right)^{3/2} e^{-Zr/a_0} \\ \Psi_{200} &= \frac{2}{\sqrt{4\pi}} \left(\frac{Z}{2a_0}\right)^{3/2} \left(1 - \frac{Zr}{2a_0}e\right) e^{-Zr/2a_0} \\ \Psi_{210} &= \sqrt{\frac{3}{4\pi}} \cos\theta \left(\frac{Zr}{2a_0}\right)^{3/2} \frac{Zr}{\sqrt{3}a_0} e^{-Zr/2a_0} \\ \Psi_{300} &= \frac{2}{\sqrt{(4\pi)}} \left(\frac{Z}{3a_0}\right)^{3/2} \left(1 - \frac{2Zr}{3a_0} + \frac{2(Zr)^2}{27a_0^2}\right) e^{-Zr/3a_0} \\ \Psi_{321} &= -\sqrt{\frac{15}{8\pi}} \sin\theta\cos\theta \ e^{i\phi} \frac{2\sqrt{2}}{27\sqrt{5}} \left(\frac{Z}{3a_0}\right)^{3/2} \left(\frac{Zr}{a_0}\right)^2 e^{-Zr/3a_0} \end{split}$$

In the 3-D picture, we have plotted

$$P(x, z) = \psi \psi^* (y = 0)$$

where  $\cos \theta = z/r$  and  $\sin \theta = x/r$ . Only  $\psi_{321}$  has the rich structure pictured.

## 3. (A)

In quantum mechanics  $\psi(x, t)$  represents the physical state of the particle/system. Usually  $\psi, \psi'$ , and  $\psi''$  are finite, single valued, and continuous. For every observable *x*, *p*, *E*, etc., there is an operator:

$$\hat{x}, \ \hat{p} = \frac{h}{i} \frac{\partial}{\partial x}, \ \hat{E} = ih \frac{\partial}{\partial t}, \ \text{etc.}$$

The expectation value of the observable is a real number:

$$\langle F \rangle = \int \phi^* \hat{F} \phi \, dx.$$

 $\psi(x, t) = \phi(x) e^{-iEt/\hbar}$  is the usual separation of variables technique.  $\phi(x)$  is normalized such that  $\int \phi^* \phi dx = 1$ . Also, the operator follows an eigenvalue equation  $\hat{F}\psi = F\psi$ .

#### 4. (E)

The quantum mechanical harmonic oscillator has energy eigenvalues.

$$E_n = \hbar \omega_0 \left( n + \frac{1}{2} \right)$$
 where  $\omega_0 = \sqrt{\frac{k}{m}}$ 

and wave functions

$$\phi(x) = \eta(x)e^{-\alpha^2 x^2/2}$$
 where  $\alpha = (mk/\hbar^2)^{1/4}$ 

is a constant and the  $\eta(x)$  are Hermite polynomials. For n = 1,

$$E_1 = \frac{3}{2}\hbar\omega_0$$
 and  $\phi_1 = Nxe^{-\alpha^2 x^2/2}$ 

By the normalization condition

$$N^2 \int_{-\infty}^{\infty} x^2 e^{-\alpha^2 x^2} dx = 1$$

Let  $t = x^2$ . Then

$$1 = N^{2} 2 \int_{0}^{\infty} t^{3/2 - 1} e^{-\alpha^{2}t} dt/2$$
$$= N^{2} \Gamma\left(\frac{3}{2}\right) / (\alpha^{2})^{3/2}$$



$$1 = N^2 \frac{1}{2}! / \alpha^3$$
$$N^2 = \alpha^3 / (\sqrt{\pi}/2)$$
$$= 2\alpha^3 / \sqrt{\pi}$$

 $N=\sqrt{2\alpha^3/\sqrt{\pi}}.$ 

Finally,

## 5. (C)

The increase in the length of the tube between the two modes is  $0.175 \text{ m} = \frac{1}{2}\lambda$ . Hence,  $\lambda = 0.350 \text{ m}$ . Now  $v = f\lambda = (1000 \text{ Hz})(0.350 \text{ m}) = 350 \text{ m/s}$ 

#### 6. (A)

The given dispersion relationship for deep water ocean waves is

$$\omega^2 = gk + ak^3$$

Solving for  $\omega$ , we get

$$\boldsymbol{\omega} = \sqrt{gk + ak^3}$$

The phase velocity is

$$v_p = \frac{\omega}{k}$$
$$= \sqrt{\frac{g}{k} + ak}$$
$$= \sqrt{\frac{g\lambda}{2\pi} + \frac{2\pi a}{\lambda}}$$

using the definition of wave number  $k = 2\pi/\lambda$ .

#### 7. **(B)**

The use of vacuum techniques is essential in many laboratory experiments. *Mechanical pumps* pump down to about  $10^{-3}$  torr (about the same as  $10^{-3}$  mm Hg). *Molecular diffusion pumps* can then take the system to  $10^{-7}$  torr. Finally, *ion pumps* are used to get to  $10^{-9}$  torr. Liquid nitrogen cold traps are used to condense volatile vapors in the system. The pressure in free space is much better than any of these values, about  $10^{-16}$  torr. Some mechanical pumps are *two stage* in design. Ion pumps are generally of the *cold cathode* or *hot filament* type.

#### 8. (A)

A one-dimensional, non-linear oscillator is usually developed as an approximation to a general potential function

$$U(x) = U(0) + U'(0)x^{1/2} U''(0)x^{2} + \dots$$

using a Taylor series. In the given problem

$$U(x) = \frac{1}{2}kx^2 + \frac{1}{4}bx^4$$

Constant terms like U(0) do not affect the results. For stable equilibrium in a symmetric potential, the odd terms are zero. The force is

 $\mathbf{F} = -\nabla U$ 

or

$$F = \frac{-\partial U}{\partial x} \text{ in } 1\text{-}D$$
$$= -kx - bx^3$$

#### 9. (B)

This is basically the soap film problem. A film has a certain surface energy per unit area  $\gamma$ . The total energy is then just  $\gamma^*$  area. Hence, minimizing the energy is tantamount to minimizing the action integral

$$A = \int 2\pi y \, ds$$
$$= \int 2\pi y \sqrt{1 + y'^2} \, dx$$

Since

$$f = y\sqrt{1 + y'^2} = f(y, y')$$

the 2nd form of Euler's equation is needed.

$$\frac{\partial f}{\partial x} - \frac{d}{dx} \left( f - y' \frac{\partial f}{\partial y'} \right) = 0 \Rightarrow f - y' \frac{\partial f}{\partial y'} = c$$

After a little work, we get

$$y = c \cosh\left(\frac{x-d}{c}\right) = \text{catenoid}$$

## 10. (A)



This is the standard pendulum problem, but in an effective local gravitational field

$$g_e = g + \frac{1}{2}g$$

$$=\frac{3}{2}g$$

By Newton's second law for rotational motion

$$\Sigma \tau = I\alpha$$

$$-mg_e l \sin \theta = I\theta''$$

$$= ml^2 \theta''$$

$$\theta'' + \frac{g_e}{l} \sin \theta = 0$$

is the equation of motion. For  $\theta \ll 1$ , a Taylor expansion gives  $\sin \theta \approx \theta$ .

Thus, 
$$\theta'' + \omega_0^2 \theta = 0$$

$$\omega_0 = \sqrt{\frac{g_\ell}{l}} = \sqrt{\frac{3g}{2l}}$$

is the angular frequency. Also,

$$\upsilon_0 = \frac{\omega_0}{2\pi} = \frac{1}{2\pi} \sqrt{\frac{3g}{2l}}$$

is the linear frequency. In other words, the problem may be solved by substituting  $g_e$  for g.

Thus,

where

## 11. (C)

The room volume is

$$V = lwh = (2.5 \text{ m}) (5 \text{ m}) (5 \text{ m})$$
  
= 62.5 m<sup>3</sup>

The smaller volume is

$$V_0 = (0.025 \text{ m})(0.05 \text{ m})(0.05 \text{ m})$$
  
=  $6.25 \times 10^{-5} \text{ m}^3$ 

The probability parameter is

$$p = \frac{V_0}{V} = 10^{-6}$$

The situation under consideration is binomial in nature

$$p(n) = \begin{pmatrix} N \\ n \end{pmatrix} p^n (1-p)^{N-n}$$

The desired probability is

$$p(N) = \frac{N!}{N!0!} p^{N} (1-p)^{0}$$
$$= (10^{-6})^{10^{27}}$$

or  $\log_{10} p(N) = -6 \times 10^{27}$ .

## 12. (C)

For gravitational 1-D motion with a resistive force the equation of motion is

$$mx'' = -mg - bx'$$
$$x'' = -(g + \gamma x'), \gamma \equiv b/m$$

or

Integrate

$$\int_{v_0}^{v} \frac{dv}{g + \gamma v} = -\int_0^t dt$$

to get

$$\frac{1}{\gamma}\ln\frac{g+\gamma v}{g+\gamma v_0} = -t$$

Exponentiate to get

$$g + \gamma v = (g + \gamma v_0)e^{-\gamma t}$$
$$v = \left(\frac{g}{\gamma} + v_0\right)e^{-\gamma t} - \frac{g}{\gamma}$$

or

$$v = \left(\frac{s}{\gamma} + v_0\right)e^{-\gamma}$$

The maximum height occurs when 
$$v = 0$$
 so that

$$e^{\gamma t} = \left(\frac{g}{\gamma} + v_0\right) / \left(\frac{g}{\gamma}\right)$$
$$t = \frac{1}{\gamma} \ln\left(1 + \frac{v_0\gamma}{g}\right)$$

or

Note for  $\gamma \ll 1$ , we get using a Taylor expansion

$$t = \frac{1}{\gamma} \frac{v_0 \gamma}{g} = \frac{v_0}{g}$$

the vacuum value.

## 13. (E)

Gauss' law for gravitation is

$$\nabla \cdot \mathbf{g} = -4\pi G \rho$$



where G = the universal constant of gravitation. Applying the divergence theorem, we get

$$\oint \gamma \cdot d\alpha = -4\pi G m_{in}$$

For an infinite line mass, the mass density is  $\lambda = m/l$ . Use a Gaussian cylinder for integration to get

$$\oint \mathbf{g} \cdot d\mathbf{a} = -4\pi G\lambda l$$
$$-g2\pi rl = -4\pi G\lambda l$$
$$\mathbf{g} = -(2\lambda G/r)\mathbf{r}$$

or

14. (B)



The basic equation to find the gravitational potential is

$$\phi = -G \int \frac{dm}{r}$$
$$= -G \int_{a}^{b} \frac{2\pi\sigma s ds}{r}$$

using the definition of mass density  $\sigma$  so that

$$dm = 2\pi\sigma s \, ds$$
$$= -2\pi\sigma G \int_a^b \frac{s \, ds}{\sqrt{s^2 + x^2}}$$

using the theorem of Pythagoras. Integrating, we obtain

$$\phi = -2\pi\sigma G \sqrt{s^2 + x^2} \begin{vmatrix} b \\ a \end{vmatrix}$$
$$= -2\pi\sigma G \left[ \sqrt{b^2 + x^2} - \sqrt{a^2 + x^2} \right]$$

Note that one could also find g since

$$\mathbf{g} = -\nabla\phi = -\frac{d\phi}{dx}\mathbf{x}$$

15. (C)

At the top of the circular path

$$\Sigma F = W - N = F_C$$

The pilot's apparent weight is

 $N_T = mg - \frac{mv^2}{r}$ 



At the bottom of the loop

$$\Sigma F = N - W = F_C$$

and the pilot's apparent weight is

$$N_B = mg + \frac{mv^2}{r}$$

We want

$$N_B = 2N_T \Rightarrow mg + rac{mv^2}{r} = 2mg - 2rac{mv^2}{r}$$

or

$$mg = 3\frac{mv^2}{r}$$
$$r = 3\frac{v^2}{g}$$

## 16. (C)

This is the standard Atwood's machine problem with  $m_1 > m_2$ . The two free-body diagrams are shown here.

1

By Newton's Second law

$$m_1g - T = m_1a$$
 and  $T - m_2g = m_2a$ 

Solving the second equation

 $T = m_2 a + m_2 g$ 



and substituting in the first:

$$m_1g - m_2a - m_2g = m_1a$$
  

$$(m_1 - m_2)g = (m_1 + m_2)a$$
  

$$a = (m_1 - m_2)g/(m_1 + m_2)$$

For  $m_1 = 4m$  and  $m_2 = m$ , we obtain.

$$a = \frac{3}{5}g$$

#### 17. (D)

This question concerns some of the basic properties of nucleons and nuclei. Protons and neutrons are nucleons. They are also fermions since they have spin  $s = \frac{1}{2}h$ . Their orbital angular momentum is integral l = 0, 1, 2, ... The total angular momentum of collections of nucleons in nuclei is

$$J = \sum_{i=1}^{A} J_i$$

and is

(i) integral for even A nuclei

(ii) half integral for odd A nuclei

and (iii) zero for even z, even N nuclei.

#### 18. (E)

We are given that

$$m = m_m, \quad r = r_m$$
  
 $a = r_{EM}, \quad M = M_E$ 

The Roche limit is the distance orbital between Earth and moon at which the tidal action of the Earth would rip the moon apart. Consider the moon as being composed of two halves. Then the attractive force is

$$F_A = G\frac{m}{2}\frac{m}{2}/r^2$$
$$= Gm^2/4r^2$$



а

 $m_2 g$ 

a

 $m_1g$ 

and the disruptive tidal force is

$$F_R = \frac{GMm}{2} \left[ \frac{1}{(a-r)^2} - \frac{1}{(a+r)^2} \right]$$

Taylor expanding

$$= \frac{GMm}{2a^2} \left[ 1 + 2\frac{r}{a} - \left(1 - 2\frac{r}{a}\right) \right]$$
$$= 2GMmr/a^3$$

Set  $F_A = F_R$  to get the Roche limit distance

 $Gm^2/4r^2 = 2 \ GMmr/a^3$  $a^3 = 8Mr^3/m$ 

Thus,

## $a = (8M/m)^{1/3}r \approx 16,000 \text{ km}$

#### 19. (D)

The electric quadrupole moment of a charge distribution is

$$A = \frac{2}{5}z(a^2 - b^2)$$

where Z = the number of protons in the nucleus a = the nuclear semi-major axis and b = the ellipse semi-minor axis

Clearly, this factor is a measure of how elliptical the nuclear charge distribution is. The eccentricity is

$$e = \sqrt{a^2 - b^2}/a \Rightarrow e^2 = (a^2 - b^2)/a^2$$

Thus,

$$Q = \frac{2}{5}Ze^2a^2$$

For a circle e = 0. The deuteron, for example, has Q = 0.003 barn.

## 20. (D)

The tukawa potential is based in the meson theory of nuclear forces. The relativistic wave equation

$$\left(\nabla^2 - \mu^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\Phi = 0$$

may be separated by

$$\Phi(\mathbf{r},t) = \phi(\mathbf{r})e^{-iEt/t}$$

to get

$$(\nabla^2 - \mu^2)\phi = 0$$

for a virtual particle. The radial solutions are  $\phi \sim -\mu r/r$ . The tukawa potential has the same form

$$U(r) = V_R e^{-k_R r} / r - V_A e^{-K_A r} / r$$

The repulsive part is

 $V_R e^{-k_R r}/r$ 

with force

$$F = \frac{-\partial}{\partial r} V_R \frac{e^{-k^R}}{r}$$
$$= \frac{V_R e^{-kR_r}}{r} \left(k_R + \frac{1}{r}\right)$$

## 21. (C)

The only force that is accelerating the crate is frictional force. Hence, by Newton's Second Law, Force = mass  $\times$  acceleration, we get friction = ma.

#### 22. (E)

The photon is a stable particle with zero mass, infinite lifetime, and spin j = 1. The electron is a stable particle with mass 0.511 MeV/c<sup>2</sup>, infinite lifetime, and spin  $j = \frac{1}{2}$ . The proton is a stable particle with mass 938.28 MeV/c<sup>2</sup>, lifetime greater than  $10^{32}$  years (perhaps infinite), and spin  $\frac{1}{2}$ . The pion  $\pi^0$  has mass 134.96 MeV/c<sup>2</sup>, half-life  $t = -8 \times 10^{-17}$  s, spin 0, and commonly decays to  $\gamma\gamma$ . The kaon K<sup>+</sup> has mass 493.67 MeV/c<sup>2</sup>, lifetime  $10^{-8}$  s, spin zero, strangeness S = 1, and commonly decays to  $\mu^+ \nu$ .

## 23. (C)

Hadrons are built of quarks whereas leptons are fundamental particles. Quarks have spin  $j = \frac{1}{2}$  and baryon number  $B = \frac{1}{3}$ . Antiquarks have the same spin, but opposite baryon number, electric charge, and isospin. u, d, s, c, b, and t are the flavors of quarks: up, down, strange, charm, bottom, and top. Some common hadron configurations are

$$\begin{array}{ll} p = uud & \bar{p} = \bar{u}\bar{u}d \\ n = udd & \bar{n} = \bar{u}\bar{u}\bar{d} & \Delta^{++} = uuu \\ \pi^+ = u\bar{d} & \pi^- = \bar{u}d & J = c\bar{c} \\ K^+ = u\bar{s} & K^- = \bar{u}s \end{array}$$

## 24. (E)

In the lab frame the atomic scattering looks like



where  $\theta = 2\theta_{lab}$  and  $\phi = \phi_{lab}$  (not shown). Particle number is conserved so that

$$\sigma(\theta_{\rm lab})d\Omega_{\rm lab} = \sigma(\theta)d\Omega$$
$$\sigma(\theta_{\rm l})2\pi\sin\theta_l \,d\theta_l = \frac{\sigma_0}{4\pi}2\pi\sin\theta \,d\theta$$
$$= \sigma_0 2\sin\theta_l\cos\theta_l \,2d\theta_l/2$$
$$\sigma(\theta_l) = \sigma_0\cos\theta_l/\pi$$

The differential cross section is thus not isotropic in the lab!

25. (C)

In radioactive decay

$$N = N_0 e^{-\lambda t}$$

follows from the assumption that the decay is a random process where the probability of one decay is  $\lambda dt$ . (This means  $dN = -\lambda N dt$ .) The half-life is related to the decay constant

$$N_0/2 = N_0 e^{-\lambda t_{1/2}} \Longrightarrow t_{1/2} = \ln(2)/\lambda$$

as is the mean life

$$\tau = \langle t \rangle = \frac{\int_0^\infty t e^{-\lambda t} dt}{\int_0^\infty e^{-\lambda t} dt}$$
$$= \lambda \left[ -\frac{t}{\lambda} e^{-\lambda t} - \frac{1}{\lambda^2} e^{-\lambda t} \right] \bigg|_0^\infty = \frac{1}{\lambda}$$

Hence, 
$$\tau = \frac{1}{\lambda} = t_{1/2} / \ln(2) = 5760 \text{ yr} / \ln(2) = 8310 \text{ yrs}$$

#### 26. (E)

Since the discovery of lasers in 1955, this laboratory technology has found diverse uses in the cutting and welding of metals, the research study of nuclear fusion, dermatology, and even art/music/entertainment. A small CO<sub>2</sub> laser can radiate at 10<sup>5</sup> times the solar intensity. The basic idea is that of optical pumping or to produce a population inversion. Suppose the atoms have two quantum states  $E_1$  and  $E_2$ . Then  $\hbar\omega = E_2 - E_1$ . Also, according to Einstein

$$\frac{dN_1}{dt} = N_1 B_{12} u(\omega)$$
$$\frac{dN_2}{dt} = N_2 A_{21} + N_2 B_{21} u(\omega)$$

At equilibrium  $N'_1 = N'_2$  so that

$$N_{1}B_{12}u = N_{2}A_{21} + N_{2}B_{21}u$$
$$\frac{N_{2}}{N_{1}} = B_{12}u(\omega)/(A_{21} + B_{21}u(\omega))$$

Thus,

#### 27. (E)

In the Raman effect, an incident beam of monochromatic light of frequency  $\omega$  induces a dipole moment in a molecule. This inelastic interaction results in scattered radiation of frequency

$$\omega'' = \omega \pm \omega'$$

depending on whether  $h\omega'$  of energy is given to or taken from the molecule. The electric field of the light interacts with the molecule. The incident light can be of any frequency whereas in fluorescence, the incident photon must be at the proper molecular absorptive frequency.

## 28. (C)

One can use the Biot-Savart law

$$d\mathbf{B} = \frac{\mu_0}{4\pi} I d\mathbf{l} \times \mathbf{s}/s^2$$
$$= \frac{\mu_0 I}{4\pi} \frac{dx \sin \theta}{s^2} \mathbf{z}$$

Р

r

1

θ

dl = dx x

and plug in the trigonometric facts

$$s = rcsc\theta$$

 $\cot \theta = \frac{-x}{r}$ 

and

so that

 $dx = r \csc^2 \theta \ d\theta$ 

to get 
$$dB = \frac{\mu_0 I}{4\pi} \frac{r \csc^2 \theta \sin \theta d\theta}{r^2 \csc^2 \theta}$$

Integrating, one obtains

$$B = \frac{\mu_0 I}{4\pi r} \int_{\theta}^{\theta'} \sin\theta \ d\theta = \frac{\mu_0 I}{4\pi r} \cos\theta \left| \begin{array}{c} \theta \\ \theta' \end{array} \right|$$
$$= \frac{\mu_0 I}{4\pi r} (\cos\theta - \cos\theta')$$

## 29. (D)

Consider the electric potential energy between the spherical shell of differential charge

$$dq = 4\pi r^2 dr \rho$$

and the central charge

$$q=\frac{4}{3}\pi r^3\rho$$

The differential potential energy is

$$dU = kqdq/r$$
$$= 16\pi^2 k\rho^2 r^4 dr/3$$

The spherical charge distribution is total electric potential energy is then

$$U = \frac{16\pi^2 k\rho^2}{3} \int_0^R r^4 dr$$
$$= \frac{16}{15}\pi^2 k\rho^2 R^5$$

Since

$$Q = \int \rho d^3 r = \frac{4}{3} \pi R^3 \rho$$

we rearrange to get

$$U = \frac{3}{5}kQ^2/R$$

## 30. (A)

Maxwell's equations in free space are

$$\nabla \cdot \mathbf{E} = 0 \quad \nabla \times \mathbf{E} = \frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \cdot \mathbf{B} \qquad \nabla \times \mathbf{B} = \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t}$$
a wave equation
$$\left( \begin{array}{c} \mathbf{E} \end{array} \right) \qquad \qquad 2^2 \quad \left( \begin{array}{c} \mathbf{E} \end{array} \right)$$

*y* 🛉

and they yield a wave equation

$$\nabla^2 \begin{pmatrix} \mathbf{E} \\ \mathbf{B} \end{pmatrix} = \mu_0 \varepsilon_0 \frac{\partial^2}{\partial t^2} \begin{pmatrix} \mathbf{E} \\ \mathbf{B} \end{pmatrix}$$

where

$$c^2 = \frac{1}{\mu_0 \varepsilon_0}$$

Ζ

A plane wave solution is

$$\mathbf{E} = \mathbf{y} E_{0y} \cos (\omega t - kx + \alpha) + \mathbf{z} E_{0z} \cos (\omega t - kx + \beta)$$

and

$$\mathbf{B} = -\mathbf{y}\frac{E_{0_z}}{c}\cos(\omega t - kx + \beta) + \mathbf{z}\frac{E_{0_y}}{c}\cos(\omega t - kx + \alpha)$$

since  $c = \omega/k$ . Now if  $\delta \equiv \beta - \alpha$ , then

$$\left(\frac{E_y}{E_{0_y}}\right)^2 + \left(\frac{E_z}{E_{0_z}}\right)^2 = \cos^2\phi + \cos^2(\delta + \phi)$$

where  $\phi \equiv \omega t - kx + \alpha$ . Clearly for  $\delta = \pm \pi/2$ , we get elliptic polarization

$$\left(\frac{E_y}{E_{0_y}}\right)^2 + \left(\frac{E_z}{E_{0_z}}\right)^2 = 1$$

## 31. (C)

Ampere's law in medium is

$$\nabla \times \mathbf{H} = \mathbf{j}$$

By Stokes' Theorem

$$\oint \mathbf{H} \cdot d\mathbf{r} = \int \nabla \times \mathbf{H} \cdot d\mathbf{a}$$

Thus

$$\oint \mathbf{H} \cdot d\mathbf{r} = \int \mathbf{j} \cdot d\mathbf{a} = I_{\rm in}$$

Integrate around a circular loop to get

$$2\pi rH = NI$$

Solve for

$$H = NI/[2\pi(a+b)/2] \text{ since } r = \frac{a+b}{2}$$
$$= NI/\pi(a+b)$$
$$B = \mu H = N\mu I/\pi(a+b)$$

Thus,

since the magnetic field B is related to the field H. Note that in the vacuum case

$$B_0 = N\mu_0 I/\pi(a+b).$$

## 32. (D)

The Fraunhofer diffraction pattern results from monochromatic light waves incident on a single slit. The condition for destructive interference is

$$d\sin\theta = n\lambda$$

Hence,



and

 $y = \frac{\lambda l}{d}$ 

gives the first minimum. The intensity pattern is

$$I = I_0 \sin^2 \left(\frac{\pi d}{\lambda l} y\right) / \left(\frac{\pi d}{\lambda l} y\right)^2$$
$$= I_0 \sin^2 \left(\frac{\pi}{2}\right) / \left(\frac{\pi}{2}\right)^2 \text{ at } y = \frac{\lambda l}{2d}$$
$$\frac{I}{I_0} = \frac{4}{\pi^2}$$

Finally,

## 33. (A)

The electric field is that of the given charge and an image charge of equal and opposite magnitude. Hence,

$$E_z = kq \left[ \frac{z - z'}{(s^2 + (z - z')^2)^{3/2}} - \frac{z + z'}{(s^2 + (z + z')^2)^{3/2}} \right]$$

The charge density is

$$\sigma = arepsilon_0 E_z igg|_{z=0} = -rac{qh/2\pi}{(s^2+h^2)^{3/2}}$$

Note that

$$\int_0^\infty \sigma s \, ds \int_0^{2\pi} d\phi = -qh \int_0^\infty s \, ds / (s^2 + h^2)^{3/2}$$
$$= qh(s^2 + h^2)^{-1/2} \begin{vmatrix} \infty \\ 0 \end{vmatrix}$$

=-q

 $-q \bullet$ 

q

z' = h

## 34. (B)

The given voltage boundary condition is

The infinite sheet has charge -q on top, as it must.

$$V(s) = \begin{cases} V_0 & s < r \\ \\ 0 & s > r \end{cases}$$

for z = 0 along the *z*-axis.

$$f(z) = \frac{V_0 z}{2p} \int \frac{s \, ds \, df}{(s^2 + z^2)^{3/2}}$$
$$= V_0 z (s^2 + z^2)^{-1/2} \begin{vmatrix} s = 0 \\ s = r \end{vmatrix}$$

taking  $\int_0^{2\pi} d\phi = 2\pi$ 

$$\phi(z) = V_0 z \left(\frac{1}{z} - \frac{1}{\sqrt{r^2 + z^2}}\right)$$
$$= V_0 \left(1 - \frac{z}{\sqrt{r^2 + z^2}}\right)$$

Now we calculate the electric field

$$E_z = -\frac{\partial f}{\partial z} = \frac{\partial}{\partial z} \left( \frac{V_0 z}{\sqrt{r^2 + z^2}} \right)$$

by the definition of the electric field  $\mathbf{E} = -\nabla \phi$ . Differentiate to get

$$E_z = V_0 \frac{(1)\sqrt{r^2 + z^2} - z(z(r^2 + z^2)^{-1/2})}{r^2 + z^2}$$
$$= V_0 \frac{r^2}{(r^2 + z^2)^{3/2}}$$

a dipole field.

#### 35. (D)

Let each rod have area of cross section A and length L. Let the end temperatures be  $T_1$  and  $T_2$  for the rod with thermal conductivity  $k_1$  and  $T_2$  and  $T_3$  for the rod with thermal conductivity  $k_2$ .

For the composite rod:  $Q/t = kA(T_1 - T_3)/(2L)$ 

For the first rod:  $Q/t = k_1 A (T_1 - T_2)/(L)$ 

For the composite rod:  $Q/t = k_2 A (T_2 - T_3)/(L)$ 

From the second and third equation:

$$(T_1 - T_2) = (1/k_1)(Q/t)L/A$$
  
 $(T_2 - T_3) = (1/k_2)(Q/t)L/A$ 

Adding these two equations:

$$(T_1 - T_3) = (1/k_1 + 1/k_2)(Q/t)L/A = [(k_{1+}k_2)/(k_1k_2)](Q/t)L/A$$
  
$$\Rightarrow Q/t = [(k_1k_2)/(k_1 + k_2)](A)(T_1 - T_3)/L = [(2k_1k_2)/(k_1 + k_2)](A)(T_1 - T_3)/(2L)$$

 $\Rightarrow$  Comparing this to the equation for the composite rod above we get

$$\Rightarrow \mathbf{k} = \frac{(2k_1k_2)}{(k_1 + k_2)}$$

36. (A)

The magnetic vector potential is

$$\mathbf{A} = \frac{\mu_0}{4\pi} \int \mathbf{j} d^3 r / r = \frac{\mu_0}{4\pi} \int \frac{I dl}{r}$$

where  $\mathbf{j}$  is the current density. By the Stokes theorem

$$\mathbf{A} = \frac{\mu_0 I}{4\pi} \int d\mathbf{a} \times \nabla \left(\frac{1}{r}\right)$$
$$= \frac{\mu_0 I}{4\pi} \int d\mathbf{a} \times \frac{\mathbf{r}}{r^2}$$

since  $d\mathbf{m} = Id\mathbf{a}$  for a small current loop.

$$= \frac{\mu_0}{4\pi} \int d\mathbf{m} \times \frac{\mathbf{r}}{r^2}$$
$$= \frac{\mu_0}{4\pi} \mathbf{m} \times \frac{\mathbf{r}}{r^2}$$

Finally,

$$\mathbf{B} = \nabla \times \mathbf{A}$$
$$= \frac{\mu_0}{4\pi} \nabla \times \left(\frac{\mathbf{m} \times \mathbf{r}}{r^2}\right)$$
$$= \frac{\mu_0}{4\pi r^3} [3(\mathbf{m} \cdot \mathbf{r})\mathbf{r} - \mathbf{m}]$$

which is the standard form for any dipole field.

## 37. (A)

This problem is best treated in cylindrical coordinates, as suggested. Therefore,

and

$$x = \rho \cos \theta, y = \rho \sin \theta$$
, and  $z = z$ 

 $x^2 + y^2 = \rho^2$ 

The potential energy is U = mgz taking U = 0 at z = 0. Also,

$$v^2 = \rho'^2 + \rho^2 \theta'^2 + z'^2$$

in cylindrical coordinates. Hence, the Lagrangian is

$$L = T - U = \frac{1}{2} mv^{2} - mgz$$
$$= \frac{1}{2} m(\rho'^{2} + \rho^{2}\theta'^{2} + z'^{2}) - mgz$$

## 38. (B)

The problem of a grounded conducting sphere in a uniform electric field has potential  $\Phi$  which satisfies Laplace's equation:

$$\nabla^2 \Phi = 0$$
 for  $a < r < \infty$ .

Also the boundary conditions are

$$\Phi_{r=a} = 0 \text{ and } \mathbf{E}_{r=\infty} = E_0 \mathbf{z}.$$
$$\Phi_{r=\infty} = -E_0 z \text{ or } -E_0 r \cos \theta$$

Thus,

since

$$E_z = -\frac{\partial \Phi}{\partial z}$$

The given solution is

$$\Phi = \left(Ar + \frac{B}{r^2}\right)\cos\theta$$

Applying the boundary conditions

$$\Phi(r = \infty) = Ar\cos\theta = -E_0 r\cos\theta$$
$$\Rightarrow A = -E_0$$
$$\Phi(r = a) = \left(Aa + \frac{B}{a^2}\right)\cos\theta = 0$$
$$\Rightarrow B = -Aa^3 = a^3E_0$$

Hence, the electric potential is

$$\Phi(r,\theta) = \left(-E_0 r + \frac{a^3 E_0}{r^2}\right) \cos \theta$$
$$= -E_0 r \left(1 - \left(\frac{a}{r}\right)^3\right) \cos \theta$$

**39.** (**B**)

Consider a system of charges and an E field. Then, since power =  $\mathbf{F} \cdot \mathbf{v}$ ,

$$\frac{dE_{\rm mech}}{dt} = \boldsymbol{\sigma} q \mathbf{v} \cdot \mathbf{E}$$

where  $\mathbf{F} = q\mathbf{E}$  has been used

$$=\int \rho \mathbf{v} \cdot \mathbf{E} d^3 r$$

passing to the continuous case

$$= \int \mathbf{j} \cdot \mathbf{B} \mathbf{E} d^3 r \equiv \int \frac{\partial W}{\partial t} d^3 r$$

where

$$\frac{\partial W}{\partial t} = \mathbf{j} \cdot \mathbf{E}$$

414

is the mechanical power density and  $\mathbf{j} = \rho \mathbf{v}$  is the current density. Let

$$u = u_E + u_M = \frac{1}{2} (\mathbf{E} \cdot \mathbf{D} + \mathbf{B} \cdot \mathbf{H})$$

be the field energy density and  $\mathbf{S} = \mathbf{E} \times \mathbf{H}$  the Poynting vector. Then

$$\nabla \cdot \mathbf{S} + \frac{\partial u}{\partial t} + \mathbf{j} \cdot \mathbf{E} = 0$$

is the differential statement of Poynting's theorem. In integral form, we have

$$\oint \mathbf{s} \cdot d\mathbf{a} + \int \frac{\partial v}{\partial t} d^3 r + \int \mathbf{j} \cdot \mathbf{E} d^3 r = 0$$

which is just the conservation of energy.

#### 40. (C)

The minimum angle of resolution for a circular lens is

$$heta_m = rac{d}{l} = 1.22 rac{\lambda}{D}$$

The relationship  $d = l\theta_m$  follows from the radian definition of angle. We are given

$$d = 1m$$
 and  $l = 10,000$  m

Thus,

$$\theta_m = 10^{-4} \text{ rad}$$

and using  $\lambda = 6000 \text{ Å}$ 

$$D = 1.22\lambda/\theta_m$$
  
= 1.22 × 6000 × 10<sup>-10</sup>/10<sup>-4</sup>  
= 0.73 × 10<sup>-2</sup> m



#### 41. (D)

We are given  $\alpha = 60^{\circ}$  and  $\theta_{i1} = 30^{\circ}$ . By Snell's law

$$n_i \sin \theta_i = n_r \sin \theta_r$$
(1) sin 30° = 1.5 sin  $\theta_r \Rightarrow \theta_{r1} = 19.47^\circ$ 

By geometry

$$\alpha = \theta_{r1} + \theta_{i2} \Longrightarrow \theta_{i2} = 60 - 19.47 = 40.53^{\circ}$$

Again apply Snell's law to get

$$1.5 \sin 40.53^\circ = 1 \sin \theta_r \Rightarrow \theta_{r2} = 77.10^\circ$$

Now the total deviation angle is

$$\delta = (\theta_{i1} - \theta_{r1}) + (\theta_{r2} - \theta_{i2})$$
$$= \theta_{i1} + \theta_{r1} - \alpha$$
$$= 30 + 77.1 - 60 = 47.1^{\circ}$$

Note that the near normal incidence prism formula *cannot* be used here.

## 42. (B)

This is the general thin lens problem. Applying the optics equation for going from one medium to another



we get

$$-\frac{n_0}{s_0} + \frac{n_r}{s'} = \frac{n_r - n_0}{R_1}$$
$$-\frac{n_r}{s'} + \frac{n_i}{s'_i} = \frac{(n_i - n_r)}{R_2}$$

and

Add the equations to get

$$-\frac{n_0}{s_0} + \frac{n_i}{s_i'} = \frac{(n_r - n_0)}{R_1} + \frac{(n_i - n_r)}{R_2}$$

To find the secondary focal length, set  $s_0 = -\infty$ 

$$\frac{n_i}{f'} = \frac{n_r - n_0}{R_1} + \frac{n_i - n_r}{R_2}$$

$$f' = \frac{n_i}{(n_r - n_0)/R_1 + (n_i - n_r)/R_2}$$

or

### 43. (C)

The 13 electrons of aluminum are arranged as

1s<sup>2</sup>2s<sup>2</sup> 2p<sup>6</sup> 3s<sup>2</sup> 3p<sup>1</sup>



 $R_2$ 

n<sub>o</sub>

R<sub>1</sub>

n<sub>i</sub>

## **DETAILED EXPLANATIONS OF ANSWERS — PRACTICE EXAM 4**

## 44. (B)

Positive logic means that the high voltage state is 1 whereas the low voltage state represents 0. The assignments are reversed for negative logic. The voltage truth table for NAND is

A	В	output
lo	lo	hi
lo	hi	hi
hi	lo	hi
hi	hi	lo

For negative logic, this becomes

A	B	output
1	1	0
1	0	0
0	1	0
0	0	1

which is A NOR B.

45. (D)

$$f' = f \frac{V \pm V_o}{V \pm V_s}$$

Therefore,

$$f' = f \frac{V - V_o}{V - V_s}$$

And since  $v_0 = v_s, f' = f$ .

## 46. (C)

The wavelength of sound is changed due to the motion of the source only. Motion of the observer does not affect the wavelength of sound. If the source is moving toward the observer, the wavelength received by the observer is

$$\lambda' = \frac{V - V_s}{f} = \lambda \left( 1 - \frac{V_s}{V} \right) = 0.343 \left( 1 - \frac{1}{2} \right) = 0.1715 \text{ m}$$

## 47. (D)

At high frequency the inductive reactance

 $X_L = \omega L$ 

is very much greater than the 10 k $\Omega$  resistance. Thus, most of the current then goes through that resistor:



Also at high frequency the capacitive reactance

 $X_c = 1/\omega C$ 

acts like a short. The equivalent circuit is a voltage divider.



## 48. (C)

Arago first discovered optical activity in 1811 in France. The plane of vibration of light undergoes a continuous rotation when passed through a number of substances (quartz, turpentine, benzil, etc.). The **E** field of the incident linear plane waves rotates about the optic axis. With respect to the source direction, a substance that causes clockwise rotation is dextro type or right handed. Levo type means left handed. Crystal quartz can be either dextro or levo depending on the crystallography.

### 49. (D)

For a plane transmission diffraction grating the condition for maxima (constructive interference) is

$$d\sin\theta = m\lambda$$

where n = 0, 1, 2, 3, ... gives the order of the spectrum. Note that for m = 0, all wavelengths are indistinguishable since  $\theta = 0$  for each of them. Differentiating, we get

$$d\cos\theta \,d\theta = md\,\lambda$$
 or  $\frac{d\theta}{d\lambda} = \frac{m}{d\cos\theta}$ 

Since  $m/d = \sin \theta / \lambda$ , the angular dispersion is

$$\frac{d\theta}{d\lambda} = \frac{1}{\lambda}\tan\theta$$
#### **DETAILED EXPLANATIONS OF ANSWERS — PRACTICE EXAM 4**

#### 50. (C)

The radii of the dark interference rings of various orders is given by

$$r = \sqrt{m\lambda R}$$

where *m* is the order of the spectrum,  $\lambda$  is the wavelength, and *R* is the radius of curvature of the planoconvex lens. This result follows from the thin film destructive interference condition.

$$2 nd = m\lambda$$
 where  $n = 1$  for air  
 $\Rightarrow d = m\lambda/2$ 

and geometry

$$R^{2} = (R - d)^{2} + r^{2}$$
$$r^{2} = 2Rd - d^{2} \approx 2 Rd$$
$$r = \sqrt{2Rd}$$
$$= \sqrt{m\lambda R}$$

Thus,

#### 51. (E)

Note that Ag is the substance. We find its work function

$$\phi = h\nu_0 = hc/\lambda_0$$
  
= 12,400 × 10<sup>-10</sup> eV m/2638 × 10<sup>-10</sup> = 4.70 eV

The energy of the incident light is

$$E = h\nu = hc/\lambda = 12,400 \times 10^{-10}/1600 \times 10^{-10} = 7.75 \text{ eV}$$

Thus, the kinetic energy of the photoelectrons is

$$T = hv - \phi = 7.75 - 4.70 = 3.05 \text{ eV}$$

Finally,

$$T = \frac{1}{2}mv^2 \Rightarrow v = \sqrt{(3.05)(2)/(511,000)c}$$
  
= 1.04 × 10<sup>6</sup> m/s

#### 52. (B)

This is a standard Compton scattering problem



We are given that

$$\lambda = 6.20 \times 10^{-10} \,\mathrm{m}$$

Hence,

$$E = \frac{hc}{\lambda} = \frac{12.4 \times 10^{-10} \text{ keV} \cdot \text{m}}{6.2x10^{-10}} = 2.0 \text{KeV}$$

The Compton Shift is

$$\phi = 180^\circ \Rightarrow \Delta \lambda = 2\lambda_c \sin^2 \frac{\phi}{2}$$
$$= 2(0.0242)(1) = 0.484 \times 10^{-10} \text{ m}$$

Thus, the photon wavelength and energy in the final state is

$$\lambda' = \lambda + \Delta \lambda = 6.2484 \times 10^{-10M}, E' = \frac{hc}{\lambda'} = 1.985 \text{ keV}$$

The Compton energy shift is then

$$\Delta E = E - E' = 0.015 \text{ keV} = 15.5 \text{ eV}$$

#### 53. (D)

Apply Newton's Second Law to get

$$mv' = -bv$$

and integrate

$$\int_{v_0}^{v} \frac{dv}{v} = -\int_0^t \frac{b}{m} dt, \ln \frac{v}{v_0} = -\frac{b}{m} t$$
$$\gamma \equiv \frac{b}{m} \Rightarrow v = v_0 e^{-\gamma t} = \frac{dx}{dt}$$

Integrate again to get

$$x = \frac{v_0}{\gamma} (1 - e^{-\gamma t})$$
$$= \frac{100}{2} (1 - e^{-5})$$

$$= 4.97 \text{ m}$$

Substituting the given information

 $t = 2.5s, \gamma = 2s^{-1}, v_0 = 100 \text{ m/s}$ 

#### 54. (E)

This is a rocket problem. By the conservation of momentum

$$md\nu = -\nu dm$$

Integrating, we obtain

$$\int_{m_0}^{m} -\frac{dm}{m} = \frac{1}{0} \int_{v_0}^{v} dv$$
$$v = v_0 + v \ln \frac{m_0}{m}$$
$$= 0.5 + 1.0 \ln \frac{2}{1}$$
$$= 1.2 \text{ km/s}$$

#### 55. (D)

In cylindrical coordinates

$$\mathbf{r} = \rho \hat{\rho} + z \hat{z}$$

and

$$\mathbf{v} = \frac{d\mathbf{r}}{dt} = \rho' \,\hat{\mathbf{\rho}} + \rho \,\theta' \,\hat{\mathbf{\theta}} + z' \hat{z} \text{ where } \frac{d\hat{\mathbf{\rho}}}{dt} = \theta' \,\hat{\mathbf{\theta}}$$

is used. Now by differentiating, one gets

$$\mathbf{a} = \frac{d\mathbf{v}}{dt} = \rho''\hat{\mathbf{\rho}} + \rho'\theta'\hat{\mathbf{\theta}} + \rho'\theta'\hat{\mathbf{\theta}} + \rho'\theta''\hat{\mathbf{\theta}} + \rho'\theta''\hat{\mathbf{\theta}} + \rho'\theta'\frac{d\mathbf{\theta}'}{dt} + z''\hat{z}$$

then use

$$\frac{d\hat{\theta}}{dt} = -\theta'\hat{\rho}$$

and collect terms

$$\mathbf{a} = (\rho'' - \rho\theta'^2)\hat{\rho} + (\rho\theta'' + 2\rho'\theta')\hat{\theta} + z''\hat{z}$$

to get the desired answer.

#### 56. (B)

The given vector is

$$\mathbf{r} = \mathbf{x} + \mathbf{y} = \mathbf{x}_1 + \mathbf{x}_2$$

The rotational transformation is

$$\begin{pmatrix} x_1' \\ x_1' \end{pmatrix} = \begin{pmatrix} \lambda_{11} & \lambda_{12} \\ \lambda_{21} & \lambda_{22} \end{pmatrix} \begin{pmatrix} x_1 \\ x_1 \end{pmatrix}$$
$$= \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} x_1 \\ x_1 \end{pmatrix}, \ \theta = 30^{\circ}$$
$$= \begin{pmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{pmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$
$$= \begin{pmatrix} 1.37 \\ 0.37 \end{pmatrix}$$
$$\mathbf{r}' = 1.37\mathbf{x}'_1 + 0.37 \mathbf{x}'_2 = (1.37, 0.37)m$$

which may be confirmed geometrically using cos 15° and sin 15°.

#### 57. (E)

Stirling's approximation to second order is desired. Use the definition of the  $\Gamma$  function:

$$N! = \Gamma(N+1) = \int_0^\infty e^{-t} t^N dt = \int_0^\infty e^{f(t)} dt$$
$$f(t) = N \ln t - t$$

Now we wish to do a Taylor expansion

$$f' = \frac{N}{t} - 1 = 0 \Rightarrow t = N$$
 maximum

$$f'' = -\frac{N}{t^2} \bigg|_{t=N} = -\frac{1}{N}$$

.

$$f(t) \approx f(N) + f'(N)(t-N) + \frac{1}{2!}f''(N)(t-N)^2$$
$$N! = e^{N \ln N - N} \int_0^\infty e^{-(t-N)^2/2N} dt$$
$$= \left(\frac{N}{e}\right)^N \sqrt{2\pi N}$$

#### 58. (A)

In microcanonical ensemble theory, the cumulative number of states for an ideal gas is

$$\Gamma(E) = \frac{\pi^{3N/2}}{N! \left(\frac{3N}{2}\right)! (2\pi\mathbf{h})^{3N}} V^N (2mE)^{3N/2}$$

The factors of

$$\pi^{3N/2} \left(\sqrt{2mE}\right)^{3N} / (3N/2)!$$

come from the volume of a 3N dimensional sphere in momentum space. Using the first order Stirling approximation, we have

$$N! = \left(\frac{N}{e}\right)^N$$
 and  $\left(\frac{3N}{2}\right)! = \left(\frac{3N}{2e}\right)^{3N/2}$ 

The entropy is

$$S = k \ln \Gamma$$
$$= Nk \ln \left[ \frac{V}{N} \left( \frac{m}{2\pi \hbar^2} \frac{2}{3} \frac{E}{N} \right)^{3/2} \right] + \frac{5}{2} Nk$$

which is the Sakur-Tetrode equation:

$$S = Nk \ln\left[(gT)^{3/2}\frac{V}{N}\right] + \frac{5}{2}Nk$$

#### 59. (E)

By the right-hand thumb rule, the wire on the x-axis creates the magnetic field in the +z direction and the sire along the y axis creates an equal amount of magnetic field in the -z direction. Hence, the net magnetic field is zero.

#### 60. (D)

The total rest and kinetic energy of the electron-positron pair must be sufficient to account for the rest energy of the resultant proton-antiproton pair. The particle reaction is

$$e^{-} + e^{+} \rightarrow p + \bar{p}$$
  
 $\sqrt{s} = E_{cm} = c^{2} \sqrt{(m_{1} + m_{T})^{2} + 2T_{\text{lab}}m_{T}}$   
 $m_{p} + m_{\bar{p}} = \sqrt{(m_{e} + m_{\bar{e}})^{2} + 2T_{e}m_{\bar{e}}}$   
 $2 \times 9.38 = \sqrt{(2 \times 0.511)^{2} + 2T_{e}(0.511)}$ 

$$T_e = 3.44 \times 10^{\circ} \text{ MeV} = 3440 \text{ GeV}$$

$$= 3.44 \text{ TeV}$$

### 61. (C)

The given Lagrangian L = T - U is

$$L = \frac{1}{2}mv^2 - q\phi + q\mathbf{A} \cdot \mathbf{v}$$

The Lagrange/Euler equation is

$$\frac{d}{dt}\left(\frac{\partial L}{\partial x'_j}\right) - \frac{\partial L}{\partial x_j} = 0$$

From this, we obtain

$$mx_{j}^{\prime\prime} = +q \left[ -\frac{\partial \phi}{\partial x_{j}} - \frac{\partial A_{j}}{\partial t} - \sum_{k} \left( \frac{\partial A_{j}}{\partial x_{k}} - \frac{\partial A_{k}}{\partial x_{j}} \right) x_{k}^{\prime} \right]$$

or

$$F_j = q[E_j + (\mathbf{v} \times \mathbf{B})_j]$$
$$\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$$

which is the Lorentz force law.

#### 62. (D)

The given nuclear reaction is

$${}^{7}_{3}\text{Li} + {}^{208}_{82}\text{Pb}$$

The closest point will be where the kinetic energy of the Li is converted entirely to potential energy.

$$T = \frac{1}{2}mv^2 = \frac{q_1q_2}{r} = \frac{3(82)e^2}{r} = \frac{246e^2}{r}$$
$$r = 246e^2/T$$
$$= 246(1.44 \text{ MeV} - \text{fm})/(50 \text{ MeV})$$
$$= 7.08 \text{ fm}$$

In the head-on collision, we thus see a transformation of KE into PE.

#### 63. (E)

Gauss's law and the definition of an electric field give

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0}, \ \mathbf{E} = -\nabla \phi \Rightarrow \nabla^2 \mathbf{E} = \begin{cases} -\frac{\rho}{\varepsilon_0} & r < R \\ 0 & r > R \end{cases}$$

In spherical coordinates

$$\nabla^2 = \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial}{\partial r} \right) = \frac{1}{r^2} \frac{\partial^2}{\partial r^2} (r)$$

We are given that

$$\phi = -\rho r^2/6\varepsilon_0 + A/r + B, r < R$$

For r > R, we guess by symmetry  $\phi = C/r + D$ . The boundary conditions then give

$$\phi(r \to \infty) = 0 \Rightarrow D = 0$$
  
$$\phi(r \to 0) \text{ finite} \Rightarrow A = 0$$
  
$$\phi(r = R) \text{ continuous} \Rightarrow B - \rho R^2/6\varepsilon_0 = C/R$$

Gauss' law in integral form gives

$$\oint \mathbf{E} \cdot d\mathbf{a} = \frac{C}{R^2} 4\pi R^2 = \frac{q}{\varepsilon_0} \Rightarrow C = \frac{q}{4\pi\varepsilon_0}$$

Thus, 
$$B = \frac{q}{4\pi\varepsilon_0 R} + \frac{\rho R^2}{G\varepsilon_0}$$

and 
$$\phi = \frac{\rho}{6\varepsilon_0} (R^2 - r^2) + \frac{q}{4\pi\varepsilon_0 R}$$
$$= \frac{\rho}{2\varepsilon_0} \left( R^2 - \frac{r^2}{3} \right)$$

$$\begin{array}{c} 2r \\ \downarrow \\ \downarrow \\ \downarrow \\ \downarrow_1 \\ \downarrow_1 \\ \downarrow_2 \end{array}$$

A matrix approach is appropriate here.

$$\begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix} = \begin{pmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \end{pmatrix} \begin{pmatrix} I_i \\ I_2 \end{pmatrix}$$
  
If  
$$I_2 = 0 \Rightarrow R_{11} = 1/(2\pi\sigma r_1) = 1/(2\pi\sigma r) \{I_1 \neq 0\}$$
$$I_1 = 0 \Rightarrow R_{22} = 1/(2\pi\sigma r_2) = 1/(2\pi\sigma r) \{I_2 \neq 0\}$$

In the same way  $R_{12} = R_{21} = 1/(2 \pi \sigma d)$ 

$$\begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix} = \frac{1}{2\pi\sigma} \begin{pmatrix} \frac{1}{r} & \frac{1}{d} \\ \frac{1}{d} & \frac{1}{r} \end{pmatrix} \begin{pmatrix} I_i \\ I_2 \end{pmatrix}$$

Now use  $I_2 = I = -I_1$ . By Ohm's law

$$\frac{(\phi_2 - \phi_1)}{I} = \frac{1}{\pi\sigma} \left(\frac{1}{r} - \frac{1}{d}\right) = R$$

#### 65. (E)

64. (D)

Newton's law of universal gravitation is

$$\mathbf{F} = -\frac{GmM}{r^2}\mathbf{r} = m\mathbf{g}$$

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We can write this as a Gauss's law for gravitation:

$$\oint \mathbf{g} \cdot d\mathbf{a} = -\frac{GM}{r^2} 4\pi r^2 = -4\pi GM \equiv -4\pi Gm_{in}$$

The divergence theorem is

$$\oint \nabla \cdot \mathbf{g} d^3 r = \oint \mathbf{g} \cdot d\mathbf{a}$$

and hence

$$\nabla \cdot \mathbf{g} \frac{4}{3}\pi r^3 = -4\pi G m_{in} \Rightarrow \nabla \cdot \mathbf{g} = -4\pi G \rho$$

This is analogous to the electric field law

$$\nabla \cdot \mathbf{E} = 4\pi k_e \rho_q$$
$$k_e = 1/4\pi\varepsilon_0$$

with

66. (A)

The action integral is

$$A = \int_{t_1}^{t_2} f(t, x, x') \, dt$$

The first variation results in  $\delta A = \int \delta t \, dt$  where

$$\delta f = \frac{\partial f}{\partial x} \delta x + \frac{\partial f}{\partial x'} \delta x'$$

gives the usual Euler-Lagrange equation.

In a similar manner, the second variation is

$$\delta^2 f = \frac{\partial^2 f}{\partial x^2} (\delta x)^2 + 2 \frac{\partial^2 f}{\partial x \partial x'} \delta x \delta x' + \frac{\partial^2 f}{\partial x'^2} (\delta x')^2$$

where  $\delta^2 A = \int \delta^2 f \, dt$  and the total variation is

$$\Delta A = \alpha \delta A + \frac{\alpha^2}{2} \delta^2 A + \dots$$

67. (E)

Snell's law from Fermat's principle of least time. The action integral

$$A = \int \frac{ds}{V}$$

is just the time t to go from P to Q.

$$A = \frac{l_1}{v_1} + \frac{l_2}{v_2} = \frac{n_1 I_1}{c} + \frac{n_2 I_2}{c}$$
$$= \frac{n_1}{c} \sqrt{a^2 + x^2} + \frac{n_2}{c} \sqrt{b^2 + (d - x)^2}$$

The first derivative condition dA/dx = 0 gives

$$n_1 \frac{x}{\sqrt{a^2 + x^2}} = n_2 \frac{d - x}{\sqrt{b^2 + (d - x)^2}}$$
$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$

or

#### 68. (A)

The usual Lorentz transformation in Minkowski space (r, *ict*) is

$$\begin{pmatrix} x_1' \\ x_4' \end{pmatrix} = \begin{pmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{pmatrix} \begin{pmatrix} x_1 \\ x_4 \end{pmatrix}$$

Using Euler's formula

$$e^{i\phi} = \cos\phi + i\sin\phi$$

one easily proves

$$\cos \phi = \frac{e^{i\phi} + e^{-i\phi}}{2} = \cosh(i\phi) = \cosh y$$
$$\sin \phi = (e^{i\phi} - e^{-i\phi})/2i = -i \sinh(y)$$

Thus, the desired result is

$$\begin{pmatrix} x'_1 \\ x'_4 \end{pmatrix} = \begin{pmatrix} \cosh y & -i \sinh y \\ \sinh y & \cosh y \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

#### 69. (D)

The total relativistic energy is

$$E = T + mc^2 = m\gamma c^2$$

where T is the relativistic kinetic energy

$$T = mc^2(\gamma - 1) = mc^2 \left(\frac{1}{\sqrt{1 - \beta^2}} - 1\right)$$

Now we use a Taylor series expansion

$$\frac{1}{\sqrt{1-x}} = 1 + \frac{1}{2}x + \frac{3}{8}x^2 + \dots$$

with  $x = \beta^2$  to get

$$T = mc^2 \left(\frac{1}{2}\beta^2 + \frac{3}{8}\beta^4\right)$$
$$= \frac{1}{2}mv^2 + \frac{3}{8}mv^2\beta^2$$

#### 70. (B)

The standard two-body nuclear reaction is

$$I + T \rightarrow E + R$$

where I is the incident nucleus, T is the target, E is the emitted particle, and R is the residual nucleus. Here the reaction is

$$\alpha + {}^{14}N \rightarrow p + {}^{17}O$$

with Q-value

$$Q = m_I + m_T - m_R - m_E$$
  
= (4.0026 + 14.0031 - 1.0078 - 16.9991) amu  
× 931.502 MeV/amu  
= 1.1 MeV

In the center of mass reference frame, this is the minimum kinetic energy needed for the reaction to occur.

#### 71. (C)

The ideal system of N spins is an example of the binomial distribution where

$$p = P(\uparrow)$$
 and  $q = 1 - p = P(\downarrow)$ 

The mean magnetic moment for one spin is

$$<\mu> = p\mu_0 + (1-p)(-\mu_0) = (2p-1)\mu_0$$

and the single spin variance is

$$\sigma^{2} = \langle (\mu - \langle \mu \rangle)^{2} \rangle = \langle \mu^{2} \rangle - \langle \mu \rangle^{2}$$
$$= p\mu_{0}^{2} + (1 - p)\mu_{0}^{2} - (2p - 1)\mu_{0}^{2}$$
$$= 4pq\mu_{0}^{2}$$

Hence, for N signs

$$= N(2p - 1)\mu_0$$
 and  $\sigma_M^2 = 4Npq\mu_0^2$ 

72. (C)

We are given that

$$p[1 \text{ event occurs in } (t, t + dt)] = r dt$$

with r = 1 Hz. One must know that the distribution is Poisson with  $\lambda = rt = 10$  being the expected number of counts in t = 10 s. Thus, the probability of 5 counts is found from

$$p(n) = \lambda^{n} e^{-\lambda}/n!$$
or is
$$p(5) = \lambda^{5} e^{-\lambda}/5!$$

$$= 10^{5} e^{-10}/5!$$

$$= 0.038$$

#### 73. (E)

Answer (A) is the postulate of Lord Kelvin and is a statement of the second law

$$\Delta S \ge 0.$$

Answer (B) is simply a restatement of choice (A). Selection (C) is the postulate of Clausius and is a statement of the second law. Answer (D) is a restatement of (C). An easy counterexample for (E) is the heating of a body, no matter what its temperature, by frictional work. Clearly also electric energy can do work and be transformed into heat in a toaster (resistor).

#### 74. (D)

For this centripetal motion laboratory experiment

$$g = 4\pi^2 mr/T^2 M$$

The standard rule for the propagation of error is

$$\Delta g = \sqrt{\left(\frac{\partial g}{\partial r}\Delta r\right)^2 + \left(\frac{\partial g}{\partial T}\Delta T\right)^2}$$
$$= \sqrt{\left(g\frac{\Delta r}{r}\right)^2 + \left(2g\frac{\Delta T}{T}\right)^2}$$

Hence, the relative error is

$$\frac{\Delta g}{g} = \sqrt{\left(\frac{\Delta r}{r}\right)^2 + 4\left(\frac{\Delta T}{T}\right)^2} = \sqrt{\left(\frac{1}{20}\right)^2 + 4\left(\frac{0.016}{0.638}\right)^2}$$

The experimental g-value is thus 970  $\pm$  69 cm/s<sup>2</sup>.

75. (E)

For the two isothermal parts of the cycle

 $\Delta U=0$ 

Hence, by the first law of thermodynamics

 $\Delta Q = \Delta W$ 

 $-Q_C = W_{CD} - \int_C^D p \ dV$ 

or

$$= \int_{V_C}^{V_D} \frac{RT_C}{V} dV = -RT_C \ln \frac{V_C}{V_D}$$

Therefore,

$$Q_C = RT_C \ln \frac{V_C}{V_D}$$

and in the same way we find

$$Q_H = RT_H \ln \frac{V_B}{V_A}$$

Thus, the total amount of heat absorbed is

$$Q_H - Q_C = RT_H \ln \frac{V_B}{V_A} - RT_C \ln \frac{V_C}{V_D}$$

76. (C)

For the simple two-level system, there are only two relevant energies

$$E_{-} = \mu_0 B$$
 and  $E_{+} = -\mu_0 B$ 

The partition function for the system is

$$Z = \sum e^{-\beta E_j}$$
$$= e^{-\beta \mu_0 B} + e^{\beta \mu_0 B}$$
$$= 2 \cosh(\beta \mu_0 B)$$

The single particle average energy is

$$\langle E \rangle = -\frac{\partial}{\partial \beta} \ln Z$$
  
=  $-2\mu_0 B \sinh(\beta \mu_0 B)/2 \cosh(\beta \mu_0 B)$   
=  $-\mu_0 B \tanh(\beta \mu_0 B)$ 

Hence, the total or N-particle energy is

$$\langle E \rangle_N = N \langle E \rangle$$
  
=  $-N\mu_0 B \tanh(\beta\mu_0 B)$ 

77. **(B)** 

The Hamiltonian of the system is

$$H(x, p) = p^2/2m + kx^2/2 = E$$

The cumulative number of states is found in the usual way:

 $\omega = \sqrt{k/m}$ 

$$\Gamma(E) = \frac{1}{2\pi\hbar} \int \theta(E-H) \, dx \, dp$$
$$= \frac{1}{2\pi\hbar} \int dx \int_{-p}^{p} dp, \ p = \sqrt{2mE - mkx^2}$$
$$= \frac{1}{2\pi\hbar} \int_{0}^{\sqrt{2E/k}} 2\sqrt{2mE} \sqrt{1 - kx^2/2E} \, dx$$
$$= \frac{E}{\hbar\omega}$$

Using

and a *u*-substitution

$$u = \sin \theta = \sqrt{k/2E}x$$

Finally, the density of states is

$$\frac{d\Gamma}{dE} = \frac{1}{h\omega}$$

Note that  $\int \theta(E - H) dx dp$  could also be found from the ellipse area formula

$$\pi ab = \pi \sqrt{\frac{2E}{k}} \sqrt{2mE} = 2\pi E/\omega$$

#### 78. (C)

The phase space orbit is easily identified as that of simple harmonic motion

$$\Sigma F = -kx = mx''$$

$$x'' = \omega_0^2 x = 0, \ \omega_0 = \sqrt{k/m}$$

$$x'x' + \omega_0^2 xx' = 0$$

$$\frac{d}{dt} \left(\frac{1}{2}x'^2 + \frac{1}{2}\omega_0^2 x^2\right) = 0$$

$$\frac{d}{dt} \left(\frac{1}{2}mx'^2 + \frac{1}{2}kx^2\right) = 0$$

The constant is the energy

$$E = \frac{1}{2}mx'^2 + \frac{1}{2}kx^2$$

which may be written as

$$\left(\frac{x'}{A}\right)^2 + \left(\frac{x}{B}\right)^2 = 1$$

with

$$A = \sqrt{2E/k}$$
 and  $B = \sqrt{2E/m}$ 

Thus, the phase space plot is an ellipse.

79. (C)

The Brackett series in hydrogen involves transitions from energy level m to energy level n = 4 resulting in the emission of a photon. The wavelength is found from

$$1/\lambda = R(1/n^2 - 1/m^2)$$

where *R* is the Rydberg constant.

The upper limit comes from m = 5

$$1/\lambda = 109,677.6 \left(\frac{1}{16} - \frac{1}{25}\right)$$

which gives  $\lambda = 40,500$  Å = 4050 nm.

Applying Snell's law to the second figure, we get



$$n_i \sin \theta_i = n_r \sin \theta_i$$

$$(1)\frac{5}{\sqrt{y^2 + 25}} = (1.3)\frac{2.5}{\sqrt{y^2 + 6.25}}$$
$$\frac{2}{\sqrt{y^2 + 25}} = \frac{1.3}{\sqrt{y^2 + 6.25}}$$

Multiplying means and extremes and squaring, we get

$$4y^2 + 25 = 1.69y^2 + 42.25$$
  
 $2.31y^2 = 17.25$   
 $y = 2.73$  cm



#### 81. (A)

The Lorentz Force is

$$\mathbf{F} = q[\mathbf{E} + (\mathbf{v} \times \mathbf{B})],$$

No deflection  $\rightarrow F = 0$ 

$$\rightarrow \overline{E} = -\overline{v} \times \overline{B} \rightarrow \text{In magnitude}, \text{E} = \text{vBsin } 90^{\circ} \rightarrow \text{v} = \text{E/B}$$

#### 82. (A)

The blackbody distribution may be expressed in various ways

$$u(\omega)d\omega = u(\lambda)d\lambda$$

$$\frac{\hbar\omega^3}{\pi^2 c^3} \frac{1}{e^{\hbar\omega\beta} - 1} d\omega = \frac{16\pi^2 \hbar c}{\lambda^5} \frac{1}{e^{\hbar c\beta/\lambda} - 1} d\lambda$$

since

 $\omega = 2\pi\upsilon = 2\pi\frac{c}{\lambda}$ 

The intensity  $u(\lambda)$  has a maximum when

 $du(\lambda)/d\lambda = 0$ 

$$-\frac{5}{\lambda^6}\frac{1}{e^x-1} + \frac{1}{\lambda^5}\frac{e^x}{(e^x-1)^2}\frac{hc\beta}{\lambda^2} = 0, \ x \equiv hc\beta/\lambda$$

Rearranging terms, we get  $e^{x}(5 - x) = 5$  which has solution x = 4.965 so that

$$\lambda_{\max}T = hc/4.97 \text{ K}$$

#### 83. (D)

The Bohr-Sommerfeld quantization rule is

$$\frac{1}{2\pi}\int p\,dy = n\mathbf{h}$$

For a ball bounding in one dimension *y*, the energy is

$$E = T + U = \frac{p^2}{2m} + mgy$$

Thus,

$$\frac{1}{2\pi} \cdot 2 \int_0^{E/my} \sqrt{2m(E-mgy)} \, dy = n \mathbf{h}$$

$$n\mathbf{h} = \frac{-2}{2\pi}\sqrt{2mE}\frac{2}{3}\left(1 - \frac{mgy}{E}\right)^{3/2}\frac{E}{my} \begin{vmatrix} y = \frac{E}{mg} \\ y = 0 \end{vmatrix}$$

Simplifying, we get

$$\frac{2}{3\pi g}\sqrt{\frac{2}{m}}E^{3/2} = n\mathbf{h} \text{ or } E_n = (9\pi^2 1^2 n^2 \mathbf{h}^2 m/8)^{1/3}$$

#### 84. (D)

The total time is a combination of the fall time plus the time for the sound wave to propagate.

$$t = \sqrt{2h/g} + h/v_s$$
$$h + v_s \sqrt{2/g}\sqrt{h} - v_s t = 0$$
$$h + 149.08\sqrt{h} - 2253.90 = 0$$

Completing the square

$$(\sqrt{h} + 74.54)^2 = 2253.90 + 5556.12$$
  
 $\sqrt{h} = -74.54 + 88.37$ 

thus

h = 191 m

85. (E)

The normalization constant for any spherical harmonic is found from the condition

$$\int |Y_{lm_l}|^2 d\Omega = 1$$

Thus or l = 1 and  $m_l = 0$ 

$$1 = \int Y_{10} Y_{10}^* d\Omega = N^2 \int_{-1}^1 \cos^2 \theta d(\cos \theta) \int_0^{2\pi} d\phi$$
$$= N^2 (2\pi) \frac{1}{3} \cos^3 \theta \left| \begin{array}{l} \theta = 0\\ \theta = \pi\\ = \frac{2\pi}{3} N^2 (1 - (-1)) = \frac{4\pi}{3} N^2 \end{array} \right|$$

Hence

$$N = \sqrt{\frac{3}{4\pi}}$$

#### **DETAILED EXPLANATIONS OF ANSWERS — PRACTICE EXAM 4**

#### 86. (B)

The degeneracy for the *H*-like atom problem is found from thinking about the quantum numbers which describe a state. n = 1, 2, 3, ... is the principal quantum number. l = 0, 1, 2, ..., n - 1 is the orbital quantum number.  $m_l = -l, -(l - 1), ..., l - 1$  is the magnetic quantum number. Hence, the degeneracy g is the number of states that have the same energy  $E_n$ :

$$g = \sum_{l=0}^{n-1} (2l+1)$$
  
=  $2\sum_{l=0}^{n-1} l + \sum_{l=0}^{n-1} 1$   
=  $2\frac{(n-1)(n)}{2} + n$   
=  $n^2$ 

#### 87. (A)

An initial, intermediate, and final picture is helpful to solve the problem.



By momentum conservation:

$$mv = (m+M)v'$$

$$v = \frac{m+M}{m}\sqrt{2gh}$$

By energy conservation:

$$\frac{1}{2}(m+M)v'^2=(m+M)gh$$

$$v' = \sqrt{gh}$$

$$h = \left(\frac{m}{m+M}v\right)^2 / 2g$$
  
=  $\left[\frac{5}{1005} 20,000\right]^2 / 2(980)$   
= 5.05 cm

88. (B)

The Lorentz transformation for coordinates is

$$x' = \lambda(x - vt)$$
$$y' = y,$$
$$z' = z$$
$$t' = \gamma \left( t - \frac{v}{c^2} x \right)$$

By differentiation one finds the velocity transformation

$$u_{x}' = (u_{x} - v)/(1 - u_{x}v/c^{2})$$
$$u_{y}' = u_{y}/\gamma(1 - u_{x}v/c^{2})$$
$$u_{z}' = u_{z}/\gamma(1 - u_{x}v/c^{2})$$

We are given that

$$\mathbf{u} = \left(\frac{1}{\sqrt{2}}c, \frac{1}{\sqrt{2}}c\right)$$
 and  $\mathbf{u}' = \left(-\frac{1}{\sqrt{2}}c, \frac{1}{\sqrt{2}}c\right)$ 

Hence,

$$-\frac{1}{\sqrt{2}} = \left(\frac{1}{\sqrt{2}} - \beta\right) \left/ \left(1 - \frac{1}{\sqrt{2}}\beta\right)$$
$$-\frac{1}{\sqrt{2}} + \frac{\beta}{2} = \frac{1}{\sqrt{2}} - \beta$$
$$\Rightarrow \beta = \frac{4}{3\sqrt{2}} = 0.94$$
$$v = \beta c = (3 \times 10^8)(0.94)$$

# $= 2.8 \times 10^8$ m/s

### 89. (C)

The Schrödinger equation for l = 0 is

$$\frac{1}{R}\frac{d}{dr}\left(r^2\frac{dR}{dr}\right) + \frac{2\mu r^2}{\hbar^2}(E - U = 0)$$

Now the wave function is given as

$$R_{10}(r) = Ne^{-Zr/a_0}$$

and the potential energy is  $U(r) = -kZe^2/r$ . Differentiating, one obtains

$$\frac{d}{dr}\left(r^2\frac{dR}{dr}\right) = -\frac{Z}{a_0}R\left(2r - \frac{Z}{a_0}r^2\right)$$

Hence, we have

$$\left(-\frac{2Z}{a_0} + \frac{2\mu \ kZe^2}{\hbar}\right)r + \left(\frac{Z^2}{a_0^2} + \frac{2\mu E}{\hbar}\right)r^2 = 0$$

Thus, the Bohr radius is

$$a_0 = \frac{\hbar^2}{k\mu e^2}$$

and the energy eigenvalue is

$$E = -\mathbf{h}^2 Z^2 / 2\mu a_0^2 = -k^2 Z^2 \mu e^4 / 2\mathbf{h}^2$$

90. (D)

The proper distance squared is

$$ds^2 = \sum_{\mu=1}^4 dx^\mu dx_\mu$$

or

$$dx^2 + dy^2 + dz^2 - c_2 dt^2$$

Hence, we have

$$\Delta s^2 = \Delta x^2 + \Delta y^2 + \Delta z^2 - c^2 \Delta t^2$$

and since  $\underline{x}(A) = (1, 2, 3, 0)$  and  $\underline{x}(B) = (2, 3, 4, 1/c)$ , we get

$$\Delta s^{2} = (2-1)^{2} + (3-2)^{2} + (4-3)^{2} - \left(\frac{1}{c} - 0\right)^{2} c^{2}$$
$$= 3 - 1$$
$$= 2$$

or finally

 $\Delta s = \sqrt{2} \text{ m}$ 

#### 91. (D)

In the Fermi gas model at zero temperature

$$\frac{dn}{d^3k} = g\frac{V}{(2\pi)^3} = \text{const.}$$

Integrate to obtain

$$\int_0^N \frac{dn}{V} = 2\frac{1}{(2\pi)^3} \int_0^{k_F} 4\pi k^2 \, dk$$

where the degeneracy g = 2 for electrons.

$$\rho = \frac{N}{V} = \frac{1}{\pi^2} \frac{1}{3} k_F^3$$
$$k_F = (3\pi^2 \rho)^{1/3} \text{\AA}^{-1}$$

Ν

or

$$\rho = d \times N_A / A = 0.971 \text{ g/cm}^3 \times \frac{6.02 \times 10^{23} \text{ mol}}{22.99 \text{ g/mol}} \times \left(\frac{10^{-8} \text{ cm}}{1 \text{ Å}}\right)^3$$
$$= 0.0254 \text{ Å}^{-3}$$

Thus,

where

$$k_F = 0.910 \times 10^{10} \,\mathrm{m}^{-1}.$$

#### 92. (B)

The angular part of the wave function is a spherical harmonic

$$Y_{lm_l} = Y_{22} = \frac{1}{4} \sqrt{\frac{15}{2\pi}} \sin^2 \theta e^{2i\phi}$$

and the radial part is

$$R_{nl} = R_{32} = \frac{2\sqrt{2}}{27\sqrt{5}} \left(\frac{Z}{3a_0}\right)^{3/2} \left(\frac{Zr}{a_0}\right)^2 e^{-Zr/3a_0}$$

The complete wave function is

$$\psi_{nlm_l}(r,\theta,\phi) = R_{nl}(r)Y_{lm_l}(\theta,\phi)$$

Fortunately,  $L_{\!\scriptscriptstyle z}$  only operates on the  $\psi$  component

$$L_z \Psi = \frac{h}{i} \frac{\partial}{\partial \phi} \Psi$$

 $=\frac{\hbar}{i}2i\Psi$ 

 $=2h\Psi$ 

Hence the angular momentum eigenvalue is  $2h = m_l h$ .

93. (C)



Newton's Second law states:

$$F_1 = mx_1'' = -kx_1 - \kappa x_1 + \kappa x_2 = -kx_1 + \kappa (x_2 - x_1)$$
$$F_2 = mx_2'' = -kx_2 - \kappa x_2 + \kappa x_1 = -kx_2 - \kappa (x_2 - x_1)$$

The solutions are

$$x_1 = A \cos(\omega t + \delta)$$
  
and 
$$x_2 = B \cos(\omega t + \delta)$$

The easiest way to find the frequencies is to add and subtract the equations to get

$$my'' = -ky, \ y = x_1 + x_2$$
$$mz'' = -(k + 2\kappa)z, \ z = x_2 - x_1$$

The symmetrical mode frequency is thus

$$\omega_s = \sqrt{\frac{k}{m}}$$

and the anti-symmetrical one is

$$\omega_a = \sqrt{(k+s\kappa)/m}$$

#### 94. (B)

Two electrons can form an l = 0 quantum state by the Pauli principle, e.g., a  $1s^2$  configuration. The current is then

$$I = \frac{q}{t} = \frac{2e}{(2\pi r/v)} = \frac{ev}{\pi r}$$

and the velocity comes from Newton's second law F = ma or

$$\frac{mv^2}{r} = \frac{ke^2}{r^2}$$

$$v = \sqrt{\frac{ke^2}{mr}}$$

$$= \sqrt{(9 \times 10^9)(1.6 \times 10^{-19})^2/(9.1 \times 10^{-31})(1 \times 10^{-10})}$$

$$= 1.59 \times 10^6 \text{m/s}$$

Therefore, the electric current is

$$I = (1.6 \times 10^{-19})(1.59 \times 10^{6})/\pi (1 \times 10^{-10})$$
$$= 8.1 \times 10^{-4} \text{ A}$$

95. (C)



By Kirchhoff's current law

$$I = I_1 - I_2$$

According to Kirchhoff's voltage law:

$$-\frac{q_1}{C} - LI'_1 - \beta I' - \frac{(q_1 - q_2)}{\gamma} = 0$$
$$-\frac{q_2}{C} - LI'_2 + \frac{(q_1 - q_2)}{\gamma} + \beta I' = 0$$

Differentiate and set

$$\kappa = 1/LC, \quad \kappa_1 = 1/L\gamma, \quad \kappa_2 = \beta/L$$

Note that  $\kappa_2$  is dimensionless. Then we find

$$I_1''(1 + \kappa_2) - \kappa_2 I_2'' + (k + \kappa_1)I_1 - \kappa_1 I_2 = 0$$
$$I_2''(1 + \kappa_2) - \kappa_2 I_1'' + (k + \kappa_1)I_2 - \kappa_1 I_1 = 0$$

Add and substract the equations letting

$$y = I_1 + I_2$$
 and  $z = I_2 - I_1$ 

to obtain

$$y'' + ky = 0$$
 and  $z'' + \frac{k + 2\kappa_1}{1 + 2\kappa_2}z = 0$ 

Thus,

$$w_a = \sqrt{(k+2\kappa_1)/(1+2\kappa_2)}$$

96. (B)

Maxwell's fourth equation is

 $\nabla \times \mathbf{B} = \mu_0 \mathbf{j}$ 

The London equation

$$\mathbf{j} = -\mathbf{A}/\mu_0 \lambda_L^2$$

is given. Hence,

$$\nabla \times \mathbf{j} = -\nabla \times \mathbf{A}/\mu_0 \lambda_L^2 = -\mathbf{B}/\mu_0 \lambda_L^2$$

Also,

$$abla imes (
abla imes \mathbf{B}) = 
abla (
abla \cdot \mathbf{B}) - 
abla^2 \mathbf{B}$$
 $abla imes \mu_0 \mathbf{j} = -
abla^2 \mathbf{B}$ 

since  $\nabla \cdot \mathbf{B} = 0$  by Gauss's law.

Thus,  $\nabla^2 \mathbf{B} = \mathbf{B}/\lambda_L^2$ .

One solution is obviously  $\mathbf{B} = 0$ . However, the desired solution is

$$B = B_0 e^{-x/\lambda}$$

The magnetic field thus penetrates into the superconductor to a penetration depth, typically  $\lambda_L \approx 200$  Å.

#### 97. (E)

The energy density of a photon gas is

$$U = \sigma T^4$$

Thus, the energy is

$$E = \int U d^3 r = \sigma V T^4$$

Now use the first law of thermodynamics:

$$dE = dQ - dW$$

$$dQ = dE + dW = dE + pdV$$

$$= \sigma T^4 dV + \frac{1}{3}\sigma T^4 dV$$

$$= \frac{4}{3}\sigma T^4 dV$$

$$dS = \frac{dQ}{T} = \frac{4}{3}\sigma T^3 dV$$

$$S = \int dS = \frac{4}{3}\sigma T^3 V$$

We have used the fact  $p = \frac{1}{3} E/V$ , the relativistic Virial theorem.

#### 98. (C)

The motion of two particles of masses  $m_1 = m_2 = m$  separated by a distance r is equivalent to that of a reduced mass particle  $\mu$  as shown. The Schrödinger equation is

$$H\Psi = E\Psi, \ H = \frac{-\mathbf{h}^2}{2\mu}\nabla^2 + U(r)$$

where U(r) = 0 for fixed *r*.

In spherical coordinates, we get

$$\frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left( \sin \theta \frac{\partial \Psi}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 \Psi}{\partial \phi^2} + \frac{2\mu E}{h^2} \Psi = 0$$



#### **DETAILED EXPLANATIONS OF ANSWERS — PRACTICE EXAM 4**

The standard separation of variables technique using  $\psi(\theta, \phi) = P(\theta)Q(\phi)$  gives

$$Q(\phi) = rac{1}{\sqrt{2\pi}} e^{im_\ell \phi}$$

and

$$\frac{1}{\sin\theta} \frac{d}{d\theta} \left( \sin\theta \frac{dP}{d\theta} \right) + \left[ l(l+1) - \frac{m_l^2}{\sin^2\theta} \right] P = 0$$

where

$$P = P_l^{m_l}(\theta)$$

is the associated Legendre function.

#### 99. (A)

The two postulates of special relativity are:

(1) True physical laws are the same in all inertial reference frames.

(2) The speed of light is a constant in vacuo regardless of source and observer relative motion.

This was Einstein's 1905 theory. In 1915, he went beyond this with the general theory of relativity for which (B)-(E) in the question are true statements.

#### 100. (E)

This is the usual relativistic rocket problem. The four momentum is

$$p^{\mu} = mu^{\mu}$$
$$= (m\gamma \mathbf{u}, m\gamma c)$$



Certainly the space component of the four momentum is conserved

$$(0, m_1 c) = (-m\lambda_v v, m\gamma_v c) + (m_2\gamma_u u, m_2\gamma_u c)$$

so that

$$0 = -m\gamma_v v + m_2 \gamma_v u$$

 $m\gamma_{\nu}v = m_2\gamma_{\mu}u$ 

or

The rest mass is not conserved since this is an inelastic collision in reverse.

Table of Information	
Rest mass of the electron	$m_e = 9.11 \times 10^{-31}$ kilogram = 9.11 × 10 <sup>-28</sup> gram
Magnitude of the electron charge	$e = 1.60 \times 10^{-19}$ coulomb = $4.80 \times 10^{-10}$ statcoulomb (esu)
Avogadro's number	$N_0 = 6.02 \times 10^{23} \text{ per mole}$
Universal gas constant	R = 8.314 joules/(mole · K)
Boltzmann's constant	$k = 1.38 \times 10^{-23}$ joule/K = $1.38 \times 10^{-16}$ erg/K
Speed of light	$c = 3.00 \times 10^8 \text{ m/s}$ = 3.00 × 10 <sup>10</sup> cm/s
Planck's constant	$h = 6.63 \times 10^{-34}$ joule $\cdot$ second = $4.14 \times 10^{-15}$ eV $\cdot$ second
Vacuum permittivity	$\varepsilon_0 = 8.85 \times 10^{-12} \text{ coulomb}^2/(\text{newton} \cdot \text{meter}^2)$
Vacuum permeability	$\mu_0 = 4\pi \times 10^{-7}$ weber/(ampere · meter)
Universal gravitational constant	$G = 6.67 \times 10^{-11} \text{ meter}^3/(\text{kilogram} \cdot \text{second}^2)$
Acceleration due to gravity	$g = 9.80 \text{ m/s}^2 = 980 \text{ cm/s}^2$
1 atmosphere pressure	1 atm = $1.0 \times 10^5$ newton/meter <sup>2</sup> = $1.0 \times 10^5$ pascals (Pa)
1 angstrom	$1 \text{ Å} = 1 \times 10^{-10} \text{ meter}$ 1 weber/m <sup>2</sup> = 1 tesla = 10 <sup>4</sup> gauss
Rydberg Constant	$R = 1.097 \times 10^7 \mathrm{m}^{-1}$

Notes



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- 2. If the installation doesn't begin automatically, from the Start Menu choose the RUN command. When the RUN dialog box appears, type d:\setup (where d is the letter of your CD-ROM drive) at the prompt and click ok.
- 3. The installation process will begin. A dialog box proposing the directory "Program Files\REA\GREPhysics" will appear. If the name and location are suitable, click ок. If you wish to specify a different name or location, type it in and click ок.
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